# Discussion of the Relevance of Transfer Admittance and Some Through Elastance Measurement Results

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***Abstract*—After a discussion on the interest of transfer admittance for EMC prediction, a method for measuring the transfer admittance of cables is presented. Some experimental results are given for coaxial cables: the through elastance was not measurable, but we show that it is at least one order of magnitude smaller than what today's theory predicts.**

## I. Introduction

It is now quite common to find specifications or data on the transfer impedance of cables, either coaxial cables or multiconductor shielded cables. However, it is also quite common to disregard the transfer admittance.

Is this justified? In other words, can one consider that the (transfer admittance) coupling caused by charges on the cable is a secondary phenomenon, that can be dropped in most calculations? This subject is discussed here in some detail, and we distinguish between what happens in the test situation and what happens in the application situation. After this discussion we present a measurement method for through elastance, and results for some classical coaxial cables.

Most of the mathematics developed below is rigorous only for coaxial cables, but our discussion (and our interest) includes multiconductor shielded cables as well.

## II. Relevance of Transfer Admittance: Test Situation

In order to better discuss the relevance of the transfer admittance coupling, we will first consider the test situation, in which a coaxial cable under test is excited by a generator connected to an external conductor and to the shield. The external conductor and the shield are the two conductors of the so-called excitation line. More precisely, we will consider a cable installed in an unspecified test setup having cylindrical symmetry, according to Fig. 1, where CUT stands for cable under test. Unspecified means here that the test setup may have any linear termination at both CUT ends, and any termination at both excitation-line ends. Let us call $Z_T$ the CUT transfer impedance, $Y_T$ the CUT transfer admittance, and $l$ the CUT length (assumed to be equal to the excitation line's length).

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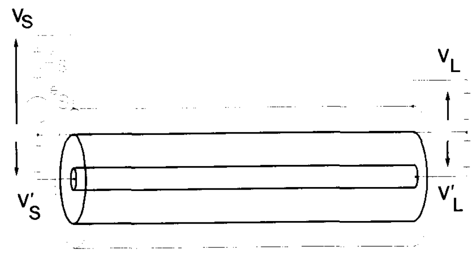


Fig. 1. The CUT (thick lines) and the excitation line.

Now, at any given frequency $\nu$ let's introduce 12 useful complex quantities:

$e_s$: the open-circuit voltage of the generator
$Z_S$: the internal impedance of the generator
$Z_O$: the characteristic impedance of the excitation line
$\rho_L$: the reflection coefficient at the far-end of the excitation line
$\rho_S$: the reflection coefficient at the near-end of the excitation line, i.e., $\rho_S = (Z_S - Z_O)(Z_S + Z_O)^{-1}$
$\gamma$: the propagation constant on the excitation line
$Z'_O$: the characteristic impedance of the CUT
$\rho'_L$: the reflection coefficient at the far-end of the CUT
$\rho'_S$: the reflection coefficient at the near-end of the CUT
$\gamma'$: the propagation constant on the CUT
$T$: defined by $T = \exp(-\gamma l)$
$T'$: defined by $T' = \exp(-\gamma' l)$.

With those notations, and assuming weak coupling, the near-end voltage $\nu'_s$ on the CUT and the far-end voltages $\nu'_L$ are easily expressed in a compact form (see Appendix), given by

$$\nu'_S = \frac{e_S/2}{Z_O + Z_S}\frac{1}{1-\rho_L\rho_S T^2}\frac{1}{1-\rho'_L\rho'_S T'^2} \cdot \left\{ \begin{array}{c} (Z_O Z'_O Y_T + Z_T)\dfrac{T'-T}{\gamma-\gamma'}(\rho_L T + \rho'_L T') \\ +(Z_O Z'_O Y_T - Z_T)\dfrac{1-TT'}{\gamma+\gamma'}(1 + TT'\rho_L\rho'_L) \end{array} \right\} \quad (1)$$

and

$$\nu'_L = \frac{e_S/2}{Z_O + Z_S}\frac{1}{1-\rho_L\rho_S T^2}\frac{1}{1-\rho'_L\rho'_S T'^2} \cdot \left\{ \begin{array}{c} (Z_O Z'_O Y_T + Z_T)\dfrac{T'-T}{\gamma-\gamma'}(1 + \rho_L\rho'_S TT') \\ +(Z_O Z'_O Y_T - Z_T)\dfrac{1-TT'}{\gamma+\gamma'}(\rho_L T + \rho'_S T') \end{array} \right\}. \quad (2)$$

For matched terminations at both ends (1) and (2), respectively, become

$$\nu'_S = \frac{e_S}{4Z_O}(Z_O Z'_O Y_T - Z_T)\frac{1 - TT'}{\gamma + \gamma'} \tag{3}$$

and

$$\nu'_L = \frac{e_S}{4Z_O}(Z_O Z'_O Y_T + Z_T)\frac{T' - T}{\gamma - \gamma'}. \tag{4}$$

Those last two equations are equivalent to the well-known Halme and Szentkuti [1], [2] equations, except that we do not use normalization in order to avoid unnecessary mathematical troubles with lossy cables (normalization involves the square root of the characteristic impedance, which is a complex number for a lossy cable).

As experiment shows that for most cables $Z_O Z'_O Y_T \ll Z_T$, one can state that it is not necessary to take into account the transfer admittance of cables in most measurements with all terminations matched.

Obviously, the measurement of $Y_T$ is feasible only if one chooses the termination in such a way that the two terms containing $Z_T$ in (1) or (2) more or less cancel each other. Two cases are of special practical interest. The first assumes that the excitation line is electrically short, matched to the generator and open-circuited at the far end, the CUT being matched at both ends. In this case:

$$\nu'_S = \nu'_L = \frac{e_S}{2} Z'_O Y_T l. \tag{5}$$

In the second case, the excitation line is electrically short and matched at both ends, the CUT being matched at one end and open-circuited at the other. The voltage must be measured at the matched terminal, and the value measured is also given by (5). Other possibilities of measuring $Y_T$ on a coaxial cable exist, for example with more than one open-ended termination. Of course, if the shield is a very good electric shield, the terms containing $Z_T$ will not compensate each other for practical length of cable, well enough for $Y_T$ to become the dominant term, and (5) is no longer valid: this will for instance always be the case for homogeneous shields, for which theory says $Y_T = 0$.

The same principles are qualitatively applicable to shielded multiconductor cables. If all internal wires and the excitation line are matched at both ends, transfer admittance will normally be negligible (that is, provided the shield is good enough). Conversely, when the excitation line or the internal wires are open-ended at one side, $Y_T$ is likely to dominate (provided the shield is not too good).

## III. Relevance of Transfer Admittance: Application Situation

This section addresses coupling on coaxial cables and multiconductor shielded cables as well. A classification including five different coupling types, numbered 1 to 5 has already been presented [3], and we will eventually refer to it in the discussion below. Type 1 coupling is the phenomenon characterized by transfer impedance. Type 2 coupling is the one responsible for transfer admittance. The other three coupling types do not appear on cables having cylindrical symmetry, and therefore do not appear in (1)–(5). It must nevertheless be remembered that all five coupling types occur on real cables, even if this paper is intentionally limited to coupling type 1 and coupling type 2.

Whenever a cable with a shield is electrically long, any excitation involving a net surface charge along a cable will also necessitate the flow of a significant common mode current. From the discussion of the previous section one can infer that in most cases coupling type 1 will dominate coupling type 2. Of course, it is likely that one may always build, on any cable configuration and at any frequency, a particular field **E** and a particular field **B** that would cancel the effects of type 1 coupling and reveal those of type 2 coupling. But this is of little interest because we are chiefly motivated by worst case figures: the worst case figure on electrically long cables may in most cases be obtain with transfer impedance alone.

We will therefore show the importance of coupling type 2 on examples involving short enough cables. The meaning of "short enough" in the previous sentence will appear more clearly in this paper's conclusion. Three situations of practical interest are presented.

1) In the first situation we consider an electrically short shielded cable surrounded by a high impedance field. The field may be such that the current flowing on the cable shield has little effect compared to the charge that appear on the shield. This situation is very similar to the test situation, in the case that leads to (5).
2) In the second situation one considers an electrically short shielded cable connected at one or both ends to high impedance devices. It is for instance a common (and good) practice to use two internal wires for conveying a signal, and to receive the signal with a differential amplifier or an isolation amplifier in the case of an analog signal, or an opto-isolator in the case of a digital signal. All these have a fairly high common-mode impedance. In this respect, it should be kept in mind that the termination's impedance must be considered at the relevant frequencies: many shielded cables have internal conductors that are intended for low-frequency signals only (say below 20 kHz), and that are often terminated in such a way that their RF impedance is not known by design.
3) In the third situation we consider an electrically short shielded cable connected to a shielded device itself isolated from ground, the cable shield being tied to the device's enclosure. This configuration is a good EMC practice in many situation of interest, because it prevents the common mode current from flowing without affecting the shield continuity. Obviously, performances in this case are essentially limited by type 2 coupling and by the stray capacitance to ground of the device's enclosure.

## IV. Measurement Method

Now that we have shown that type 2 coupling is indeed needed for EMC design, let us say a few words on how to specify it. The transfer admittance is the quantity that naturally appears in transmission line calculations and results such as (1)–(5). However, the transfer admittance is a characteristic of both the cable and its surroundings (the test setup in the case of measurements). Standardization bodies such as IEC now advocate to characterize coaxial cables with the use of the through elastance $K_T$ that depends only on the cable itself [1], [2]. It is related to the transfer admittance $Y_T$ by

$$K_T = \frac{Y_T}{j\omega C_1 C_2} \tag{6}$$

where $C_1$ is the per-unit-length capacitance of the outer circuit and $C_2$ is the per-unit-length capacitance between the two conductors of the coaxial cable. Obviously, $Y_T$ is dependant on the measurement setup because $C_1$ will differ from one test setup to the other, if their transverse dimensions are different. However, it is not easy to extend the definition of the through elastance to multiconductor cables, and we need to introduce a new quantity $\zeta_R$, called the radial electric coupling coefficient. The radial electric coupling coefficient is dimensionless, and defined as

$$\zeta_R = \frac{Y_T}{j\omega C_1}. \tag{7}$$

It has the advantage of being a quantity dependant on the cable only, and also to have a clear physical significance: it may be interpreted as the ratio of the per-unit-length current injected into the inner conductor on the per-unit-length displacement current impinging on the shield.

Our measurement setup for through elastance is shown in Fig. 2, and is designed for the 10 kHz to 10 MHz region. The excitation signal is produced by the tracking generator of our Advantest R3361A spectrum analyzer, at the level of 100 dB($\mu$V). A small amplifier increases this level to about 116 dB($\mu$V). This level is then fed into our Excem AMP 1 amplifier, installed inside a shielded room. This amplifier provides about 54 dB of voltage gain, with an output impedance of 51 k$\Omega$||3 pF. This setup is particularly suitable for our purpose, because it delivers about 316 V rms into our high-impedance measurement cell, with low-cost equipment and no unnecessary power to dissipate. The cell itself is simply a 500 mm long cupper tube of 22 mm diameter in the center of which the CUT is installed with appropriate spacers. The tube being isolated, its impedance to ground is mainly caused by its capacitance to the CUT. The full voltage mentioned above at the output of the AMP 1 amplifier is only available at lower frequencies (<100 kHz), because of the reduced cell to ground impedance at high frequencies.

The CUT itself may be terminated in two different ways. For reference measurements, the floating termination is a short-circuit, and the $N$ plug installed in the shielded room's wall is modified in such a way that it does not connect the CUT's screen to the shielded room (a clean gap of about 2 mm is present). In that way the total displacement current on the CUT flows into the CUT's inner conductor. When we make a reference measurement, a 40 dB attenuator is installed between the CUT connector and the Sonoma Instrument model 310 amplifier. The output signal of this amplifier is directed to

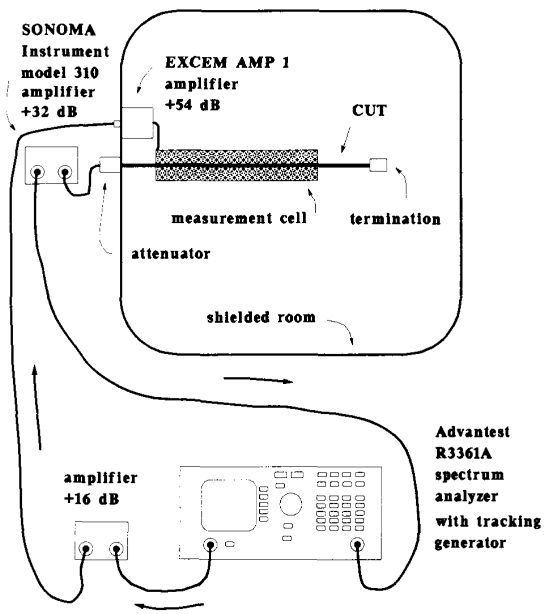


Fig. 2. Our measurement setup.

the signal input of the spectrum analyzer. For attenuation measurements, the floating termination is a 50 $\Omega$ termination, and the plug in the shielded room's wall is a normal one. The 40 dB attenuator is also removed and the signal goes directly to the Sonoma Instrument amplifier, which provides 32 dB of gain with a noise figure of 1.8 dB.

Our signal-to-noise ratio was good enough for our using the peak detector of the spectrum analyzer (no averaging) with a resolution bandwidth of 100 Hz below 100 kHz, and of 1 kHz above.

For $Z'_O = 50\ \Omega$ cables, the radial electric coupling coefficient $\zeta_R$ can be computed from the calibration and attenuation measurements, as

$$\zeta_R \approx 2\frac{\left(\dfrac{\nu_{S\mathrm{ref}}}{\nu'_{S\mathrm{ref}}} - 1\right)}{\left(\dfrac{\nu_{S\mathrm{att}}}{\nu'_{S\mathrm{att}}} - 1\right)} \tag{8}$$

where

$\nu_{S\mathrm{ref}}$: the voltage delivered to the cell during reference measurement

$\nu'_{S\mathrm{ref}}$: the voltage measured at the cell output during reference measurement

$\nu_{S\mathrm{att}}$: the voltage delivered to the cell during attenuation measurement

$\nu'_{S\mathrm{att}}$: the voltage measured at the cell output during attenuation measurement.

At frequencies not too high, the 50 $\Omega$ impedance of the attenuator is small enough to be negligible compared to the cell-to-CUT capacitive impedance, and one may consider that $\nu_{S\mathrm{ref}}$ is almost equal to $\nu_{S\mathrm{att}}$ because the AMP 1 amplifier sees almost the same load impedance. Assuming also that

$\nu'_{Satt} \ll \nu_{Satt}$ and $\nu'_{Sref} \ll \nu_{Sref}$, we get

$$\zeta_R \approx 2\frac{\nu'_{Satt}}{\nu'_{Sref}}. \tag{9}$$

As our measurement setup did not allow the measurement of the phase, we did not implement (8) but (9), therefore expecting some limitations in accuracy at upper frequencies.

Also, it should be noted that (8) and (9) stem from an approximation of (1) of zeroth order in $\gamma l$ and $\gamma' l$, from which the transfer impedance $Z_T$ is excluded. A first-order approximation in $\gamma l$ and $\gamma' l$ would have given for (9)

$$2\frac{\nu'_{Satt}}{\nu'_{Sref}} \approx \zeta_R\left(1 - \gamma l - \gamma'\frac{l}{2}\right) - \frac{Z_T}{100\Omega}l. \tag{10}$$

Now we must also take into account that the common-mode current on the cable due to injection of displacement current flows not only on the cell's length $l = 50$ cm but also on the length $l_2 = 7$ cm of cable between the cell and the $N$ connector, and then on the $N$ connector and the $N$ plug. Neglecting the connector and plug transfer impedances, and removing the unnecessary correction factor in the transfer admittance term, we get

$$2\frac{\nu'_{Satt}}{\nu'_{Sref}} \approx \zeta_R - \frac{Z_T}{50\Omega}\left(\frac{l}{2} + l_2\right). \tag{11}$$

Obviously, we wish the second term to be much smaller than the first.

## V. Results

If we apply (9) on our measurements, we obtain what could be called an "experimental" radial electric coupling coefficient. As all cable tested have a capacitance of nearly 100 pF/m the conversion to "experimental" through elastance is simply a product by this quantity. The use of "experimental" means that we do not necessarily consider that those values represent the true value, until consistency with $Z_T$ values is checked with (11), that is, until we are sure that we do not measure a parasitic signal related to transfer impedance.

We must also consider that if one accepts conventional shielding theory [1], [2], [4], $Y_T$ is given by

$$Y_T = j\omega C_{12} \tag{12}$$

where $C_{12}$ is the through capacitance between the CUT's inner conductor and the cell. The radial electric coupling coefficient and the through elastance should therefore be frequency independant.

Figs. 3 to 6 show our results for RG58C/U (single braid), RG223 (double braid), RG213 (single braid), and RG214 (double braid) cables. The manufacturer of the cables we used provides a comprehensive documentation [5] including $Z_T$ versus frequency plots for each cable type that we used, measured according to the IEC triaxial method. Comparing our measured values to the $Z_T$ plots shows that what we have on Figs. 3 to 6 may be the result of the effect of

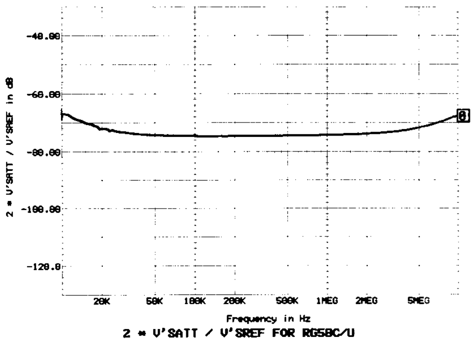


Fig. 3. "Experimental" radial electric coupling coefficient measured on RG58C/U.

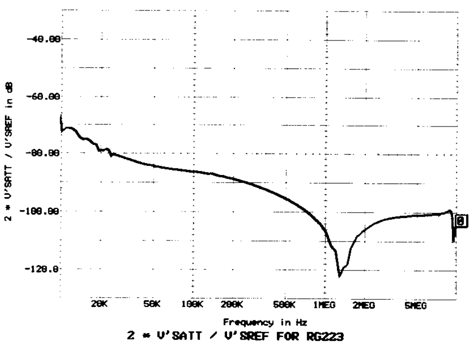


Fig. 4. "Experimental" radial electric coupling coefficient measured on RG223.

type 1 coupling only (the match is very close!). The increase of our measured value below 50 kHz is a measurement artifact due to the increase of the input impedance of the Sonoma Instrument amplifier at low frequency (ac-coupled inputs). Some saturation in the AMP 1 amplifier caused by variation in the measurement chain gain explain small anomalous variations on Figs. 4 to 6 below 30 kHz. The noise figure of the front-end amplifier was so good that the noise floor of the instrumentation was never reached, except for the bottom of the deep on the RG223 plot.

What can we conclude from those curves? Obviously we have obtained upper bounds for the value of the radial electric coupling coefficient and through elastance of the four cables. Those values are given in Table I, and were taken as the lowest value found in the curves increased by 6 dB as a safety margin for possible accidental cancellation of the two terms in (11).

Those values deserve a comment. The only published values for $K_T$ that we are aware of come from Vance [4], and they are computed values for single-braid cable. For RG58 cable he gives 66 m/$\mu$F or 36 dB(m/$\mu$F). For RG213 he gives 16 m/$\mu$F or 24 dB(m/$\mu$F). From our measurements those values are at least one order of magnitude (25 dB) too high.

As a matter of fact these were the values that we expected to find, and the measurement cell was designed in such a way that it would have appropriate rejection (at least 30 dB below

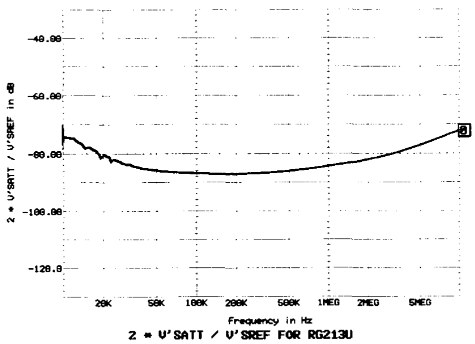


Fig. 5. "Experimental" radial electric coupling coefficient measured on RG213U.

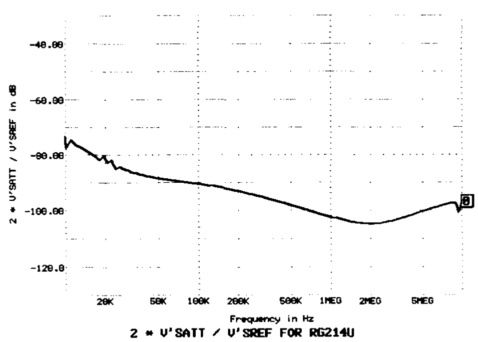


Fig. 6. "Experimental" radial electric coupling coefficient measured on RG214U.

TABLE I

| Type of Cable | $\zeta_R$ (in dB) | $K_T$ (in dB(m/$\mu$F)) |
|---|---|---|
| RG58C/U | $<-70$ | $<10$ |
| RG223 | $<-110$ | $<-30$ |
| RG213U | $<-81$ | $<-1$ |
| RG214U | $<-100$ | $<-20$ |

type 2 coupling for single-braid cables) of type 1 coupling for us to achieve the measurement of $\zeta_R$. We did not consider that the cables would behave much better for the protection against type 2 coupling than what theory predicts!

## VI. Conclusion

Our most important result is that on the kind of coaxial cables that we tested, on any length of cable matched at both ends, larger than 0.5 m, transfer admittance can be neglected, and cannot even be measured.

This applies to the test situation and the application situation.

This is in fact a very good news. However, it does not mean that for those cables, type 2 coupling is always negligible: a cable with internal wires connected to high impedance at both ends and submitted to a high-impedance field would be much more sensitive to type 2 coupling.

If one wants to be able to find the value of radial electric coupling coefficient or the through elastance of the cables considered here, (11) tells us that one possibility is to implement a setup similar to the one used here, but with a shorter cable length. One could probably try to implement a quadraxial test fixture similar to the one proposed in [6] with a length of CUT of 5 cm or less, but we are not sure that reduction in cable length would be enough.

Another possibility would be to implement high-impedance terminations at both ends of the CUT, and on the excitation line. This solution would probably require high-impedance, high-level attenuators. One could also implement time domain measurements [7], for which a cheap high-voltage transient generator would be used.

An other interesting question is related to the use of a standard cable for the calibration of the test setup. Such a good standard cable could be a thin metallic tube with holes of well defined dimensions: the transfer admittance of such a structure is known from analytical formulas [8]. However, if a very thin metallic layer is used, it would need a substrate, the permittivity of which would affect the transfer admittance in a complex way: the matter is therefore more complex than the design of a standard cable for transfer impedance.

## Appendix
### Derivation of (1) and (2)

On the disturbing line, taking the origin on the generator side of the cable, the voltage and currents along the disturbing line are

$$\nu(x) = e_S \frac{Z_O}{Z_O + Z_S} \frac{1}{1 - \rho_L \rho_S T^2} (e^{-\gamma x} + \rho_L T^2 e^{\gamma x}) \quad (13)$$

$$i(x) = e_S \frac{1}{Z_O + Z_S} \frac{1}{1 - \rho_L \rho_S T^2} (e^{-\gamma x} - \rho_L T^2 e^{\gamma x}) \quad (14)$$

if one assumes that the CUT is matched at both ends, voltages at both termination can be computed as

$$\overline{\nu}'(0) = \frac{1}{2} \int_0^l [Z'_O Y_T \nu(x) - Z_T i(x)] e^{-\gamma' x} dx \quad (15)$$

$$\overline{\nu}'(l) = \frac{1}{2} \int_0^l [Z'_O Y_T \nu(x) + Z_T i(x)] e^{-\gamma'(l-x)} dx \quad (16)$$

which leads to

$$\overline{\nu}'(0) = \frac{e_S/2}{Z_O + Z_S} \frac{1}{1 - \rho_L \rho_S T^2} \cdot \left\{ \begin{array}{c} (Z_O Z'_O Y_T + Z_T) \dfrac{T' - T}{\gamma - \gamma'} \rho_L T \\ + (Z_O Z'_O Y_T - Z_T) \dfrac{1 - TT'}{\gamma + \gamma'} \end{array} \right\} \quad (17)$$

$$\overline{\nu}'(l) = \frac{e_S/2}{Z_O + Z_S} \frac{1}{1 - \rho_L \rho_S T^2} \cdot \left\{ \begin{array}{c} (Z_O Z'_O Y_T + Z_T) \dfrac{T' - T}{\gamma - \gamma'} \\ + (Z_O Z'_O Y_T - Z_T) \dfrac{1 - TT'}{\gamma + \gamma'} \rho_L T \end{array} \right\}. \quad (18)$$

If the CUT is not matched at both ends, then reflections must be accounted for and voltages at both terminations become

$$\nu_S' = \frac{1}{1-\rho_L'\rho_S'T'^2}(\overline{\nu}'(0)+\rho_L'T'\overline{\nu}'(l)) \tag{19}$$

$$\nu_L' = \frac{1}{1-\rho_L'\rho_S'T'^2}(\overline{\nu}'(l)+\rho_S'T'\overline{\nu}'(0)). \tag{20}$$

Formulas (17)–(20) directly lead to (1) and (2).

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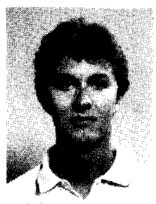
**Daniel Givord** was born in 1944.

In 1968, in joined the Etablissement Technique Central de l'Armement (ETCA), Arcueil, France. Most of his work has been focused on the improvement of measurement techniques for the transfer impedance of cables and connectors.



**Pascal Vallet** was born in 1969.

From 1988 to 1993, he worked at the ETCA, Arcueil, France, where his interests included measurements of the shielding characteristics of cables. Since September 1993, he has been with the LETTI (Laboratoire d'Etude des Transmissions Ionosphériques), Orsay, France, where his work is related to radar systems.