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The Open-Circuit Voltage of a Planar Wire Loop Antenna Used for Reception

FRÉDÉRIC BROYDÉ[1], and EVELYNE CLAVELIER[2]

[1]Eurexcem, 12 chemin des Hauts de Clairefontaine, 78580 Maule, France
[2]Excem, 12 chemin des Hauts de Clairefontaine, 78580 Maule, France

Corresponding author: Frédéric Broydé (e-mail: fredbroyde@eurexcem.com).

ABSTRACT Using "electromagnetic field" to designate an electric field and a magnetic field which satisfy Maxwell's equations, we define a decomposition of an arbitrary incident time-harmonic electromagnetic field into four elementary time-harmonic electromagnetic fields $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$. The decomposition is the basis of a formula that gives the open-circuit voltage of an arbitrary planar wire loop antenna used for reception. This formula is applicable to any incident field configuration, and valid at any frequency at which the thin wire approximation applies. It separates the response of the antenna into three parts, one of which may be viewed as the intended response of the antenna. Our analysis teaches that $\mathcal{F}_C$ and $\mathcal{F}_D$ have no effect on the antenna, and how $\mathcal{F}_A$ and $\mathcal{F}_B$ excite the antenna. It allows us to better understand the characteristics and limitations of a planar wire loop antenna used as a measuring antenna or as a direction finder.

INDEX TERMS Antenna theory, loop antenna, receiving antenna, electromagnetic compatibility, EMC, measuring antenna.

I. INTRODUCTION

This article is a greatly expanded version of the material presented in [1].

An electrically small loop antenna may be used for measuring a magnetic component of an electromagnetic field [2]. For accurate measurements using a circular loop antenna, the feeder (i.e., feed line) of the antenna is often connected to the loop by utilizing a shielded-loop configuration, which prevents common-mode currents on the feeder from affecting the antenna's response [3, Sec. 5-4], [4, Sec. 11.8]. The shield of a shielded-loop antenna operates like an unshielded loop antenna, so that results derived for loop antennas can be adapted to shielded loop antennas.

Loop antennas and shielded-loop antennas are used for electromagnetic compatibility (EMC) testing in laboratories, and for various types of outdoor measurements such as site surveys, proof of performance of fixed antennas, propagation measurements and spectrum monitoring [5, Sec. 13 Par. 41], [6]–[12]. Loop antennas that need not be electrically small are also used in direction-finding systems and high-frequency ground-wave radars [5, Sec. 12], [13]–[15].

According to a simplistic analysis, an open-circuit voltage at the port of an electrically small single-turn loop antenna used for reception is given by $j\omega A\,\mu_0 H_{NA}$ where ω is the radian frequency of an incident electromagnetic field and H_{NA} is the magnetic field normal to the plane of the loop, averaged over the area A of the loop [3, Sec. 5-2]. A more elaborate analysis is available for electrically small single-turn square or circular loop antennas subject to an incident uniform plane wave. It provides an approximate formula that includes a response of the loop antenna to the polarization for which H_{NA} is zero [2], [4, Sec. 11.7]. Unfortunately, this analysis only applies to an incident uniform plane wave, or to several such waves if the formula is used repeatedly. Thus, it does not provide a general picture of what is sensed by the antenna if it is subject to an arbitrary incident electromagnetic field. For instance, it does not apply to the standard-field calibration technique, in which the circular loop antenna of a field-strength meter to be calibrated is excited by a coaxial circular loop placed at an electrically short distance [6]. For instance, it does not apply to measurement configurations in which a planar loop antenna used for EMC measurements lies in the near-field of a device under test. For instance, it does not apply to outdoor measurements where multiple sources and scatterers at unknown locations, as well as passive nearby objects, contribute to the measured signal.

Broadly speaking, the purpose of this paper is to investigate some general properties of a planar wire loop antenna (which need neither be electrically small, nor circular) used for receiving an incident electromagnetic field (which need not be a uniform plane wave), to better understand its characteristics and limitations as a measuring antenna or as

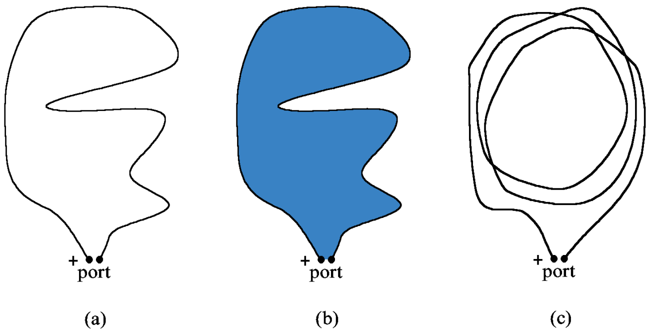

FIGURE 1. A non-self-intersecting planar loop antenna (a), the surface inside the single loop formed by this antenna and the arbitrary path over the gap (b), and a self-intersecting planar loop antenna (c).

a direction finder. We shall use "electromagnetic field" to designate the ordered pair $\mathcal{F} = (\mathbf{E}, \mathbf{H})$ of an electric field $\mathbf{E}$ and a magnetic field $\mathbf{H}$ which satisfy Maxwell's equations in a specified region.

A new decomposition of an arbitrary incident time-harmonic electromagnetic field into 4 elementary time-harmonic electromagnetic fields (ETHEFs) is defined in Section III. A new and exact formula based on the ETHEF decomposition, obtained in Section IV, provides the open-circuit voltage at the port of a planar wire loop antenna having any shape, structure and size, subject to an arbitrary incident time-harmonic electromagnetic field. Alternative formulas are proposed and discussed in Section V.

The ETHEF decomposition is applied to some simple incident electromagnetic fields, in Section VI and Section VII. Sections VIII and IX present three general methods for the direct computation of the ETHEF decomposition of an arbitrary incident electromagnetic field, and examples.

II. ASSUMPTIONS AND KNOWN RESULTS

A. ASSUMPTIONS ABOUT THE LOOP ANTENNA

We use the traditional rectangular coordinate system $Oxyz$, of unit vectors $\mathbf{u}_x$, $\mathbf{u}_y$ and $\mathbf{u}_z$. We use "planar wire loop antenna" to designate an antenna made of a single thin wire lying in the plane $z = 0$ and made of an electrical conductor. This thin wire forms a curve, the ends of which are connected to the terminals of the antenna port, which lie in the plane $z = 0$. The physical space between these terminals is the gap. Also in the plane $z = 0$, an arbitrary path over the gap is added to this curve, to obtain a closed path.

A front view of a non-self-intersecting planar loop antenna is shown in Fig. 1(a). For this type of loop antenna, the closed path forms a non-self-intersecting continuous loop in the plane $z = 0$, which bounds the surface shown in Fig. 1(b).

A front view of a self-intersecting planar loop antenna is shown in Fig. 1(c). We assume that there is no electric contact at the crossing points which appear in the front view. This implies that, at such a crossing point, the crossing arcs of the wire are separated in the z direction. Thus, the antenna is not strictly planar, but we nevertheless assume that the extent of the loop antenna in the z direction is so small that it may be ignored in our computations. For this type of loop antenna, the aforementioned closed path has multiple turns. We will see in Section IV how we can count the turns and identify the corresponding loops.

B. RECEPTION BY A THIN WIRE ANTENNA

For any wire antenna, the open-circuit voltage e_{ant} of the antenna used for reception is given by

$$e_{ant} = -\frac{1}{I_0} \iiint\limits_{\text{Antenna}} \mathbf{J}_t \cdot \mathbf{E}_i \, dv \,, \tag{1}$$

where $\mathbf{J}_t$ is the current density in the antenna if it is used for emission and a current I_0 flows into the positive terminal of the antenna port, where $\mathbf{E}_i$ is the incident electric field, and where dv is a volume element [16, Sec. 13.06]. This formula is based on reciprocity, so that it assumes a reciprocal medium. In the case of a thin-wire antenna along which a curvilinear abscissa s is defined, we obtain

$$e_{ant} = -\int\limits_{\text{Antenna}} \frac{i(s)}{I_0} \mathbf{u}_t \cdot \mathbf{E}_i(s) \, ds \,, \tag{2}$$

where $i(s)$ is the current in the thin wire if the antenna is used for emission and a current I_0 flows into the positive terminal of the antenna port, where $\mathbf{u}_t$ is a unit vector tangent to the thin wire, its direction being the direction of positive current, where $\mathbf{E}_i(s)$ is $\mathbf{E}_i$ at the curvilinear abscissa s, and where ds is the length element.

More precisely, $\mathbf{E}_i$ is the electric field produced by some external field source, in the absence of the wire antenna, but in the presence of passive nearby objects, if they exist. According to the proof of (1) provided in Appendix A, which is based on the method explained in [16, Sec. 13.06], there is no interaction between the field source and the wire antenna, since the same impressed current density represents the field source in the presence of the wire antenna, and the field source in the absence of the wire antenna. Thus, this proof is applicable if this interaction may be ignored, for instance in the case of a sufficiently remote field source of finite size.

Another proof of (1), provided in Appendix B, is new and applicable to a field source which may interact with the wire antenna, because the incident electric field is produced by a transmitting antenna (TA) coupled to a generator of internal impedance Z_G. In this case, $\mathbf{J}_t$ and I_0 are determined in the presence of the passive nearby objects and of the inactive field source (that is, the TA coupled to the impedance Z_G), and $\mathbf{E}_i$ is the electric field which would be caused by the TA excited by the generator, in the presence of the passive nearby objects but in the absence of the wire antenna.

Based on Appendix A and Appendix B, we can assert that:

- (1) and (2) are applicable regardless of the distance between the field source and the wire antenna;
- (1) is accurate if the gap is narrow and if $\mathbf{J}_t$ and I_0 are determined in the presence of the inactive field source and of passive nearby objects, if they exist; and
- (2) is accurate if the wire forming the wire antenna is thin, if the gap is narrow, and if $i(s)$ and I_0 are determined in the presence of the inactive field source and of the passive nearby objects, if they exist.

C. TE AND TM COMPONENTS

We consider an arbitrary incident time-harmonic electromagnetic field $\mathcal{F}_i = (\mathbf{E}_i, \mathbf{H}_i)$, in an homogeneous and source-free region characterized by the complex permittivity ϵ and the complex permeability μ, so that $k = \omega(\epsilon\mu)^{1/2}$, where the power 1/2 means the principal square root. It is well known that $\mathcal{F}_i$ can be expressed as the sum of two components: an electromagnetic field $\mathcal{F}_{TM} = (\mathbf{E}_{TM}, \mathbf{H}_{TM})$ transverse magnetic (TM) to z; and an electromagnetic field $\mathcal{F}_{TE} = (\mathbf{E}_{TE}, \mathbf{H}_{TE})$ transverse electric (TE) to z [17, Ch. 13], [18, Sec. 3.12]. Thus, we write

$$\mathcal{F}_i = \mathcal{F}_{TE} + \mathcal{F}_{TM} \,. \tag{3}$$

For $\mathcal{F}_{TM}$, we have

$$\mathbf{H}_{TM} = \nabla \times (\psi_{TM} \mathbf{u}_z) = \nabla \psi_{TM} \times \mathbf{u}_z \tag{4}$$

and

$$\mathbf{E}_{TM} = \frac{1}{j\omega\epsilon} \nabla \times \nabla \times (\psi_{TM} \mathbf{u}_z) \,, \tag{5}$$

where the scalar distribution ψ_{TM} has the dimensions of current and satisfies the Helmholtz equation

$$\nabla^2 \psi_{TM} + k^2 \psi_{TM} = 0 \,. \tag{6}$$

Let us use E_{TMx}, E_{TMy} and E_{TMz} to denote the rectangular coordinates of $\mathbf{E}_{TM}$, and H_{TMx}, H_{TMy} and H_{TMz} to denote the rectangular coordinates of $\mathbf{H}_{TM}$. We have

$$E_{TMx} = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi_{TM}}{\partial x \partial z} \,, \tag{7}$$

$$E_{TMy} = \frac{1}{j\omega\epsilon} \frac{\partial^2 \psi_{TM}}{\partial y \partial z} \,, \tag{8}$$

$$E_{TMz} = -\frac{1}{j\omega\epsilon} \left(\frac{\partial^2 \psi_{TM}}{\partial x^2} + \frac{\partial^2 \psi_{TM}}{\partial y^2} \right) \,, \tag{9}$$

$$H_{TMx} = \frac{\partial \psi_{TM}}{\partial y} \,, \tag{10}$$

$$H_{TMy} = -\frac{\partial \psi_{TM}}{\partial x} \tag{11}$$

and

$$H_{TMz} = 0 \,. \tag{12}$$

For $\mathcal{F}_{TE}$, we have

$$\mathbf{E}_{TE} = -\nabla \times (\psi_{TE} \mathbf{u}_z) = -\nabla \psi_{TE} \times \mathbf{u}_z \tag{13}$$

and

$$\mathbf{H}_{TE} = \frac{1}{j\omega\mu} \nabla \times \nabla \times (\psi_{TE} \mathbf{u}_z) \,, \tag{14}$$

where the scalar distribution ψ_{TE} has the dimensions of voltage and satisfies the Helmholtz equation

$$\nabla^2 \psi_{TE} + k^2 \psi_{TE} = 0 \,. \tag{15}$$

Let us use E_{TEx}, E_{TEy} and E_{TEz} to denote the rectangular coordinates of $\mathbf{E}_{TE}$, and H_{TEx}, H_{TEy} and H_{TEz} to denote the rectangular coordinates of $\mathbf{H}_{TE}$. We have

$$E_{TEx} = -\frac{\partial \psi_{TE}}{\partial y} \,, \tag{16}$$

$$E_{TEy} = \frac{\partial \psi_{TE}}{\partial x} \,, \tag{17}$$

$$E_{TEz} = 0 \,, \tag{18}$$

$$H_{TEx} = \frac{1}{j\omega\mu} \frac{\partial^2 \psi_{TE}}{\partial x \partial z} \,, \tag{19}$$

$$H_{TEy} = \frac{1}{j\omega\mu} \frac{\partial^2 \psi_{TE}}{\partial y \partial z} \tag{20}$$

and

$$H_{TEz} = -\frac{1}{j\omega\mu} \left(\frac{\partial^2 \psi_{TE}}{\partial x^2} + \frac{\partial^2 \psi_{TE}}{\partial y^2} \right) \,. \tag{21}$$

The existence of a decomposition of $\mathcal{F}_i$ into a TM-to-z component (the electromagnetic field $\mathcal{F}_{TM}$) and a TE-to-z component (the electromagnetic field $\mathcal{F}_{TE}$) does not entail that this decomposition is unique. This is because $\mathcal{F}_i$ may comprise an electromagnetic field that is both TE to z and TM to z, which may therefore be either a part of $\mathcal{F}_{TM}$ or of $\mathcal{F}_{TE}$ [19, Sec. 10.3]. We will use this observation in the definition of ψ_{TE} in Section VI, and later in Section VIII.F.

III. THE ETHEFS OF AN INCIDENT FIELD

A. MIRROR-SYMMETRIC AND MIRROR-ANTISYMMETRIC PARTS

We consider an arbitrary incident time-harmonic electromagnetic field $\mathcal{F}_i$, in a region which includes a part of the plane $z = 0$. Let $\mathcal{F}_m$ be the result of the transformation of $\mathcal{F}_i$ under reflection in the plane $z = 0$. Let $\mathbf{E}_i(x, y, z)$ and $\mathbf{H}_i(x, y, z)$ be the electric field and the magnetic field of $\mathcal{F}_i$, respectively, at a point of rectangular coordinates (x, y, z); and $\mathbf{E}_m(x, y, z)$ and $\mathbf{H}_m(x, y, z)$ be the electric field and the magnetic field of $\mathcal{F}_m$, respectively, at this point. We know that $\mathcal{F}_m$ is an electromagnetic field which satisfies [20, Sec. 6.11]:

$$\mathbf{E}_m(x, y, -z) = \mathbf{E}_i(x, y, z) - 2 \left(\mathbf{E}_i(x, y, z) \cdot \mathbf{u}_z \right) \mathbf{u}_z \tag{22}$$

and

$$\mathbf{H}_m(x, y, -z) = - \left[\mathbf{H}_i(x, y, z) - 2 \left(\mathbf{H}_i(x, y, z) \cdot \mathbf{u}_z \right) \mathbf{u}_z \right] \,. \tag{23}$$

Let us use $E_{ix}(x, y, z)$, $E_{iy}(x, y, z)$ and $E_{iz}(x, y, z)$ to denote the rectangular coordinates of $\mathbf{E}_i(x, y, z)$; $E_{mx}(x, y, z)$, $E_{my}(x, y, z)$ and $E_{mz}(x, y, z)$ the rectangular coordinates of $\mathbf{E}_m(x, y, z)$; $H_{ix}(x, y, z)$, $H_{iy}(x, y, z)$ and $H_{iz}(x, y, z)$ the rectangular coordinates of $\mathbf{H}_i(x, y, z)$; and $H_{mx}(x, y, z)$, $H_{my}(x, y, z)$ and $H_{mz}(x, y, z)$ the rectangular coordinates of $\mathbf{H}_m(x, y, z)$. Eq. (22) and (23) are equivalent to

$$E_{mx}(x, y, -z) = E_{ix}(x, y, z) \,, \tag{24}$$

$$E_{my}(x, y, -z) = E_{iy}(x, y, z) \,, \tag{25}$$

$$E_{mz}(x, y, -z) = -E_{iz}(x, y, z) \,, \tag{26}$$

$$H_{mx}(x, y, -z) = -H_{ix}(x, y, z) \,, \tag{27}$$

$$H_{my}(x, y, -z) = -H_{iy}(x, y, z) \tag{28}$$

and

$$H_{mz}(x, y, -z) = H_{iz}(x, y, z) \,. \tag{29}$$

We can now define the mirror-symmetric part of $\mathcal{F}_i$ as

$$\mathcal{F}_{MS} = \frac{\mathcal{F}_i + \mathcal{F}_m}{2} \tag{30}$$

and the mirror-antisymmetric part of $\mathcal{F}_i$ as

$$\mathcal{F}_{AS} = \frac{\mathcal{F}_i - \mathcal{F}_m}{2} \,. \tag{31}$$

Being linear combinations of electromagnetic fields, $\mathcal{F}_{MS}$ and $\mathcal{F}_{AS}$ are electromagnetic fields. Since

$$\mathcal{F}_i = \mathcal{F}_{MS} + \mathcal{F}_{AS} \,, \tag{32}$$

we have obtained a decomposition of $\mathcal{F}_i$ into two electromagnetic fields.

Let us use $\mathbf{E}_{MS}(x, y, z)$ and $\mathbf{H}_{MS}(x, y, z)$ to denote the electric field and the magnetic field of $\mathcal{F}_{MS}$, respectively, at a point of rectangular coordinates (x, y, z). Let us use $E_{MSx}(x, y, z)$, $E_{MSy}(x, y, z)$ and $E_{MSz}(x, y, z)$ to denote the rectangular coordinates of $\mathbf{E}_{MS}(x, y, z)$; and $H_{MSx}(x, y, z)$, $H_{MSy}(x, y, z)$ and $H_{MSz}(x, y, z)$ to denote the rectangular coordinates of $\mathbf{H}_{MS}(x, y, z)$. By (24)–(30), we have

$$E_{MSx}(x, y, 0) = E_{ix}(x, y, 0) \,, \tag{33}$$

$$E_{MSy}(x, y, 0) = E_{iy}(x, y, 0) \,, \tag{34}$$

$$E_{MSz}(x, y, 0) = 0 \,, \tag{35}$$

$$H_{MSx}(x, y, 0) = H_{MSy}(x, y, 0) = 0 \tag{36}$$

and

$$H_{MSz}(x, y, 0) = H_{iz}(x, y, 0) \,. \tag{37}$$

Let us use $\mathbf{E}_{AS}(x, y, z)$ and $\mathbf{H}_{AS}(x, y, z)$ to denote the electric field and the magnetic field of $\mathcal{F}_{AS}$, respectively, at a point of rectangular coordinates (x, y, z). Let us use $E_{ASx}(x, y, z)$, $E_{ASy}(x, y, z)$ and $E_{ASz}(x, y, z)$ to denote the rectangular coordinates of $\mathbf{E}_{AS}(x, y, z)$; and $H_{ASx}(x, y, z)$, $H_{ASy}(x, y, z)$ and $H_{ASz}(x, y, z)$ to denote the rectangular coordinates of $\mathbf{H}_{AS}(x, y, z)$. By (24)–(29) and (31), we have

$$E_{ASx}(x, y, 0) = E_{ASy}(x, y, 0) = 0 \,, \tag{38}$$

$$E_{ASz}(x, y, 0) = E_{iz}(x, y, 0) \,, \tag{39}$$

$$H_{ASx}(x, y, 0) = H_{ix}(x, y, 0) \,, \tag{40}$$

$$H_{ASy}(x, y, 0) = H_{iy}(x, y, 0) \tag{41}$$

and

$$H_{ASz}(x, y, 0) = 0 \,. \tag{42}$$

B. DEFINITION OF THE ETHEFS

We consider an arbitrary incident time-harmonic electromagnetic field $\mathcal{F}_i$, in an homogeneous and source-free region which includes a part of the plane $z = 0$, and which is characterized by the complex permittivity ϵ, and the complex permeability μ. For an arbitrary vector $\mathbf{v}$, we use v_z to denote $\mathbf{v} \cdot \mathbf{u}_z$, and $\mathbf{v}_\perp$ to denote the vector

$$\mathbf{v}_\perp = \mathbf{v} - (\mathbf{v} \cdot \mathbf{u}_z)\mathbf{u}_z = (\mathbf{v} \cdot \mathbf{u}_x)\mathbf{u}_x + (\mathbf{v} \cdot \mathbf{u}_y)\mathbf{u}_y \,. \tag{43}$$

Likewise, we introduce the operator $\nabla_\perp$ which, in cartesian form, is

$$\nabla_\perp = \mathbf{u}_x \frac{\partial}{\partial x} + \mathbf{u}_y \frac{\partial}{\partial y} \tag{44}$$

so that

$$\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \,. \tag{45}$$

We first apply the TM/TE decomposition, to obtain $\mathcal{F}_{TM}$ and $\mathcal{F}_{TE}$. By (4) and (7)–(12), $\mathcal{F}_{TM}$ is given by:

$$\mathbf{E}_{TM\perp} = \frac{1}{j\omega\epsilon} \frac{\partial}{\partial z} \nabla_\perp \psi_{TM} \,, \tag{46}$$

$$E_{TMz} = -\frac{1}{j\omega\epsilon} \nabla_\perp^2 \psi_{TM} \,, \tag{47}$$

$$\mathbf{H}_{TM\perp} = \nabla \psi_{TM} \times \mathbf{u}_z = \nabla_\perp \psi_{TM} \times \mathbf{u}_z \tag{48}$$

and

$$H_{TMz} = 0 \,. \tag{49}$$

By (13) and (16)–(21), $\mathcal{F}_{TE}$ is given by:

$$\mathbf{E}_{TE\perp} = -\nabla \psi_{TE} \times \mathbf{u}_z = -\nabla_\perp \psi_{TE} \times \mathbf{u}_z \,, \tag{50}$$

$$E_{TEz} = 0 \,, \tag{51}$$

$$\mathbf{H}_{TE\perp} = \frac{1}{j\omega\mu} \frac{\partial}{\partial z} \nabla_\perp \psi_{TE} \tag{52}$$

and

$$H_{TEz} = -\frac{1}{j\omega\mu} \nabla_\perp^2 \psi_{TE} \,. \tag{53}$$

We define the four ETHEFs of $\mathcal{F}_i$ as follows:

- the first ETHEF is the mirror-symmetric part of $\mathcal{F}_{TE}$, denoted by $\mathcal{F}_A = (\mathbf{E}_A, \mathbf{H}_A)$;
- the second ETHEF is the mirror-symmetric part of $\mathcal{F}_{TM}$, denoted by $\mathcal{F}_B = (\mathbf{E}_B, \mathbf{H}_B)$;
- the third ETHEF is the mirror-antisymmetric part of $\mathcal{F}_{TE}$, denoted by $\mathcal{F}_C = (\mathbf{E}_C, \mathbf{H}_C)$; and
- the fourth ETHEF is the mirror-antisymmetric part of $\mathcal{F}_{TM}$, denoted by $\mathcal{F}_D = (\mathbf{E}_D, \mathbf{H}_D)$.

By (3) and (32), we have

$$\mathcal{F}_i = \mathcal{F}_A + \mathcal{F}_B + \mathcal{F}_D + \mathcal{F}_C \,, \tag{54}$$

which provides a decomposition of $\mathcal{F}_i$ into four components, each of which could exist independently of the others.

By (33)–(42) and (46)–(53), in the plane $z = 0$, we have

$$\mathbf{E}_A = -\nabla_\perp \psi_{TE} \times \mathbf{u}_z \,, \tag{55}$$

$$\mathbf{H}_A = -\frac{\mathbf{u}_z}{j\omega\mu} \nabla_\perp^2 \psi_{TE} \,, \tag{56}$$

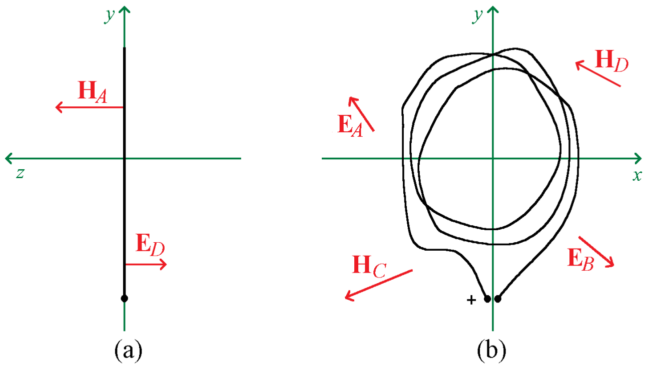

FIGURE 2. The planar loop antenna of Fig. 1(c), and the nonzero vectors given by (55)-(62), each represented at an arbitrary observation point in the plane $z = 0$, as if it was a real vector having the dimensions of length. The vectors which are orthogonal to the plane $z = 0$ are shown in the right side view of the loop antenna (a). The vectors which lie in the plane $z = 0$ are shown in the front view of the loop antenna (b).

$$\mathbf{E}_B = \frac{1}{j\omega\epsilon}\frac{\partial}{\partial z}\nabla_\perp \psi_{TM}\,, \tag{57}$$

$$\mathbf{H}_B = \mathbf{0}\,, \tag{58}$$

$$\mathbf{E}_C = \mathbf{0}\,, \tag{59}$$

$$\mathbf{H}_C = \frac{1}{j\omega\mu}\frac{\partial}{\partial z}\nabla_\perp \psi_{TE}\,, \tag{60}$$

$$\mathbf{E}_D = -\frac{\mathbf{u}_z}{j\omega\epsilon}\nabla_\perp^2 \psi_{TM} \tag{61}$$

and

$$\mathbf{H}_D = \nabla_\perp \psi_{TM} \times \mathbf{u}_z\,. \tag{62}$$

Since $\nabla_\perp \psi_{TM}$ and $\nabla_\perp \psi_{TE}$ are normal to $\mathbf{u}_z$, it follows that, in the plane $z = 0$:

- $\mathcal{F}_A$ has a magnetic field which is normal to this plane, whereas $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$ have each a magnetic field which is parallel to this plane, or a null vector; and
- $\mathcal{F}_A$ and $\mathcal{F}_B$ each have an electric field which is parallel to this plane, whereas $\mathcal{F}_C$ and $\mathcal{F}_D$ have each an electric field which is normal to this plane, or a null vector.

The vectors given by (55)–(62) are represented in Fig. 2. Using (54)–(62), we find that, in the plane $z = 0$,

$$\mathbf{H}_A = (\mathbf{H}_i \cdot \mathbf{u}_z)\mathbf{u}_z \tag{63}$$

and

$$\mathbf{E}_D = (\mathbf{E}_i \cdot \mathbf{u}_z)\mathbf{u}_z\,. \tag{64}$$

Thus, even though $\mathcal{F}_{TM}$ and $\mathcal{F}_{TE}$ need not be uniquely defined, $\mathbf{H}_A$, $\mathbf{H}_B$, $\mathbf{E}_C$ and $\mathbf{E}_D$ are uniquely defined in the plane $z = 0$. Also, $\mathbf{E}_A + \mathbf{E}_B$ and $\mathbf{H}_C + \mathbf{H}_D$ are uniquely defined everywhere, because they are the electric field of the mirror-symmetric part of $\mathcal{F}_i$, and the magnetic field of the mirror-antisymmetric part of $\mathcal{F}_i$, respectively.

IV. OPEN-CIRCUIT VOLTAGE OF A PLANAR WIRE LOOP ANTENNA OF ANY SHAPE AND SIZE

By (2) and (54), the open-circuit voltage of the planar wire loop antenna of Section II-A, having a thin wire and a narrow gap, used for reception of $\mathcal{F}_i$, is given by

$$e_{ant} = -\oint \frac{i(s)}{I_0}\mathbf{u}_t \cdot (\mathbf{E}_A + \mathbf{E}_B + \mathbf{E}_C + \mathbf{E}_D)\, ds\,, \tag{65}$$

where the integration path is closed by the arbitrary path over the gap, in which $i(s) = 0$. For reasons which will become apparent later, the arbitrary path over the gap should be as short as possible, for instance a straight line, but this is not required. The positive orientation of the integration path, which corresponds to the orientation of $\mathbf{u}_t$, goes from the positive terminal of the antenna port to the negative terminal of the antenna port along the thin wire, and then from the negative terminal to the positive terminal along the arbitrary path over the gap. Using (59) and (61), we get

$$e_{ant} = -\oint \frac{I_0 + (i(s) - I_0)}{I_0}\mathbf{u}_t \cdot (\mathbf{E}_A + \mathbf{E}_B)\, ds\,. \tag{66}$$

The integration path has a single turn if and only if the integration path forms a non-self-intersecting continuous loop in the plane $z = 0$. If the integration path has a single turn, let $\mathcal{A}$ be the part of the plane $z = 0$ that is bounded by the integration path. $\mathcal{F}_A$ and $\mathcal{F}_B$ being electromagnetic fields, we have $\nabla \times \mathbf{E}_A = -j\omega\mu\mathbf{H}_A$ and $\nabla \times \mathbf{E}_B = -j\omega\mu\mathbf{H}_B$. Thus, if the integration path has a single turn, it follows from Stoke's theorem that

$$\oint \mathbf{u}_t \cdot (\mathbf{E}_A + \mathbf{E}_B)\, ds = -j\omega\mu\kappa \iint_{\mathcal{A}} (\mathbf{H}_A + \mathbf{H}_B) \cdot \mathbf{u}_z\, da\,, \tag{67}$$

where $\kappa = 1$ if the orientation of the integration path agrees with the orientation $\mathbf{u}_z$ of the surface $\mathcal{A}$, and $\kappa = -1$ in the opposite case. It follows from (58), that, if the integration path has a single turn, we have

$$\oint \mathbf{u}_t \cdot (\mathbf{E}_A + \mathbf{E}_B)\, ds = -j\omega\mu\kappa \iint_{\mathcal{A}} \mathbf{H}_A \cdot \mathbf{u}_z\, da\,. \tag{68}$$

If the integration path forms a self-intersecting continuous loop in the plane $z = 0$, we use the following algorithm: (step 1) we start from the positive terminal of the antenna port and set the turn index n to 0; (step 2) moving along the integration path in the positive direction, as soon as a loop is formed with the part of the integration path already traveled and not erased, n is incremented by 1, this loop is labelled “loop n”, the part of the plane $z = 0$ which is bounded by loop n is denoted by $\mathcal{A}(n)$, we posit $\kappa_n = 1$ if the orientation of the integration path agrees with the orientation $\mathbf{u}_z$ of the surface $\mathcal{A}(n)$, or $\kappa_n = -1$ in the opposite case, and the line integral over loop n is, using Stoke's theorem, replaced with a surface integral over $\mathcal{A}(n)$; (step 3) loop n is erased from the integration path for the rest of the algorithm; (step 4) if the end of the integration path is not reached, we go back to step 2, otherwise we go to step 5; and (step 5) we set the number of turns, denoted by N, to n and end the algorithm.

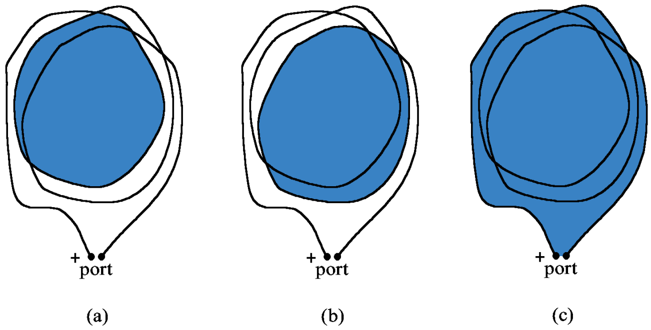

FIGURE 3. The planar loop antenna of Fig. 1(c), and the 3 surfaces bounded by a loop obtained using the proposed algorithm.

For the planar loop antenna of Fig. 1(c), an implementation of this algorithm is represented in Fig. 3, in which (a) shows $\mathcal{A}(1)$; (b) shows $\mathcal{A}(2)$; and (c) shows $\mathcal{A}(3)$. In this example, we have $N = 3$ and $\kappa_1 = \kappa_2 = \kappa_3$. Using Fig. 2(b), we find $\kappa_1 = \kappa_2 = \kappa_3 = -1$. We note that the algorithm would not work if the antenna was not planar. The algorithm leads us to the general result

$$\oint \mathbf{u}_t \cdot (\mathbf{E}_A + \mathbf{E}_B)\, ds = -j\omega\mu \sum_{n=1}^{N} \kappa_n \iint_{\mathcal{A}(n)} \mathbf{H}_A \cdot \mathbf{u}_z \, da \,. \quad (69)$$

Using (69) in (66), we obtain

$$e_{ant} = j\omega\mu \sum_{n=1}^{N} \kappa_n \iint_{\mathcal{A}(n)} \mathbf{H}_A \cdot \mathbf{u}_z \, da - \oint \frac{i(s) - I_0}{I_0} \mathbf{u}_t \cdot (\mathbf{E}_A + \mathbf{E}_B)\, ds \,, \quad (70)$$

which, for a single loop or identical loops, becomes

$$e_{ant} = j\omega\mu N \kappa \iint_{\mathcal{A}} \mathbf{H}_A \cdot \mathbf{u}_z \, da - \oint \frac{i(s) - I_0}{I_0} \mathbf{u}_t \cdot (\mathbf{E}_A + \mathbf{E}_B)\, ds \,. \quad (71)$$

In (70)–(71), the line integral is zero if $i(s)$ is uniform along the thin wire and if the length of the arbitrary path over the gap is zero. Thus, it may be ignored if the frequency is sufficiently low to allow us to consider that $i(s) \simeq I_0$ everywhere along the thin wire, and if the length of the arbitrary path over the gap is sufficiently short. This is why the arbitrary path over the gap should be as short as possible. As said in Section III, $\mathbf{H}_A$ and $\mathbf{E}_A + \mathbf{E}_B$ are uniquely defined in the plane $z = 0$, so that (70) and (71) are uniquely defined. In (70)–(71), the contributions of $\mathbf{H}_A$ and $\mathbf{E}_A$ are related because they correspond to the effect of $\mathcal{F}_A$, determined by ψ_{TE}, whereas the contribution of $\mathbf{E}_B$, which corresponds to the effect of $\mathcal{F}_B$, determined by ψ_{TM}, is completely independent from the contributions of $\mathbf{H}_A$ and $\mathbf{E}_A$.

The decomposition of $\mathcal{F}_i$ into four ETHEFs is particularly relevant to planar wire loop antennas which are intended to, or expected to, be mainly responsive to a magnetic field orthogonal to the plane of the antenna (such as most electrically small planar wire loop antennas), since, in this context:

- the ETHEF $\mathcal{F}_A$ causes the intended or expected response of the antenna;
- the ETHEF $\mathcal{F}_B$ may cause an unwanted or unexpected response of the antenna; and
- the ETHEFs $\mathcal{F}_C$ and $\mathcal{F}_D$ cannot cause any response of the antenna.

The use of $\mathbf{H}_A$, $\mathbf{E}_A$ and $\mathbf{E}_B$ in (70) and (71) shows how $\mathcal{F}_A$ and $\mathcal{F}_B$ excite the antenna.

V. ALTERNATIVE FORMULATIONS

In spite of (70) and (71), computing $\mathbf{H}_A$, $\mathbf{E}_A$ and $\mathbf{E}_B$ is not required to obtain e_{ant}, since, by (54)–(62), we also have

$$e_{ant} = j\omega\mu \sum_{n=1}^{N} \kappa_n \iint_{\mathcal{A}(n)} \mathbf{H}_{TE} \cdot \mathbf{u}_z \, da - \oint \frac{i(s) - I_0}{I_0} \mathbf{u}_t \cdot (\mathbf{E}_{TE} + \mathbf{E}_{TM})\, ds \quad (72)$$

if we use the decomposition of $\mathcal{F}_i$ into $\mathcal{F}_{TE}$ and $\mathcal{F}_{TM}$, and

$$e_{ant} = j\omega\mu \sum_{n=1}^{N} \kappa_n \iint_{\mathcal{A}(n)} \mathbf{H}_i \cdot \mathbf{u}_z \, da - \oint \frac{i(s) - I_0}{I_0} \mathbf{u}_t \cdot \mathbf{E}_i \, ds \quad (73)$$

if we use no decomposition of $\mathcal{F}_i$. If $\mathbf{H}_A$, $\mathbf{E}_A$ and $\mathbf{E}_B$ are not known, (72) or (73) may be easier to compute than (71). However, they require information that is not needed to determine e_{ant}, because, at $z = 0$, $\mathbf{H}_i$ and $\mathbf{E}_i$ contain more information about $\mathcal{F}_i$ than $\mathbf{H}_{TE}$, $\mathbf{E}_{TE}$ and $\mathbf{E}_{TM}$, which contain more information about $\mathcal{F}_i$ than $\mathbf{H}_A$, $\mathbf{E}_A$ and $\mathbf{E}_B$, the unnecessary information being ignored in (72) and (73).

Another formulation is directly based on the Maxwell-Faraday equation $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$. Using the algorithm of Section IV, we obtain

$$\oint \mathbf{u}_t \cdot \mathbf{E}\, ds = -j\omega\mu \sum_{n=1}^{N} \kappa_n \iint_{\mathcal{A}(n)} \mathbf{H} \cdot \mathbf{u}_z \, da \,, \quad (74)$$

where $\mathbf{E}$ is the total electric field and $\mathbf{H}$ is the total magnetic field. Since $\mathbf{u}_t \cdot \mathbf{E}$ is zero along the conducting wire and

$$e_{ant} = -\int_{\text{Gap}} \mathbf{u}_t \cdot \mathbf{E}\, ds \,, \quad (75)$$

we get

$$e_{ant} = j\omega\mu \sum_{n=1}^{N} \kappa_n \iint_{\mathcal{A}(n)} \mathbf{H} \cdot \mathbf{u}_z \, da \,. \quad (76)$$

$\mathbf{H}$ exists in the presence of the open-circuited planar loop antenna and of $\mathcal{F}_i$. The current i_R induced by $\mathcal{F}_i$ in the open-circuited planar loop antenna produces a magnetic field $\mathbf{H}_R$, and we have $\mathbf{H} = \mathbf{H}_i + \mathbf{H}_R$. In general, there is no simple way of computing i_R and $\mathbf{H}_R$, so that (76) cannot be directly used to determine e_{ant}. At low enough frequencies however, i_R is very small and we can use $\mathbf{H} \simeq \mathbf{H}_i$ in (76), to obtain the same result as (73) with $i(s) \simeq I_0$, in line with the "simplistic analysis" mentioned in Section I.

VI. APPLICATION TO AN INCIDENT PLANE WAVE

Let $\mathbf{k}_i = k_{ix}\mathbf{u}_x + k_{iy}\mathbf{u}_y + k_{iz}\mathbf{u}_z$ be an arbitrary complex wave vector such that

$$\mathbf{k}_i \cdot \mathbf{k}_i = k^2 = \omega^2 \epsilon \mu \,. \tag{77}$$

Let i_{TM} be an arbitrary complex number having the dimensions of current, v_{TM} be an arbitrary complex number having the dimensions of voltage, and $\mathbf{e}_{TEM}$ be an arbitrary complex vector having the dimensions of electric field and such that $\mathbf{e}_{TEM} \cdot \mathbf{u}_z = 0$. Here, i_{TM}, v_{TM} and $\mathbf{e}_{TEM}$ are independent of the observation point. The radius vector of the observation point being denoted by $\mathbf{r}$, we posit

$$\psi_{TM} = i_{TM}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{78}$$

so that (6) is satisfied. We posit

$$\psi_{TE} = \begin{cases} (\mathbf{e}_{TEM} \times \mathbf{u}_z) \cdot \mathbf{r}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} & \text{if } k_{ix} = k_{iy} = 0 \\ v_{TE}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} & \text{else} \,, \end{cases} \tag{79}$$

so that (15) is satisfied in both cases. The first case of (79) corresponds to a wave which is TE to z and TM to z (i.e., TEM to z), arbitrarily considered as a TE wave.

Here, $\mathcal{F}_i$ is a general time-harmonic plane wave in an homogeneous and source-free region characterized by the complex permittivity ϵ and the complex permeability μ. It is discussed in [18, Sec 2.11] and [18, Sec 4.2], where it is explained that the real part of $\mathbf{k}_i$ indicates the direction of propagation. Let $k_{iz} = \mathbf{k}_i \cdot \mathbf{u}_z$ and $\mathbf{k}_{i\perp}$ be the vector defined by letting $\mathbf{v} = \mathbf{k}_i$ in (43). Using (46)–(53), we find that the TE and TM components of $\mathcal{F}_i$ are, if $\mathbf{k}_{i\perp} \neq \mathbf{0}$, given by

$$\mathbf{E}_{TE} = j v_{TE}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{80}$$

$$\mathbf{H}_{TE} = \frac{v_{TE}}{j\omega\mu} \left[(\mathbf{k}_{i\perp} \cdot \mathbf{k}_{i\perp})\mathbf{u}_z - k_{iz}\mathbf{k}_{i\perp} \right] e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{81}$$

$$\mathbf{E}_{TM} = \frac{i_{TM}}{j\omega\epsilon} \left[(\mathbf{k}_{i\perp} \cdot \mathbf{k}_{i\perp})\mathbf{u}_z - k_{iz}\mathbf{k}_{i\perp} \right] e^{-j\mathbf{k}_i \cdot \mathbf{r}} \tag{82}$$

and

$$\mathbf{H}_{TM} = -j i_{TM}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,. \tag{83}$$

If $\mathbf{k}_{i\perp} = \mathbf{0}$, the TE and TM components of $\mathcal{F}_i$ are:

$$\mathbf{E}_{TE} = \mathbf{e}_{TEM}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,; \tag{84}$$

$$\mathbf{H}_{TE} = \frac{-k_{iz}}{\omega\mu}\, \mathbf{e}_{TEM} \times \mathbf{u}_z\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,; \tag{85}$$

and

$$\mathbf{E}_{TM} = \mathbf{0} \text{ and } \mathbf{H}_{TM} = \mathbf{0} \,, \tag{86}$$

as if we had used $\psi_{TM} = 0$ instead of (78) when $\mathbf{k}_{i\perp} = \mathbf{0}$.

For any value of $\mathbf{k}_i$, we have

$$\mathbf{E}_{TE} \cdot \mathbf{k}_i = 0 \quad \text{and} \quad \mathbf{H}_{TE} = \frac{\mathbf{k}_i}{\omega\mu} \times \mathbf{E}_{TE} \tag{87}$$

and

$$\mathbf{E}_{TM} \cdot \mathbf{k}_i = 0 \quad \text{and} \quad \mathbf{H}_{TM} = \frac{\mathbf{k}_i}{\omega\mu} \times \mathbf{E}_{TM} \,, \tag{88}$$

so that $\mathcal{F}_{TE}$, $\mathcal{F}_{TM}$ and $\mathcal{F}_i$ are TEM to $\mathbf{k}_i$. This does not entail an orthogonality of $\mathbf{E}_{TE}$, $\mathbf{H}_{TE}$, $\mathbf{E}_{TM}$ or $\mathbf{H}_{TM}$ to the direction of propagation, except in the case where $\mathbf{k}_i$ is real, or, more generally, in the case where $\mathbf{k}_i$ is collinear to its real part, that is the case of a uniform plane wave.

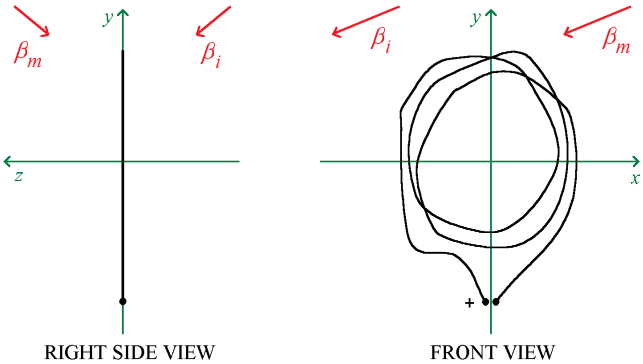

FIGURE 4. The planar loop antenna of Fig. 1(c), and the vectors $\boldsymbol{\beta}_i$ and $\boldsymbol{\beta}_m$, each represented at an arbitrary observation point in space, as if they had the dimensions of length. They are independent of the observation point.

According to (80)–(86), $\mathcal{F}_{TE}$ and $\mathcal{F}_{TM}$ are plane waves of wave vector $\mathbf{k}_i$, like $\mathcal{F}_i$. In contrast, $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$ are not plane waves, since, according to Section III, each of them is a linear combination of the incident plane wave of wave vector $\mathbf{k}_i$ and a plane wave of wave vector $\mathbf{k}_m$ given by

$$\mathbf{k}_m = \mathbf{k}_i - 2(\mathbf{k}_i \cdot \mathbf{u}_z)\mathbf{u}_z = k_{ix}\mathbf{u}_x + k_{iy}\mathbf{u}_y - k_{iz}\mathbf{u}_z \,. \tag{89}$$

Fig. 4 shows the planar loop antenna of Fig. 1(c) lying in the plane $z = 0$, the real part $\boldsymbol{\beta}_i = \mathrm{Re}(\mathbf{k}_i)$ of the wave vector $\mathbf{k}_i$, and the real part $\boldsymbol{\beta}_m = \mathrm{Re}(\mathbf{k}_m)$ of the wave vector $\mathbf{k}_m$. The values of $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$ everywhere in space are provided in Appendix C. Using (55)–(62) and (78)–(79), or the results of Appendix C, we obtain, in the plane $z = 0$,

$$\mathbf{E}_A = \begin{cases} \mathbf{e}_{TEM}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} & \text{if } \mathbf{k}_{i\perp} = \mathbf{0} \\ j v_{TE}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} & \text{if } \mathbf{k}_{i\perp} \neq \mathbf{0} \,, \end{cases} \tag{90}$$

$$\mathbf{H}_A = \frac{v_{TE}}{j\omega\mu} (\mathbf{k}_{i\perp} \cdot \mathbf{k}_{i\perp})\mathbf{u}_z\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{91}$$

$$\mathbf{E}_B = \frac{-i_{TM}}{j\omega\epsilon}\, k_{iz}\mathbf{k}_{i\perp}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{92}$$

$$\mathbf{H}_C = \begin{cases} \dfrac{k_{iz}}{\omega\mu}\, \mathbf{u}_z \times \mathbf{e}_{TEM}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} & \text{if } \mathbf{k}_{i\perp} = \mathbf{0} \\ \dfrac{-v_{TE}}{j\omega\mu}\, k_{iz}\mathbf{k}_{i\perp}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} & \text{if } \mathbf{k}_{i\perp} \neq \mathbf{0} \,, \end{cases} \tag{93}$$

$$\mathbf{E}_D = \frac{i_{TM}}{j\omega\epsilon} (\mathbf{k}_{i\perp} \cdot \mathbf{k}_{i\perp})\mathbf{u}_z\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \tag{94}$$

and

$$\mathbf{H}_D = -j i_{TM}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{95}$$

which, combined with (58)–(59), completely define the ETHEFs. Appendix D provides results which are simpler than (90)–(95), in the special case of a real $\mathbf{k}_i$.

Let $\mathbf{E}_{i0}$ be $\mathbf{E}_i$ at the origin, and $\mathbf{H}_{i0}$ be $\mathbf{H}_i$ at the origin. We observe that, based on (3) and (80)–(85), or on (54) and (90)–(95): if $\mathbf{k}_{i\perp} = \mathbf{0}$, we have

$$\mathbf{e}_{TEM} = \mathbf{E}_{i0} \,; \tag{96}$$

whereas, if $\mathbf{k}_{i\perp} \neq \mathbf{0}$, we get

$$v_{TE} = -j\,\frac{\mathbf{E}_{i0}\cdot(\mathbf{k}_{i\perp}\times\mathbf{u}_z)}{\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp}} = j\omega\mu\,\frac{\mathbf{H}_{i0}\cdot\mathbf{u}_z}{\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp}} \qquad (97)$$

and

$$i_{TM} = j\,\frac{\mathbf{H}_{i0}\cdot(\mathbf{k}_{i\perp}\times\mathbf{u}_z)}{\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp}} = j\omega\epsilon\,\frac{\mathbf{E}_{i0}\cdot\mathbf{u}_z}{\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp}}\,. \qquad (98)$$

Thus, since $\mathbf{H}_{i0} = \mathbf{k}_i \times \mathbf{E}_{i0}/(\omega\mu)$, we find that the knowledge of $\mathbf{k}_i$ and $\mathbf{E}_{i0}$ is sufficient to directly determine the ETHEFs in the plane $z = 0$, using (90)–(98).

VII. SOME OTHER PARTICULAR INCIDENT FIELDS

A. SUPERPOSITION OF PLANE WAVES

$\mathcal{F}_i$ is sometimes a known superposition of time-harmonic plane waves. For instance, this superposition may consist of a primary plane wave and its reflection on a conducting plane, or of an integral of plane waves providing a plane-wave spectrum representation [21, Sec. 19.2], [22].

The results of Section VI are applicable to any complex $\mathbf{k}_i$ satisfying (77), so that all possible uniform or evanescent plane waves are included. It follows that, if $\mathcal{F}_i$ is a known superposition of time-harmonic plane waves, we can apply the results of Section VI to each of them. Thus, $\mathcal{F}_A$ in the plane $z = 0$ is obtained as a superposition of electromagnetic fields each given by (90)–(91) applied to one of the time-harmonic plane waves, $\mathcal{F}_B$ is obtained as a superposition of electromagnetic fields each given by (58) and (92) applied to one of the time-harmonic plane waves, etc.

B. MAGNETIC DIPOLE MOMENT NORMAL TO THE PLANE OF THE PLANAR WIRE LOOP ANTENNA

The electric field produced by a magnetic dipole of moment $\mathbf{m} = m\mathbf{u}_z$ lying at the origin is [21, Sec 15.5], [23, Sec 8.6]

$$\begin{aligned}\mathbf{E} &= \frac{-k^2\eta}{4\pi r}\left(1+\frac{1}{jkr}\right)\mathbf{u}_r\times\mathbf{m}\,e^{-jkr}\\ &= \frac{k^2 m\eta}{4\pi r}\left(1+\frac{1}{jkr}\right)\sin\theta\,\mathbf{u}_\varphi\,e^{-jkr}\,, \end{aligned}\qquad (99)$$

where $\eta = (\mu/\epsilon)^{1/2}$, and where r, θ and φ are the usual spherical coordinates of the observation point, the unit vectors of the spherical coordinates being denoted by $\mathbf{u}_r$, $\mathbf{u}_\theta$ and $\mathbf{u}_\varphi$.

Let $\mathcal{F}_i$ be the electromagnetic field produced by a magnetic dipole of moment $\mathbf{m} = m\mathbf{u}_z$ lying at $x = x_S$, $y = y_S$ and $z = z_S$. By (99), this spherical wave is such that

$$\begin{aligned}\mathbf{E}_i = \frac{k^2 m\eta}{4\pi D^2}\left(1+\frac{1}{jkD}\right)&[-(y-y_S)\mathbf{u}_x\\ &+(x-x_S)\mathbf{u}_y]\,e^{-jkD}\,,\end{aligned}\qquad (100)$$

where

$$D = \sqrt{(x-x_S)^2+(y-y_S)^2+(z-z_S)^2}\,. \qquad (101)$$

Thus, $\mathcal{F}_i$ produced by the magnetic dipole is TE to z, so that we may write $\mathcal{F}_i = \mathcal{F}_{TE}$. Consequently, by the definition of the ETHEFs, we have, in the plane $z = 0$,

$$\mathbf{E}_A = \mathbf{E}_i \;\text{ and }\; \mathbf{E}_B = \mathbf{0}\,, \qquad (102)$$

which, combined with (63) is sufficient to directly determine the fields used in (70) and (71), from the known $\mathbf{E}_i$ and $\mathbf{H}_i$.

C. CIRCULAR TRANSMITTING LOOP PARALLEL TO THE PLANE OF THE PLANAR WIRE LOOP ANTENNA

As said in the introduction, a possible calibration technique for a field-strength meter is the standard-field method, in which the circular loop antenna of the field-strength meter to be calibrated is excited by a coaxial single-turn circular transmitting loop antenna [6]. For instance, the National Bureau of Standards (NBS) used a transmitting loop of radius $r_T = 10$ cm, up to 50 MHz, the distance d between the planes of the transmitting loop antenna and of the field-strength meter's loop antenna ranging from 1.5 m to 3 m [24]. The calibration was based on the "formula of Greene", that is formula (24) of [25], which takes propagation between the loops into account to compute an average normal component of $\mathbf{H}_i$, but assumes a uniform current in the transmitting loop.

We therefore consider the electromagnetic field $\mathcal{F}_i$ produced by a single-turn circular transmitting loop antenna, in which a uniform current flows, this circular transmitting loop antenna being parallel to the plane $z = 0$, its center lying at $x = x_S$, $y = y_S$ and $z = z_S$. We first consider a point P that is not located on the straight line D of equations $x = x_S$ and $y = y_S$. Let Π be the plane that contains this straight line and point P. The transformation that combines a mirror reflection in the plane Π and a charge inversion is a symmetry present in the causes of the polar vector $\mathbf{E}_i$ caused by the uniform current. Curie's principle states that, when some causes produce some effects, any symmetry present in the causes must also be present in the effects, and any asymmetry present in the effects must also be present in the causes [26, Sec. IV]. It follows from Curie's principle that $\mathbf{E}_i$ is orthogonal to Π at point P. If $P \in D$, we can use two orthogonal planes containing this straight line to conclude that $\mathbf{E}_i$ is null at point P. Thus, $\mathcal{F}_i$ is TE to z.

In an actual single-turn circular transmitting loop antenna of radius r_T, the current is not exactly uniform, but it is practically uniform if r_T is sufficiently small with respect to the wavelength λ. Thus, in this case, $\mathcal{F}_i$ is practically TE to z, so that we may write $\mathcal{F}_i \approx \mathcal{F}_{TE}$. It follows that, in the plane $z = 0$,

$$\mathbf{E}_A \approx \mathbf{E}_i \;\text{ and }\; \mathbf{E}_B \approx \mathbf{0}\,, \qquad (103)$$

which, combined with (63) is sufficient to approximately determine the fields used in (70) and (71), if $\mathbf{E}_i$ and $\mathbf{H}_i$ are known. Thus, the effects of $\mathcal{F}_B$ are completely ignored in the standard-field calibration method.

This method has a worse problem, though, because the formula of Greene uses a value of e_{ant} given by the "simplistic analysis" of Section I, or equivalently by (73) under the assumption $i(s) = I_0$, whereas the loop antenna of the field-strength meter need not be small enough for this assumption to be accurate at the highest frequency of intended operation.

The standard-field calibration method is also referred to as the "standard transmitting loop" method. It is described in some current EMC standards [27, Sec. N.2.2], [28, Sec. 7].

VIII. METHOD OF COMPUTING THE ETHFS

A. GOAL

If we know $\mathcal{F}_i$, we have identified two ways of computing $\mathcal{F}_A$ and $\mathcal{F}_B$ in the plane $z = 0$, in order to be able to use (70)–(71). The first path, presented in Section V or Section VI-A, is applicable if $\mathcal{F}_i$ is a known plane wave, or a superposition of known plane waves. The second path, exemplified in Sections VI-B and VI-C, applies to particular forms of $\mathcal{F}_i$.

We now want to compute $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$ under broad assumptions. Let $\mathcal{D} \subset \mathbb{R}^3$ be a bounded domain, homogeneous, of complex permittivity ϵ and complex permeability μ. Let $\partial\mathcal{D}$ be the boundary of $\mathcal{D}$, and $\overline{\mathcal{D}}$ be the closure $\mathcal{D} \cup \partial\mathcal{D}$ of $\mathcal{D}$. Let $\mathcal{O}$ be an open set containing $\overline{\mathcal{D}}$. We assume that $\mathcal{F}_i$ is source-free in $\mathcal{D}$, known in $\overline{\mathcal{D}}$, and that $\mathbf{E}_i$ and $\mathbf{H}_i$ are both of class C^2 in $\mathcal{D}$ and of class C^1 in $\mathcal{O}$.

B. FIRST STEP

Using (6) and (9) in $\mathcal{D}$, we get

$$E_{TMz} = \frac{1}{j\omega\epsilon}\left(\frac{\partial^2\psi_{TM}}{\partial z^2} + k^2\psi_{TM}\right), \qquad (104)$$

and it follows from (3), (18) and (104) that the distribution ψ_{TM} satisfies

$$\frac{\partial^2\psi_{TM}}{\partial z^2} + k^2\psi_{TM} = j\omega\epsilon E_{iz}, \qquad (105)$$

where $\mathbf{E}_i \cdot \mathbf{u}_z$ is denoted by E_{iz}.

Likewise, using (15) and (21) in $\mathcal{D}$, we get

$$H_{TEz} = \frac{1}{j\omega\mu}\left(\frac{\partial^2\psi_{TE}}{\partial z^2} + k^2\psi_{TE}\right), \qquad (106)$$

and it follows from (3), (12) and (106) that the distribution ψ_{TE} satisfies

$$\frac{\partial^2\psi_{TE}}{\partial z^2} + k^2\psi_{TE} = j\omega\mu H_{iz}, \qquad (107)$$

where $\mathbf{H}_i \cdot \mathbf{u}_z$ is denoted by H_{iz}.

Since $\mathcal{F}_i$ is known, E_{iz} and H_{iz} are known, so that (105) and (107) are ordinary differential equations in the variable z. To solve them, we assume that there exists a simply connected and bounded surface $\mathcal{T} \subset \mathbb{R}^2$, a real $z_N < 0$ and a real $z_P > 0$ such that

$$((x, y, z) \in \mathcal{D}) \Leftrightarrow ((x, y) \in \mathcal{T} \text{ and } z_N < z < z_P). \qquad (108)$$

$\mathcal{T}$ is a domain of $\mathbb{R}^2$, of boundary $\partial\mathcal{T}$ and closure $\overline{\mathcal{T}} = \mathcal{T} \cup \partial\mathcal{T}$. Let g be a continuous function defined in $\overline{\mathcal{D}}$, and $g(x, y, z)$ be the value of g at a point of rectangular coordinates (x, y, z).

We define the operator $\mathfrak{P}$ such that the value of $\mathfrak{P}g$ at a point of rectangular coordinates $(x, y, z) \in \overline{\mathcal{D}}$ is given by

$$(\mathfrak{P}g)(x, y, z) = e^{jkz}\int_0^z e^{-2jkv}\int_0^v g(x, y, u)e^{jku}\,du\,dv\,. \qquad (109)$$

$\mathfrak{P}g$ is of class C^2. Omitting the (x, y, z) dependencies, we find

$$\frac{\partial(\mathfrak{P}g)}{\partial z} = jk\mathfrak{P}g + e^{-jkz}\int_0^z g e^{jku}\,du\,, \qquad (110)$$

and

$$\frac{\partial^2(\mathfrak{P}g)}{\partial z^2} = -k^2\mathfrak{P}g + g\,. \qquad (111)$$

Using $g = j\omega\epsilon E_{iz}$, we find that $j\omega\epsilon\,\mathfrak{P}E_{iz}$ is a particular solution of (105) in $\mathcal{D}$. Since ψ_{TM} satisfies (105), it must be given by a general solution of (105). Solutions of the corresponding homogeneous differential equation being of the form $C(x, y)e^{\pm jkz}$, a ψ_{TM} providing $\mathbf{E}_{TM}$ and $\mathbf{H}_{TM}$ of class C^1 in $\mathcal{D}$ is given by

$$\psi_{TM} = C_1(x, y)e^{jkz} + C_2(x, y)e^{-jkz} + j\omega\epsilon\,\mathfrak{P}E_{iz} \qquad (112)$$

in $\overline{\mathcal{D}}$, where C_1 and C_2 are unknown complex functions of $(x, y) \in \overline{\mathcal{T}}$, of class C^2 in $\mathcal{T}$ and of class C^1 in $\overline{\mathcal{T}}$.

Using $g = j\omega\mu H_{iz}$, we find that $j\omega\mu\,\mathfrak{P}H_{iz}$ is a particular solution of (107) in $\mathcal{D}$. Since ψ_{TE} satisfies (107), a ψ_{TE} providing $\mathbf{E}_{TE}$ and $\mathbf{H}_{TE}$ of class C^1 in $\mathcal{D}$ is given by

$$\psi_{TE} = C_3(x, y)e^{jkz} + C_4(x, y)e^{-jkz} + j\omega\mu\,\mathfrak{P}H_{iz} \qquad (113)$$

in $\overline{\mathcal{D}}$, where C_3 and C_4 are unknown complex functions of $(x, y) \in \overline{\mathcal{T}}$, of class C^2 in $\mathcal{T}$ and of class C^1 in $\overline{\mathcal{T}}$.

C. SECOND STEP

It follows from (3), (47) and (51) that ψ_{TM} satisfies

$$\nabla_\perp^2\psi_{TM} = -j\omega\epsilon E_{iz} \qquad (114)$$

in $\mathcal{D}$. It follows from (3), (49) and (53) that ψ_{TE} satisfies

$$\nabla_\perp^2\psi_{TE} = -j\omega\mu H_{iz} \qquad (115)$$

in $\mathcal{D}$. We observe that, if ψ_{TM} is given by (112) and satisfies (114), then it also satisfies (6) and (105) in $\mathcal{D}$. Likewise, if ψ_{TE} is given by (113) and satisfies (115), then it also satisfies (15) and (107) in $\mathcal{D}$.

Let f be a function of class C^2 defined in $\mathcal{D}$. We have

$$\frac{\partial}{\partial z}\left(\left[\frac{\partial f}{\partial z} - jkf\right]e^{jkz}\right) = \left[\frac{\partial^2 f}{\partial z^2} + k^2 f\right]e^{jkz}, \qquad (116)$$

so that

$$\begin{aligned} e^{-2jkz}\int_0^z &\left[\frac{\partial^2 f}{\partial z^2}(x, y, u) + k^2 f(x, y, u)\right]e^{jku}\,du \\ &= \left[\frac{\partial f}{\partial z}(x, y, z) - jkf(x, y, z)\right]e^{-jkz} \\ &\quad - \left[\frac{\partial f}{\partial z}(x, y, 0) - jkf(x, y, 0)\right]e^{-2jkz}\,. \end{aligned} \qquad (117)$$

Since

$$\frac{\partial}{\partial z}\left(fe^{-jkz}\right) = \left[\frac{\partial f}{\partial z} - jkf\right]e^{-jkz}, \qquad (118)$$

it follows from (117) that

$$\begin{aligned} \int_0^z e^{-2jkv}\int_0^v &\left[\frac{\partial^2 f}{\partial z^2}(x, y, u) + k^2 f(x, y, u)\right]e^{jku}\,du\,dv \\ &= f(x, y, z)e^{-jkz} - f(x, y, 0) \\ &+ \frac{1}{2jk}\left[\frac{\partial f}{\partial z}(x, y, 0) - jkf(x, y, 0)\right]\left(e^{-2jkz} - 1\right). \end{aligned} \qquad (119)$$

Consequently, we have

$$\left(\mathfrak{P}\left[\frac{\partial^2 f}{\partial z^2}+k^2 f\right]\right)(x,y,z)$$
$$=f(x,y,z)-\frac{1}{2}\left(e^{jkz}+e^{-jkz}\right)f(x,y,0)$$
$$-\frac{1}{2jk}\left(e^{jkz}-e^{-jkz}\right)\frac{\partial f}{\partial z}(x,y,0)\,. \quad (120)$$

Since $\mathcal{F}_i=(\mathbf{E}_i,\mathbf{H}_i)$ is an electromagnetic field, $\mathbf{E}_i$ and $\mathbf{H}_i$ satisfy the vector wave equations $\nabla^2\mathbf{E}_i+k^2\mathbf{E}_i=\mathbf{0}$ and $\nabla^2\mathbf{H}_i+k^2\mathbf{H}_i=\mathbf{0}$ in $\mathcal{D}$ [18, Sec. 2-1]. It follows that E_{iz} and H_{iz} satisfy the Helmholz equations $\nabla^2 E_{iz}+k^2E_{iz}=0$ and $\nabla^2 H_{iz}+k^2H_{iz}=0$ in $\mathcal{D}$. Consequently, if we now assume that $f=E_{iz}$ or $f=H_{iz}$, we get

$$\nabla^2_\perp(\mathfrak{P}f)=\mathfrak{P}(\nabla^2_\perp f)=-\mathfrak{P}\left[\frac{\partial^2 f}{\partial z^2}+k^2 f\right]\,. \quad (121)$$

Combining (120) and (121), we obtain

$$\nabla^2_\perp(\mathfrak{P}f)(x,y,z)$$
$$=-f(x,y,z)+\frac{1}{2}\left(e^{jkz}+e^{-jkz}\right)f(x,y,0)$$
$$+\frac{1}{2jk}\left(e^{jkz}-e^{-jkz}\right)\frac{\partial f}{\partial z}(x,y,0)\,, \quad (122)$$

in which $f=E_{iz}$ or $f=H_{iz}$.

It follows from (112) and (122) that the condition (114) is satisfied everywhere in $\mathcal{D}$ if and only if

$$\nabla^2_\perp C_1(x,y)+\frac{j\omega\epsilon}{2}\left[E_{iz}+\frac{1}{jk}\frac{\partial E_{iz}}{\partial z}\right](x,y,0)=0 \quad (123)$$

and

$$\nabla^2_\perp C_2(x,y)+\frac{j\omega\epsilon}{2}\left[E_{iz}-\frac{1}{jk}\frac{\partial E_{iz}}{\partial z}\right](x,y,0)=0 \quad (124)$$

for any $(x,y)\in\mathcal{T}$.

Likewise, it follows from (113) and (122) that the condition (115) is satisfied everywhere in $\mathcal{D}$ if and only if

$$\nabla^2_\perp C_3(x,y)+\frac{j\omega\mu}{2}\left[H_{iz}+\frac{1}{jk}\frac{\partial H_{iz}}{\partial z}\right](x,y,0)=0 \quad (125)$$

and

$$\nabla^2_\perp C_4(x,y)+\frac{j\omega\mu}{2}\left[H_{iz}-\frac{1}{jk}\frac{\partial H_{iz}}{\partial z}\right](x,y,0)=0 \quad (126)$$

for any $(x,y)\in\mathcal{T}$.

D. THIRD STEP

We have obtained four partial differential equations in 2 dimensions (2D): the unknown functions C_1, C_2, C_3 and C_4 are determined by the 2D Poisson equations (123), (124), (125) and (126), respectively, and by appropriate conditions at the boundary $\partial\mathcal{T}$ of $\mathcal{T}$.

We are not yet ready to recite these boundary conditions, and, in the remainder of this Section VIII, "arbitrary" shall refer to a result which need not satisfy or be based on appropriate boundary conditions. We observe that, since results of (112)–(113) based on arbitrary solutions of (123)–(126) will satisfy (104)–(107), appropriate boundary conditions can only relate to values of $\mathbf{E}_{i\perp}$ and $\mathbf{H}_{i\perp}$, where we have used the notation (43). Using (3), (46), (48), (50) and (52), we find that, everywhere in $\mathcal{D}$, these values are given by

$$\mathbf{E}_{i\perp}=\frac{1}{j\omega\epsilon}\frac{\partial}{\partial z}\nabla_\perp\psi_{TM}-\nabla_\perp\psi_{TE}\times\mathbf{u}_z \quad (127)$$

and

$$\mathbf{H}_{i\perp}=\nabla_\perp\psi_{TM}\times\mathbf{u}_z+\frac{1}{j\omega\mu}\frac{\partial}{\partial z}\nabla_\perp\psi_{TE}\,. \quad (128)$$

For any $\alpha\in\{1,\ldots,4\}$, we will:

- use $C_{A\alpha}$ and $C_{B\alpha}$ to denote two complex functions of class C^2 in $\mathcal{T}$ and of class C^1 in $\overline{\mathcal{T}}$, such that $C_{A\alpha}$ and $C_{B\alpha}$ are each an arbitrary solution of the 2D Poisson equation for C_α, that is an arbitrary solution of (123) if $\alpha=1$, or of (124) if $\alpha=2$, or of (125) if $\alpha=3$, or of (126) if $\alpha=4$; and
- observe that the difference $Q_\alpha=C_{B\alpha}-C_{A\alpha}$ is a complex function of class C^2 in $\mathcal{T}$ and of class C^1 in $\overline{\mathcal{T}}$, which satisfies the 2D Laplace equation $\nabla^2_\perp Q_\alpha=0$.

Let ψ_{ATM} be the right-hand-side of (112) if we use C_{A1} in the place of C_1 and C_{A2} in the place of C_2, ψ_{BTM} be the right-hand-side of (112) if we use C_{B1} in the place of C_1 and C_{B2} in the place of C_2, ψ_{ATE} be the right-hand-side of (113) if we use C_{A3} in the place of C_3 and C_{A4} in the place of C_4, and ψ_{BTE} be the right-hand-side of (113) if we use C_{B3} in the place of C_3 and C_{B4} in the place of C_4.

Let $\mathbf{E}_{A\perp}$ be the right-hand-side of (127) if we use ψ_{ATM} in the place of ψ_{TM} and ψ_{ATE} in the place of ψ_{TE}, $\mathbf{E}_{B\perp}$ be the right-hand-side of (127) if we use ψ_{BTM} in the place of ψ_{TM} and ψ_{BTE} in the place of ψ_{TE}, $\mathbf{H}_{A\perp}$ be the right-hand-side of (128) if we use ψ_{ATM} in the place of ψ_{TM} and ψ_{ATE} in the place of ψ_{TE}, and $\mathbf{H}_{B\perp}$ be the right-hand-side of (128) if we use ψ_{BTM} in the place of ψ_{TM} and ψ_{BTE} in the place of ψ_{TE}.

$\mathbf{E}_{A\perp}$ and $\mathbf{H}_{A\perp}$ are arbitrary in the sense that they are based on the arbitrary solutions C_{A1} to C_{A4}. Likewise, $\mathbf{E}_{B\perp}$ and $\mathbf{H}_{B\perp}$ are arbitrary in the sense that they are based on the arbitrary solutions C_{B1} to C_{B4}. The difference $\mathbf{E}_{B\perp}-\mathbf{E}_{A\perp}$ induced by the differences Q_1 to Q_4 is given by

$$\mathbf{E}_{B\perp}-\mathbf{E}_{A\perp}=\eta\left(\nabla_\perp Q_1e^{jkz}-\nabla_\perp Q_2e^{-jkz}\right)$$
$$-\left(\nabla_\perp Q_3e^{jkz}+\nabla_\perp Q_4e^{-jkz}\right)\times\mathbf{u}_z\,, \quad (129)$$

and the difference $\mathbf{H}_{B\perp}-\mathbf{H}_{A\perp}$ induced by the differences Q_1 to Q_4 is given by

$$\mathbf{H}_{B\perp}-\mathbf{H}_{A\perp}=\left(\nabla_\perp Q_1e^{jkz}+\nabla_\perp Q_2e^{-jkz}\right)\times\mathbf{u}_z$$
$$+\frac{1}{\eta}\left(\nabla_\perp Q_3e^{jkz}-\nabla_\perp Q_4e^{-jkz}\right)\,. \quad (130)$$

The differences $\mathbf{E}_{B\perp}-\mathbf{E}_{A\perp}$ and $\mathbf{H}_{B\perp}-\mathbf{H}_{A\perp}$ correspond to: a TEM-to-z wave propagating in the direction of $-\mathbf{u}_z$, caused by Q_1; a TEM-to-z wave propagating in the direction of $\mathbf{u}_z$, caused by Q_2; another TEM-to-z wave propagating in the direction of $-\mathbf{u}_z$, caused by Q_3; and another TEM-to-z wave propagating in the direction of $\mathbf{u}_z$, caused by Q_4.

E. FOURTH STEP

This raises two questions: can we obtain the same TEM-to-z wave from Q_1 or Q_3? and can we obtain the same TEM-to-z wave from Q_2 or Q_4? To address these questions, we introduce, for any $\alpha \in \{1, \dots, 4\}$, the differential forms δP_α defined by

$$\delta P_\alpha = -\frac{\partial Q_\alpha}{\partial y} dx + \frac{\partial Q_\alpha}{\partial x} dy \,. \tag{131}$$

Q_α being a solution of the 2D Laplace equation in $\mathcal{T}$, we have

$$\frac{\partial}{\partial y}\left(-\frac{\partial Q_\alpha}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial Q_\alpha}{\partial x}\right) , \tag{132}$$

so that δP_α is an exact differential which can be used to define a function P_α of class C^1 in $\overline{\mathcal{T}}$, as

$$P_\alpha(x, y) = \int_{\Gamma(x,y)} \delta P_\alpha \,, \tag{133}$$

where $\Gamma(x, y)$ is an arbitrary path in $\mathcal{T}$, from the origin to (x, y). We have $P_\alpha(0, 0) = 0$ and $dP_\alpha = \delta P_\alpha$. To define P_α, we have implemented an approach similar to the one which may be used to define harmonic conjugate functions, with a notable difference that Q_α and P_α are complex whereas harmonic conjugate functions are real [29, Sec. 28-3].

Q_α being of class C^2 in $\mathcal{T}$, it follows from $dP_\alpha = \delta P_\alpha$ and (131) that P_α is of class C^2 and a solution of the 2D Laplace equation in $\mathcal{T}$. Consequently, we may posit

$$Q_3 = \eta P_1 \text{ and } Q_4 = -\eta P_2 \,, \tag{134}$$

or, alternatively,

$$Q_1 = -\frac{P_3}{\eta} \text{ and } Q_2 = \frac{P_4}{\eta} \,. \tag{135}$$

It follows from (131) that, in both cases, we have:

$$\frac{\partial Q_3}{\partial x} = -\eta \frac{\partial Q_1}{\partial y} \text{ and } \frac{\partial Q_3}{\partial y} = \eta \frac{\partial Q_1}{\partial x} \tag{136}$$

and

$$\frac{\partial Q_4}{\partial x} = \eta \frac{\partial Q_2}{\partial y} \text{ and } \frac{\partial Q_4}{\partial y} = -\eta \frac{\partial Q_2}{\partial x} \,, \tag{137}$$

which may also be written

$$\nabla_\perp Q_3 = -\eta \nabla_\perp Q_1 \times \mathbf{u}_z \tag{138}$$

and

$$\nabla_\perp Q_4 = \eta \nabla_\perp Q_2 \times \mathbf{u}_z \,. \tag{139}$$

Using (138)–(139) in (129)–(130), we find that either one of the assumptions (134) or (135) leads us to

$$\mathbf{E}_{B\perp} - \mathbf{E}_{A\perp} = \mathbf{0} \tag{140}$$

and

$$\mathbf{H}_{B\perp} - \mathbf{H}_{A\perp} = \mathbf{0} \,. \tag{141}$$

In the case of the assumption (134), we have found Q_3 and Q_4 that compensate the effect of specified Q_1 and Q_2, respectively, in (129)–(130). In the case of the assumption (135), we have found Q_1 and Q_2 which compensate the effect of specified Q_3 and Q_4, respectively, in (129)–(130). Thus, we can obtain: the same TEM-to-z wave from Q_1 and Q_3; and the same TEM-to-z wave from Q_2 and Q_4.

F. FIFTH STEP

For any $\alpha \in \{1, \dots, 4\}$, we will:

- use $C_{U\alpha}$ to denote a complex functions of class C^2 in $\mathcal{T}$ and of class C^1 in $\overline{\mathcal{T}}$, such that $C_{U\alpha}$ is an arbitrary solution of the 2D Poisson equation for C_α;
- use $Q_{U\alpha}$ to denote a complex function of class C^2 in $\mathcal{T}$ and of class C^1 in $\overline{\mathcal{T}}$, that satisfies the 2D Laplace equation $\nabla^2 Q_{U\alpha} = 0$; and
- use C_α to denote the solution $C_\alpha = C_{U\alpha} + Q_{U\alpha}$ of the 2D Poisson equation for C_α.

We want to adjust $Q_{U1}, \dots, Q_{U4}$ in such a way that $C_1, \dots, C_4$ satisfy appropriate boundary conditions. All properties of $Q_1, \dots, Q_4$ established in Section VIII.D are applicable to $Q_{U1}, \dots, Q_{U4}$, respectively. It follows that $Q_{U1}, \dots, Q_{U4}$ correspond to TEM-to-z waves, a TEM-to-z wave caused by Q_{U1} or Q_{U2} being a part of $\mathcal{F}_{TM}$ whereas a TEM-to-z wave caused by Q_{U3} or Q_{U4} is a part of $\mathcal{F}_{TE}$. It follows from Sections VIII.E that: the same TEM-to-z wave can be obtained from Q_{U1} and Q_{U3}; and that the same TEM-to-z wave can be obtained from Q_{U2} and Q_{U4}. Thus, such a TEM-to-z wave may be a part of $\mathcal{F}_{TM}$ or a part of $\mathcal{F}_{TE}$, a point which was previously mentioned in Section II.C without detailed explanations.

It follows that we can for instance posit $Q_{U1} = 0$ and $Q_{U2} = 0$, or $Q_{U1} = 0$ and $Q_{U4} = 0$, or $Q_{U2} = 0$ and $Q_{U3} = 0$, or $Q_{U3} = 0$ and $Q_{U4} = 0$. In other words, among solutions $C_1, \dots, C_4$ which satisfy appropriate boundary conditions, it is possible to use arbitrary solutions for C_1 and C_2, or for C_1 and C_4, or for C_2 and C_3, or for C_3 and C_4.

G. SIXTH STEP

By (109)–(110), for any $(x, y) \in \overline{\mathcal{T}}$, we have

$$(\mathfrak{P} E_{iz})(x, y, 0) = 0 \text{ and } \frac{\partial(\mathfrak{P} E_{iz})}{\partial z}(x, y, 0) = 0 \tag{142}$$

and

$$(\mathfrak{P} H_{iz})(x, y, 0) = 0 \text{ and } \frac{\partial(\mathfrak{P} H_{iz})}{\partial z}(x, y, 0) = 0 \,. \tag{143}$$

Thus, it follows from (3), (46), (48), (50), (52), (112) and (113) that, for any $(x, y) \in \overline{\mathcal{T}}$, we have

$$\begin{aligned} \mathbf{E}_i(x, y, 0) &= \eta \left(\nabla_\perp C_1 - \nabla_\perp C_2\right) \\ &\quad - \left(\nabla_\perp C_3 + \nabla_\perp C_4\right) \times \mathbf{u}_z + E_{iz}(x, y, 0)\, \mathbf{u}_z \end{aligned} \tag{144}$$

and

$$\begin{aligned} \mathbf{H}_i(x, y, 0) &= \left(\nabla_\perp C_1 + \nabla_\perp C_2\right) \times \mathbf{u}_z \\ &\quad + \frac{1}{\eta} \left(\nabla_\perp C_3 - \nabla_\perp C_4\right) + H_{iz}(x, y, 0)\, \mathbf{u}_z \,. \end{aligned} \tag{145}$$

Consequently, we have

$$\begin{aligned} \mathbf{E}_{i\perp}(x, y, 0) &- \eta\, \mathbf{H}_{i\perp}(x, y, 0) \times \mathbf{u}_z \\ &= 2\eta\, \nabla_\perp C_1 - 2\nabla_\perp C_3 \times \mathbf{u}_z \end{aligned} \tag{146}$$

and

$$\begin{aligned} \mathbf{E}_{i\perp}(x, y, 0) &+ \eta\, \mathbf{H}_{i\perp}(x, y, 0) \times \mathbf{u}_z \\ &= -2\eta\, \nabla_\perp C_2 - 2\nabla_\perp C_4 \times \mathbf{u}_z \,. \end{aligned} \tag{147}$$

We are now ready to determine some appropriate boundary conditions. Let the boundary $\partial\mathcal{T}$ of $\mathcal{T}$ be a closed curve of class C^0 in $\mathbb{R}^2$ and a finite union of curves of class C^1 (so that a finite number of angular points are permitted), on which we everywhere define an outward unit normal vector $\mathbf{n}$, such that $\mathbf{n}\cdot\mathbf{u}_z = 0$. In a first method, we use Section VIII.F to state that we can use arbitrary solutions for C_3 and C_4, satisfying the homogeneous Dirichlet boundary conditions

$$\forall (x,y)\in\partial\mathcal{T},\ \ C_3(x,y) = K_3 \tag{148}$$

and

$$\forall (x,y)\in\partial\mathcal{T},\ \ C_4(x,y) = K_4\,, \tag{149}$$

where $K_3 \in \mathbb{C}$ and $K_4 \in \mathbb{C}$ are arbitrary constants. The Dirichlet problem corresponding to (125) and (148) has a unique solution, and the Dirichlet problem corresponding to (126) and (149) has a unique solution [30, Ch. 6], [31, Ch. 4]. On $\partial\mathcal{T}$, since C_3 and C_4 are constant, $\nabla_\perp C_3$ and $\nabla_\perp C_4$ are normal to $\partial\mathcal{T}$. Consequently, we have

$$\forall (x,y)\in\partial\mathcal{T},\ \ (\nabla_\perp C_3(x,y)\times\mathbf{u}_z)\cdot\mathbf{n} = 0 \tag{150}$$

and

$$\forall (x,y)\in\partial\mathcal{T},\ \ (\nabla_\perp C_4(x,y)\times\mathbf{u}_z)\cdot\mathbf{n} = 0\,. \tag{151}$$

Using (150)–(151) in (146)–(147), we obtain the inhomogeneous Neumann boundary conditions

$$\forall (x,y)\in\partial\mathcal{T},\ \ [\nabla_\perp C_1(x,y)]\cdot\mathbf{n} = \frac{1}{2\eta}\left[\mathbf{E}_{i\perp}(x,y,0) - \eta\,\mathbf{H}_{i\perp}(x,y,0)\times\mathbf{u}_z\right]\cdot\mathbf{n} \tag{152}$$

and

$$\forall (x,y)\in\partial\mathcal{T},\ \ [\nabla_\perp C_2(x,y)]\cdot\mathbf{n} = -\frac{1}{2\eta}\left[\mathbf{E}_{i\perp}(x,y,0) + \eta\,\mathbf{H}_{i\perp}(x,y,0)\times\mathbf{u}_z\right]\cdot\mathbf{n}\,. \tag{153}$$

Using (123)–(124), we show in Appendix E that the boundary condition (152) satisfies the compatibility condition

$$\oint_{\partial\mathcal{T}} [\nabla_\perp C_1]\cdot\mathbf{n}\,ds = \iint_{\mathcal{T}} \nabla_\perp^2 C_1\,da \tag{154}$$

and that the boundary condition (153) satisfies the compatibility condition

$$\oint_{\partial\mathcal{T}} [\nabla_\perp C_2]\cdot\mathbf{n}\,ds = \iint_{\mathcal{T}} \nabla_\perp^2 C_2\,da\,, \tag{155}$$

in which ds is a length element of $\partial\mathcal{T}$, and da is a surface element of $\mathcal{T}$. It follows that the Neumann problem corresponding to (123) and (152) has solutions which differ by a complex constant, and that the Neumann problem corresponding to (124) and (153) has solutions which differ by a complex constant [30, Ch. 6], [31, Ch. 4].

In the first method, we have used arbitrary Dirichlet boundary conditions for C_3 and C_4 and Neumann boundary conditions for C_1 and C_2, such conditions being appropriate for computing a possible ψ_{TE} and a possible ψ_{TM}.

We can go the other way around. In a second method, instead of positing (148) and (149), we assert that Section VIII.F allows us to specify the homogeneous Dirichlet boundary conditions

$$\forall (x,y)\in\partial\mathcal{T},\ \ C_1(x,y) = K_1 \tag{156}$$

and

$$\forall (x,y)\in\partial\mathcal{T},\ \ C_2(x,y) = K_2\,, \tag{157}$$

where $K_1 \in \mathbb{C}$ and $K_2 \in \mathbb{C}$ are arbitrary constants. The Dirichlet problem corresponding to (123) and (156) has a unique solution, and the Dirichlet problem corresponding to (124) and (157) has a unique solution. On $\partial\mathcal{T}$, since C_1 and C_2 are constant, we get

$$\forall (x,y)\in\partial\mathcal{T},\ \ (\nabla_\perp C_1(x,y)\times\mathbf{u}_z)\cdot\mathbf{n} = 0 \tag{158}$$

and

$$\forall (x,y)\in\partial\mathcal{T},\ \ (\nabla_\perp C_2(x,y)\times\mathbf{u}_z)\cdot\mathbf{n} = 0\,. \tag{159}$$

We then observe that (144) and (145) allow us to write

$$\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z + \eta\,\mathbf{H}_{i\perp}(x,y,0) = 2\eta\,\nabla_\perp C_1\times\mathbf{u}_z + 2\nabla_\perp C_3 \tag{160}$$

and

$$\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z - \eta\,\mathbf{H}_{i\perp}(x,y,0) = -2\eta\,\nabla_\perp C_2\times\mathbf{u}_z + 2\nabla_\perp C_4\,. \tag{161}$$

Using (158)–(159) in (160)–(161), we obtain the inhomogeneous Neumann boundary conditions

$$\forall (x,y)\in\partial\mathcal{T},\ \ [\nabla_\perp C_3(x,y)]\cdot\mathbf{n} = \frac{1}{2}\left[\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z + \eta\,\mathbf{H}_{i\perp}(x,y,0)\right]\cdot\mathbf{n} \tag{162}$$

and

$$\forall (x,y)\in\partial\mathcal{T},\ \ [\nabla_\perp C_4(x,y)]\cdot\mathbf{n} = \frac{1}{2}\left[\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z - \eta\,\mathbf{H}_{i\perp}(x,y,0)\right]\cdot\mathbf{n}\,. \tag{163}$$

Using (125)–(126), we show in Appendix E that the boundary condition (162) satisfies the compatibility condition

$$\oint_{\partial\mathcal{T}} [\nabla_\perp C_3]\cdot\mathbf{n}\,ds = \iint_{\mathcal{T}} \nabla_\perp^2 C_3\,da \tag{164}$$

and that the boundary condition (163) satisfies the compatibility condition

$$\oint_{\partial\mathcal{T}} [\nabla_\perp C_4]\cdot\mathbf{n}\,ds = \iint_{\mathcal{T}} \nabla_\perp^2 C_4\,da\,. \tag{165}$$

It follows that the Neumann problem corresponding to (125) and (162) has solutions which differ by a complex constant, and that the Neumann problem corresponding to (126) and (163) has solutions which differ by a complex constant.

The first method and the second method have both led us to derive boundary conditions that are appropriate for computing a possible ψ_{TE} and a possible ψ_{TM}.

H. FINAL STEP AND COMMENTS

Once a possible ψ_{TE} and a possible ψ_{TM} are obtained, possible $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$ can be computed everywhere in $\overline{\mathcal{D}}$, by utilizing their definitions (shown in Section III.B). Using (55)–(64), we obtain, in the plane $z = 0$,

$$\mathbf{E}_A = -\left(\nabla_\perp C_3 + \nabla_\perp C_4\right) \times \mathbf{u}_z \,, \tag{166}$$

$$\mathbf{H}_A = H_{iz} \mathbf{u}_z \,, \tag{167}$$

$$\mathbf{E}_B = \eta \left(\nabla_\perp C_1 - \nabla_\perp C_2\right) \,, \tag{168}$$

$$\mathbf{H}_C = \frac{1}{\eta} \left(\nabla_\perp C_3 - \nabla_\perp C_4\right) \,, \tag{169}$$

$$\mathbf{E}_D = E_{iz} \mathbf{u}_z \tag{170}$$

and

$$\mathbf{H}_D = \left(\nabla_\perp C_1 + \nabla_\perp C_2\right) \times \mathbf{u}_z \,, \tag{171}$$

which, combined with (58)–(59), completely define the ETHEFs in this plane.

To allow us to compute a possible ψ_{TE} and a possible ψ_{TM} everywhere in $\overline{\mathcal{D}} = \overline{\mathcal{T}} \times [z_N, z_P]$, both methods presented in Section VIII.G only use: the knowledge of E_{iz} and H_{iz} everywhere in $\overline{\mathcal{D}}$; and the knowledge of the normal component $\mathbf{E}_{i\perp} \cdot \mathbf{n}$ of $\mathbf{E}_{i\perp}$ and of the tangential component $(\mathbf{H}_{i\perp} \times \mathbf{u}_z) \cdot \mathbf{n}$ of $\mathbf{H}_{i\perp}$ everywhere in $\partial\mathcal{T}$ in the case of the first method, or the knowledge of the normal component $\mathbf{H}_{i\perp} \cdot \mathbf{n}$ of $\mathbf{H}_{i\perp}$ and of the tangential component $(\mathbf{E}_{i\perp} \times \mathbf{u}_z) \cdot \mathbf{n}$ of $\mathbf{E}_{i\perp}$ everywhere in $\partial\mathcal{T}$ in the case of the second method. At this stage, there is a possibility of checking that $\mathcal{F}_{TE}$ and $\mathcal{F}_{TM}$ resulting from the computed ψ_{TE} and ψ_{TM} satisfy (3), that is $\mathbf{E}_i = \mathbf{E}_{TE} + \mathbf{E}_{TM}$ and $\mathbf{H}_i = \mathbf{H}_{TE} + \mathbf{H}_{TM}$.

In either method, four Poisson equations must be solved in $\mathcal{T}$, two with homogeneous Dirichlet boundary conditions, and two with inhomogeneous Neumann boundary conditions.

This computational burden may be decreased if we only need to compute the ETHEFs. To achieve this reduction, we can implement an alternative method in which either ψ_{TM} or ψ_{TE} is computed. Let us for instance choose to only compute ψ_{TM}. Based on Section VIII.F, we can use arbitrary solutions for C_1 and C_2, corresponding to any convenient boundary conditions, or to no explicit boundary condition. This allows us to compute ψ_{TM} everywhere in $\overline{\mathcal{D}}$, and then $\mathbf{E}_{TM}$ and $\mathbf{H}_{TM}$ everywhere in $\overline{\mathcal{D}}$. We can then use (3) to compute $\mathbf{E}_{TE} = \mathbf{E}_i - \mathbf{E}_{TM}$ and $\mathbf{H}_{TE} = \mathbf{H}_i - \mathbf{H}_{TM}$ everywhere in $\overline{\mathcal{D}}$, to obtain the ETHEFs everywhere in $\overline{\mathcal{D}}$.

In this alternative method, to obtain possible $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$ everywhere in $\overline{\mathcal{D}}$, we need the knowledge of $\mathbf{E}_i$ and $\mathbf{H}_i$ everywhere in $\overline{\mathcal{D}}$.

The methods of Section VIII.G and the alternative method are suitable for the direct computation of the ETHEF decomposition of any incident electromagnetic field. They are general and direct in the sense that they directly use the fields $\mathbf{E}_i$ and $\mathbf{H}_i$ of the incident electromagnetic field $\mathcal{F}_i = (\mathbf{E}_i, \mathbf{H}_i)$, without preliminary step (such as the computation of a plane wave spectrum representation).

IX. SOME ETHF COMPUTATION EXAMPLES

A. GOAL

Let $\mathbf{k}_i = k_{ix}\mathbf{u}_x + k_{iy}\mathbf{u}_y + k_{iz}\mathbf{u}_z$ be an arbitrary complex wave vector which satisfies (77). Let $\mathbf{E}_{i0}$ be an arbitrary complex vector having the dimensions of electric field and such that $\mathbf{E}_{i0} \cdot \mathbf{k}_i = 0$. Let $\mathbf{r} = x\mathbf{u}_x + y\mathbf{u}_y + z\mathbf{u}_z$ be the radius vector of the observation point. We consider an incident electromagnetic field $\mathcal{F}_i = (\mathbf{E}_i, \mathbf{H}_i)$ in $\mathcal{D}$, such that

$$\mathbf{E}_i = \mathbf{E}_{i0}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \tag{172}$$

and

$$\mathbf{H}_i = \frac{\mathbf{k}_i}{\omega\mu} \times \mathbf{E}_i \,, \tag{173}$$

which is a plane wave, not necessarily uniform. We want to implement the three methods proposed in Section VIII to compute $\mathcal{F}_{TM}$, $\mathcal{F}_{TE}$ and the ETHEFs.

B. FIRST PROBLEM

In this first problem, we assume $\mathbf{k}_i = \pm k\mathbf{u}_z$, so that $\mathcal{F}_i$ is TEM to z. In $\overline{\mathcal{D}}$, we have: $E_{iz} = 0$; $\partial E_{iz}/\partial z = 0$; $H_{iz} = 0$; and $\partial H_{iz}/\partial z = 0$. It follows that $\mathfrak{P} E_{iz} = 0$, that $\mathfrak{P} H_{iz} = 0$ and that (123)–(126) are Laplace equations.

We select the second method of Section VIII.G, $K_1 = 0$ and $K_2 = 0$, so that

$$\psi_{TM} = 0 \,. \tag{174}$$

Using (162)–(163), and

$$\mathbf{H}_i = \frac{\pm 1}{\eta}\, \mathbf{u}_z \times \mathbf{E}_{i0}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{175}$$

we obtain the Neumann boundary conditions

$$\forall (x, y) \in \partial\mathcal{T}, \;\; [\nabla_\perp C_3(x, y)] \cdot \mathbf{n} = \frac{1 \mp 1}{2} [\mathbf{E}_{i0} \times \mathbf{u}_z] \cdot \mathbf{n} \tag{176}$$

and

$$\forall (x, y) \in \partial\mathcal{T}, \;\; [\nabla_\perp C_4(x, y)] \cdot \mathbf{n} = \frac{1 \pm 1}{2} [\mathbf{E}_{i0} \times \mathbf{u}_z] \cdot \mathbf{n} \,. \tag{177}$$

We find that

$$C_3 = \frac{1 \mp 1}{2} (\mathbf{E}_{i0} \times \mathbf{u}_z) \cdot \mathbf{r} \tag{178}$$

is a solution of the Neumann problem corresponding to (125) and (176), and

$$C_4 = \frac{1 \pm 1}{2} (\mathbf{E}_{i0} \times \mathbf{u}_z) \cdot \mathbf{r} \tag{179}$$

is a solution of the Neumann problem corresponding to (126) and (177). It follows that a possible ψ_{TE} is given by

$$\psi_{TE} = (\mathbf{E}_{i0} \times \mathbf{u}_z) \cdot \mathbf{r}\, e^{\mp jkz} \,, \tag{180}$$

which may also be written

$$\psi_{TE} = (\mathbf{e}_{TEM} \times \mathbf{u}_z) \cdot \mathbf{r}\, e^{-j\mathbf{k}_i \cdot \mathbf{r}} \,, \tag{181}$$

where $\mathbf{e}_{TEM}$ is used to denote $\mathbf{E}_{i0}$.

$\mathcal{F}_{TM}$, $\mathcal{F}_{TE}$ and the ETHEFs resulting from (174) and (181) were investigated in Section VI.

C. SECOND PROBLEM

In this second problem, we assume that $\mathbf{k}_{i\perp} \neq \mathbf{0}$. We first compute $\mathfrak{P}E_{iz}$ and $\mathfrak{P}H_{iz}$. By (172), we have

$$\int_0^v E_{iz}(x,y,u)e^{jku}\,du = \mathbf{E}_{i0}\cdot\mathbf{u}_z\,e^{-j(k_{ix}x+k_{iy}y)}\frac{e^{j(k-k_{iz})v}-1}{j(k-k_{iz})} \quad (182)$$

and

$$\int_0^z e^{-2jkv}\int_0^v E_{iz}(x,y,u)e^{jku}\,du\,dv = \mathbf{E}_{i0}\cdot\mathbf{u}_z \times \frac{e^{-j(k_{ix}x+k_{iy}y)}}{k-k_{iz}}\left[\frac{e^{-j(k+k_{iz})z}-1}{k+k_{iz}} - \frac{e^{-2jkz}-1}{2k}\right]. \quad (183)$$

By (109) we obtain

$$(\mathfrak{P}E_{iz})(x,y,z) = \mathbf{E}_{i0}\cdot\mathbf{u}_z\;e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \times\left[\frac{e^{-jk_{iz}z}}{k^2-k_{iz}^2} - \frac{e^{jkz}}{2k(k+k_{iz})} - \frac{e^{-jkz}}{2k(k-k_{iz})}\right]. \quad (184)$$

Likewise, using (109) and (173), we get

$$(\mathfrak{P}H_{iz})(x,y,z) = (\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\;e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \times\frac{1}{\omega\mu}\left[\frac{e^{-jk_{iz}z}}{k^2-k_{iz}^2} - \frac{e^{jkz}}{2k(k+k_{iz})} - \frac{e^{-jkz}}{2k(k-k_{iz})}\right]. \quad (185)$$

Using (172)–(173) in (123)–(126), we obtain the Poisson equations:

$$\nabla_\perp^2 C_1 + \frac{j\omega\epsilon}{2}\left[1-\frac{k_{iz}}{k}\right]\mathbf{E}_{i0}\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} = 0\,, \quad (186)$$

$$\nabla_\perp^2 C_2 + \frac{j\omega\epsilon}{2}\left[1+\frac{k_{iz}}{k}\right]\mathbf{E}_{i0}\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} = 0\,, \quad (187)$$

$$\nabla_\perp^2 C_3 + \frac{j}{2}\left[1-\frac{k_{iz}}{k}\right](\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} = 0 \quad (188)$$

and

$$\nabla_\perp^2 C_4 + \frac{j}{2}\left[1+\frac{k_{iz}}{k}\right](\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} = 0\,. \quad (189)$$

D. FIRST SOLUTION TO THE SECOND PROBLEM

To treat the second problem, we select the alternative method of Section VIII.H, and decide to use the arbitrary solutions

$$C_1 = \frac{j\omega\epsilon}{2(k_{ix}^2+k_{iy}^2)}\left[1-\frac{k_{iz}}{k}\right]\mathbf{E}_{i0}\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \quad (190)$$

and

$$C_2 = \frac{j\omega\epsilon}{2(k_{ix}^2+k_{iy}^2)}\left[1+\frac{k_{iz}}{k}\right]\mathbf{E}_{i0}\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \quad (191)$$

of (186)–(187), which may be written

$$C_1 = \frac{j\omega\epsilon}{2k(k+k_{iz})}\mathbf{E}_{i0}\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \quad (192)$$

and

$$C_2 = \frac{j\omega\epsilon}{2k(k-k_{iz})}\mathbf{E}_{i0}\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\,. \quad (193)$$

Using (184), (192) and (193) in (112), we obtain

$$\psi_{TM} = j\omega\epsilon\,\mathbf{E}_{i0}\cdot\mathbf{u}_z\;e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\;\frac{e^{-jk_{iz}z}}{k^2-k_{iz}^2}\,. \quad (194)$$

It is remarkable that this simple solution results from the fact that a cancellation has occurred between all terms containing the factor $\exp(jkz)$ or the factor $\exp(-jkz)$ in (112). If we introduce

$$i_{TM} = \frac{j\omega\epsilon\,\mathbf{E}_{i0}\cdot\mathbf{u}_z}{\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp}}\,, \quad (195)$$

we obtain

$$\psi_{TM} = i_{TM}\;e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,. \quad (196)$$

$\mathcal{F}_{TM}$, $\mathcal{F}_B$ and $\mathcal{F}_D$ resulting from (196) were investigated in Section VI. $\mathcal{F}_{TE}$ and then $\mathcal{F}_A$ and $\mathcal{F}_C$ can be determined as explained in Section VIII.H.

E. SECOND SOLUTION TO THE SECOND PROBLEM

To treat the second problem differently, we now select the first method of Section VIII.G. We must solve two Dirichlet problems, defined by (188)-(189) and some homogeneous boundary conditions. We choose $K_3 = 0$ and $K_4 = 0$. Thus, the unique solution of the Dirichlet problem for C_3 is

$$C_3 = Q_3 + \frac{j}{2k(k+k_{iz})}(\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\,, \quad (197)$$

where Q_3 is given by the 2D Laplace equation $\nabla_\perp^2 Q_3 = 0$ with the inhomogeneous Dirichlet boundary condition

$$\forall(x,y)\in\partial\mathcal{T},\quad Q_3(x,y) = \frac{-j}{2k(k+k_{iz})}(\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\,, \quad (198)$$

and the unique solution of the Dirichlet problem for C_4 is

$$C_4 = Q_4 + \frac{j}{2k(k-k_{iz})}(\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\,, \quad (199)$$

where Q_4 is given by the 2D Laplace equation $\nabla_\perp^2 Q_4 = 0$ with the inhomogeneous Dirichlet boundary condition

$$\forall(x,y)\in\partial\mathcal{T},\quad Q_4(x,y) = \frac{-j}{2k(k-k_{iz})}(\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\,e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\,. \quad (200)$$

Using (185), (197) and (199) in (113), we obtain

$$\psi_{TE} = Q_3(x,y)e^{jkz} + Q_4(x,y)e^{-jkz} + j\,(\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z\;e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\;\frac{e^{-jk_{iz}z}}{k^2-k_{iz}^2}\,. \quad (201)$$

If we introduce

$$v_{TE} = j\,\frac{(\mathbf{k}_i\times\mathbf{E}_{i0})\cdot\mathbf{u}_z}{\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp}} = -j\,\frac{\mathbf{E}_{i0}\cdot(\mathbf{k}_{i\perp}\times\mathbf{u}_z)}{\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp}}\,, \quad (202)$$

we obtain

$$\psi_{TE} = Q_3e^{jkz} + Q_4e^{-jkz} + v_{TE}\;e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,. \quad (203)$$

According to the first method of Section VIII.G, we must now solve two Neumann problems, defined by (186)-(187) and the inhomogeneous Neumann boundary conditions (152)-(153). In the present case, these boundary conditions are

$$\forall(x,y) \in \partial\mathcal{T},\ \ [\nabla_\perp C_1(x,y)] \cdot \mathbf{n} = \frac{1}{2\eta}\left[\left(1-\frac{k_{iz}}{k}\right)\mathbf{E}_{i0} + (\mathbf{E}_{i0}\cdot\mathbf{u}_z)\frac{\mathbf{k}_i}{k}\right]\cdot\mathbf{n}\, e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \tag{204}$$

and

$$\forall(x,y) \in \partial\mathcal{T},\ \ [\nabla_\perp C_2(x,y)] \cdot \mathbf{n} = \frac{-1}{2\eta}\left[\left(1+\frac{k_{iz}}{k}\right)\mathbf{E}_{i0} - (\mathbf{E}_{i0}\cdot\mathbf{u}_z)\frac{\mathbf{k}_i}{k}\right]\cdot\mathbf{n}\, e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}. \tag{205}$$

Thus, a solution of the Neumann problem for C_1 is

$$C_1 = Q_1 + \frac{j\omega\epsilon}{2k(k+k_{iz})}\,\mathbf{E}_{i0}\cdot\mathbf{u}_z\, e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}, \tag{206}$$

where Q_1 is given by the 2D Laplace equation $\nabla_\perp^2 Q_1 = 0$ with the inhomogeneous Neumann boundary condition

$$\forall(x,y) \in \partial\mathcal{T},\ \ [\nabla_\perp Q_1(x,y)] \cdot \mathbf{n} = \frac{1}{2\eta}e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \times \left[\left(1-\frac{k_{iz}}{k}\right)\mathbf{E}_{i0} + (\mathbf{E}_{i0}\cdot\mathbf{u}_z)\frac{k_{iz}\mathbf{k}_{i\perp}}{k(k+k_{iz})}\right]\cdot\mathbf{n}, \tag{207}$$

and a solution of the Neumann problem for C_2 is

$$C_2 = Q_2 + \frac{j\omega\epsilon}{2k(k-k_{iz})}\,\mathbf{E}_{i0}\cdot\mathbf{u}_z\, e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}, \tag{208}$$

where Q_2 is given by the 2D Laplace equation $\nabla_\perp^2 Q_2 = 0$ with the inhomogeneous Neumann boundary condition

$$\forall(x,y) \in \partial\mathcal{T},\ \ [\nabla_\perp Q_2(x,y)] \cdot \mathbf{n} = \frac{-1}{2\eta}e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}} \times \left[\left(1+\frac{k_{iz}}{k}\right)\mathbf{E}_{i0} + (\mathbf{E}_{i0}\cdot\mathbf{u}_z)\frac{k_{iz}\mathbf{k}_{i\perp}}{k(k-k_{iz})}\right]\cdot\mathbf{n}. \tag{209}$$

Using (184), (206) and (208) in (112), we obtain

$$\psi_{TM} = Q_1(x,y)e^{jkz} + Q_2(x,y)e^{-jkz} + j\omega\epsilon\,\mathbf{E}_{i0}\cdot\mathbf{u}_z\ e^{-j\mathbf{k}_{i\perp}\cdot\mathbf{r}}\ \frac{e^{-jk_{iz}z}}{k^2-k_{iz}^2}. \tag{210}$$

If we use i_{TM} defined by (195), we obtain

$$\psi_{TM} = Q_1 e^{jkz} + Q_2 e^{-jkz} + i_{TM}\ e^{-j\mathbf{k}_i\cdot\mathbf{r}}. \tag{211}$$

This completes the computations of the first method of Section VIII.G. It has provided a possible ψ_{TE} and a possible ψ_{TM} that are different from the ones used in Section VI, ψ_{TM} being also different from the one obtained in Section IX.D.

Since we assume that $\mathbf{k}_{i\perp} \neq \mathbf{0}$ in this second problem, we see that, $\forall(x,y) \in \overline{\mathcal{T}}$, $\mathbf{E}_i(x,y,z)$ and $\mathbf{H}_i(x,y,z)$ have a single nonzero Fourier component with respect to the variable z, at the wave number $-k_{iz} \notin \{-k,k\}$. It follows that the TEM-to-z waves propagating in the direction of $-\mathbf{u}_z$ caused by Q_1 and Q_3 must cancel out in $\mathcal{F}_i$, and that the TEM-to-z waves propagating in the direction of $\mathbf{u}_z$ caused by Q_2 and Q_4 must cancel out in $\mathcal{F}_i$. Consequently, it would be possible to simultaneously remove the terms containing Q_3 and Q_4 in (203) and the terms containing Q_1 and Q_2 in (211). $\mathcal{F}_{TM}$, $\mathcal{F}_{TE}$ and the ETHEFs resulting from (203) and (211) modified in this manner were investigated in Section VI.

X. CONCLUSION

We have analyzed the response of a planar wire loop antenna used for reception, defined as its open-circuit voltage. This article is therefore relevant to antenna theory and applications where accuracy is important, among which electromagnetic compatibility, electromagnetic field measurements, antenna calibration, direction finding, etc.

We have presented and used a new and specialized decomposition of an arbitrary incident time-harmonic electromagnetic field $\mathcal{F}_i$ into the ETHEFs $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$, which was shown to be useful to study the response of an arbitrary planar wire loop antenna used for reception.

We have obtained a formula (70) which gives the response of the arbitrary planar wire loop antenna receiving $\mathcal{F}_i$. This formula is applicable to any incident field configuration, and valid at any frequency at which the thin wire approximation applies. It separates the response of the antenna into three parts: a surface integral of $\mathbf{H}_A$, a line integral of $\mathbf{E}_A$ and a line integral of $\mathbf{E}_B$. This decomposition shows that, at any frequency, only $\mathcal{F}_A$ and $\mathcal{F}_B$ excite the antenna.

It is possible, especially at low frequencies, to consider that $\mathcal{F}_A$ causes the intended response of the antenna, while $\mathcal{F}_B$ may cause an unwanted response. In (70) the effect of $\mathcal{F}_A$ is subdivided into the surface integral of $\mathbf{H}_A$, which may be viewed as the intended response of the antenna, and the line integral of $\mathbf{E}_A$, which can be viewed as a correction term for the gap width and the nonuniformity of the high-frequency current distribution, since this line integral vanishes if the current is uniform over the integration path. We have also compared (70) to an alternative formulation (76).

This paper is directed at a planar wire loop antenna used as a measuring antenna or as a direction finder. It was recognized a long time ago that, if the antenna is not very small (e.g., a circular loop antenna of diameter less than 0.01λ, if $\mathcal{F}_i$ is a uniform plane wave [2]), the goal "measuring a magnetic component of $\mathcal{F}_i$", used in the introduction section, is not consistent with the actual capabilities of the loop antenna. This paper teaches that, up to larger antenna sizes, a reasonable purpose of the measurement is to obtain information about $\mathcal{F}_A$, in the presence of unwanted effects of $\mathcal{F}_B$ (and possibly of $\mathcal{F}_C$ or $\mathcal{F}_D$, since an actual antenna is different from the theoretical planar wire antenna that we have assumed).

If we except calibration procedures, little information about $\mathcal{F}_i$ is typically available before a measurement, so that the question of computing $\mathcal{F}_A$ and $\mathcal{F}_B$ does not arise. In contrast, in the context of a calibration operation during which the planar wire loop antenna is used for reception, a computation of $\mathcal{F}_A$ and $\mathcal{F}_B$ is possible and useful to analyze the calibration. This is why we have carefully explained how this calculation can be carried out.

APPENDIX A

This appendix presents a proof of (1), based on the approach used in [16, Sec. 13.06]. Here, the field source is represented by an impressed current density $\mathbf{J}_{FS}$. We assume that the medium surrounding the wire antenna is reciprocal. We assume that the field source is contained within a finite volume, so that we can use the field reciprocity relation given by [16, Eq. (13-36)], which will be referred to as the "Lorentz reciprocity theorem" [18, Sec. 3.8], [32, Sec. 13.1].

In a configuration A (CA), we have: the open-circuited wire antenna; passive nearby objects, if they exist; and the impressed current density $\mathbf{J}_{FS}$. The electric field in CA is denoted by $\mathbf{E}_{ca}$. In a configuration B (CB), we have: the wire antenna coupled to a current source delivering an impressed current density $\mathbf{J}_0$; and the passive nearby objects, if they exist. In CB, the electric field is denoted by $\mathbf{E}_{cb}$, and the current density in the antenna is denoted by $\mathbf{J}_t$.

Applying the Lorentz reciprocity theorem to CA and CB, we get

$$\iiint \mathbf{E}_{ca} \cdot \mathbf{J}_0 \, dv = \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{FS} \, dv \,, \tag{212}$$

which leads us to

$$-e_{ant} I_0 = \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{FS} \, dv \,, \tag{213}$$

where e_{ant} is the open-circuit voltage of the wire antenna used for reception in CA, and I_0 is the current flowing into the positive terminal of the antenna port, in CB.

In a configuration C (CC), we have: the passive nearby objects, if they exist; and the impressed current density $\mathbf{J}_{FS}$. The electric field in CC is denoted by $\mathbf{E}_i$. In a configuration D (CD), we have: the passive nearby objects, if they exist; the impressed current density $\mathbf{J}_0$; and an additional impressed current density equal to $\mathbf{J}_t$. In CD, the electric field is equal to $\mathbf{E}_{cb}$, because the additional impressed current density is equal to the current density in the antenna in CB.

Applying the Lorentz reciprocity theorem to CC and CD, we get

$$\iiint \mathbf{E}_i \cdot (\mathbf{J}_0 + \mathbf{J}_t) \, dv = \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{FS} \, dv \,. \tag{214}$$

For a narrow gap, the contribution of $\mathbf{E}_i \cdot \mathbf{J}_0$ to the left-hand side of (214) may be ignored, so that

$$\iiint \mathbf{E}_i \cdot \mathbf{J}_t \, dv \simeq \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{FS} \, dv \,. \tag{215}$$

Using (213) and (215), we obtain

$$e_{ant} \simeq -\frac{1}{I_0} \iiint_{\text{Antenna}} \mathbf{J}_t \cdot \mathbf{E}_i \, dv \,. \tag{216}$$

Thus, we see that, in this Appendix A:

- $\mathbf{E}_i$ is the electric field produced by the field source, in the absence of the wire antenna, but in the presence of the passive nearby objects, if they exist; and
- (1) is accurate if the gap is narrow and if $\mathbf{J}_t$ and I_0 are determined in the presence of the passive nearby objects, if they exist.

APPENDIX B

This appendix presents a new proof of (1), based on more general assumptions than the ones used in Appendix A. Here, the field source is represented by a reciprocal transmitting antenna (TA) coupled to a generator of internal impedance Z_G, "reciprocal antenna" meaning an antenna to which we could apply the Lorentz reciprocity theorem if it was used in free space [32, Sec. 13.1]. We assume that the TA is contained within a finite volume and that the medium surrounding the wire antenna is reciprocal, so that we can use the Lorentz reciprocity theorem.

In a configuration A (CA), we have: the open-circuited wire antenna; passive nearby objects, if they exist; and the active field source. The active field source is composed of the TA having its port coupled to an impedance Z_G connected in parallel with a current source delivering an impressed current density $\mathbf{J}_{ca}$. The electric field in CA is denoted by $\mathbf{E}_{ca}$. In a configuration B (CB), we have: the wire antenna coupled to a current source delivering an impressed current density $\mathbf{J}_0$; the passive nearby objects, if they exist; and the inactive field source. The inactive field source is composed of the TA having its port coupled to the impedance Z_G. In CB, the electric field is denoted by $\mathbf{E}_{cb}$, and the current density in the antenna is denoted by $\mathbf{J}_t$.

Applying the Lorentz reciprocity theorem to CA and CB, we get

$$\iiint \mathbf{E}_{ca} \cdot \mathbf{J}_0 \, dv = \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{ca} \, dv \,, \tag{217}$$

which leads us to

$$-e_{ant} I_0 = \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{ca} \, dv \,, \tag{218}$$

where e_{ant} is the open-circuit voltage of the wire antenna used for reception in CA, and I_0 is the current flowing into the positive terminal of the antenna port, in CB.

In a configuration C (CC), we have: the passive nearby objects, if they exist; and the active field source defined as in CA. The electric field in CC is denoted by $\mathbf{E}_i$. In a configuration D (CD), we have: the passive nearby objects, if they exist; the impressed current density $\mathbf{J}_0$; an additional impressed current density equal to $\mathbf{J}_t$; and the inactive field source defined as in CB. In CD, the electric field is equal to $\mathbf{E}_{cb}$, because the additional impressed current density is equal to the current density in the antenna in CB.

Applying the Lorentz reciprocity theorem to CC and CD, we get

$$\iiint \mathbf{E}_i \cdot (\mathbf{J}_0 + \mathbf{J}_t) \, dv = \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{ca} \, dv \,. \tag{219}$$

For a narrow gap, the contribution of $\mathbf{E}_i \cdot \mathbf{J}_0$ to the left-hand side of (219) may be ignored, so that

$$\iiint \mathbf{E}_i \cdot \mathbf{J}_t \, dv \simeq \iiint \mathbf{E}_{cb} \cdot \mathbf{J}_{ca} \, dv \,. \tag{220}$$

Using (218) and (220), we obtain

$$e_{ant} \simeq -\frac{1}{I_0} \iiint_{\text{Antenna}} \mathbf{J}_t \cdot \mathbf{E}_i \, dv \,. \tag{221}$$

Thus, we see that, in this Appendix B:

- $\mathbf{E}_i$ is the electric field produced by the field source, in the absence of the wire antenna, but in the presence of the passive nearby objects, if they exist; and
- (1) is accurate if the gap is narrow and if $\mathbf{J}_t$ and I_0 are determined in the presence of the inactive field source and of the passive nearby objects, if they exist.

APPENDIX C

In this Appendix C, we use the assumptions of Section VI, where $\mathbf{k}_i$ is an arbitrary complex vector that satisfies (77). Using (24)-(31), (80)-(86) and (89), we find that, in the case of an incident plane wave, $\mathcal{F}_A$, $\mathcal{F}_B$, $\mathcal{F}_C$ and $\mathcal{F}_D$ everywhere in space are, if $\mathbf{k}_{i\perp} \neq \mathbf{0}$, given by

$$\mathbf{E}_A = j v_{TE}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} + e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2}\,, \tag{222}$$

$$\mathbf{H}_A = \frac{v_{TE}}{2j\omega\mu} \Big([(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z - k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_i\cdot\mathbf{r}} + [(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z + k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_m\cdot\mathbf{r}} \Big)\,, \tag{223}$$

$$\mathbf{E}_B = \frac{i_{TM}}{2j\omega\epsilon} \Big([(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z - k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_i\cdot\mathbf{r}} - [(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z + k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_m\cdot\mathbf{r}} \Big)\,, \tag{224}$$

$$\mathbf{H}_B = -j i_{TM}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} - e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2}\,, \tag{225}$$

$$\mathbf{E}_C = j v_{TE}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} - e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2}\,, \tag{226}$$

$$\mathbf{H}_C = \frac{v_{TE}}{2j\omega\mu} \Big([(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z - k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_i\cdot\mathbf{r}} - [(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z + k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_m\cdot\mathbf{r}} \Big)\,, \tag{227}$$

$$\mathbf{E}_D = \frac{i_{TM}}{2j\omega\epsilon} \Big([(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z - k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_i\cdot\mathbf{r}} + [(\mathbf{k}_{i\perp}\cdot\mathbf{k}_{i\perp})\mathbf{u}_z + k_{iz}\mathbf{k}_{i\perp}]\, e^{-j\mathbf{k}_m\cdot\mathbf{r}} \Big) \tag{228}$$

and

$$\mathbf{H}_D = -j i_{TM}\, \mathbf{k}_{i\perp} \times \mathbf{u}_z \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} + e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2}\,, \tag{229}$$

whereas, if $\mathbf{k}_{i\perp} = \mathbf{0}$, we obtain

$$\mathbf{E}_A = \mathbf{e}_{TEM} \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} + e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2}\,, \tag{230}$$

$$\mathbf{H}_A = \frac{-k_{iz}}{\omega\mu}\, \mathbf{e}_{TEM} \times \mathbf{u}_z \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} - e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2}\,, \tag{231}$$

$$\mathbf{E}_B = \mathbf{0} \quad\text{and}\quad \mathbf{H}_B = \mathbf{0}\,, \tag{232}$$

$$\mathbf{E}_C = \mathbf{e}_{TEM} \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} - e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2}\,, \tag{233}$$

$$\mathbf{H}_C = \frac{-k_{iz}}{\omega\mu}\, \mathbf{e}_{TEM} \times \mathbf{u}_z \, \frac{e^{-j\mathbf{k}_i\cdot\mathbf{r}} + e^{-j\mathbf{k}_m\cdot\mathbf{r}}}{2} \tag{234}$$

and

$$\mathbf{E}_D = \mathbf{0} \text{ and } \mathbf{H}_D = \mathbf{0}\,. \tag{235}$$

APPENDIX D

Let (r, θ, φ) be the spherical coordinates associated with the rectangular coordinate system $Oxyz$, the unit vectors of the spherical coordinates being denoted by $\mathbf{u}_r$, $\mathbf{u}_\theta$ and $\mathbf{u}_\varphi$. In this Appendix D, we use the assumptions of Section VI, and we further assume that $\mathbf{k}_i$ is real and

$$\mathbf{k}_i = -k\, \mathbf{u}_{ri}\,, \tag{236}$$

in which $\mathbf{u}_{ri}$ is the unit vector in the direction $\theta = \theta_i$ and $\varphi = \varphi_i$. Thus, $\mathbf{k}_i$ is an arbitrary real vector that satisfies (77), and $\mathcal{F}_i$ is a uniform time-harmonic plane wave propagating from the direction $\theta = \theta_i$ and $\varphi = \varphi_i$.

Let (ρ, φ, z) be the cylindrical coordinates associated with the rectangular coordinate system $Oxyz$, the unit vectors of the cylindrical coordinates being $\mathbf{u}_\rho$, $\mathbf{u}_\varphi$, $\mathbf{u}_z$. Let $\mathbf{u}_{\rho i}$ be the value of $\mathbf{u}_{\rho i}$ for the direction $\theta = \theta_i$ and $\varphi = \varphi_i$. We have

$$\mathbf{k}_{i\perp} = -k \sin\theta_i\, \mathbf{u}_{\rho i}\,. \tag{237}$$

In the case where $\mathbf{k}_{i\perp} \neq \mathbf{0}$, or equivalently in the case in which $\theta_i \neq p\pi$, where p is an integer, we can define $E_{i0\theta}$ and $E_{i0\varphi}$ such that $\mathbf{E}_{i0} = E_{i0\theta}\mathbf{u}_{\theta i} + E_{i0\varphi}\mathbf{u}_{\varphi i}$, and it follows from (97)–(98) and (237) that

$$v_{TE} = \frac{E_{i0\varphi}}{jk\sin\theta_i} \tag{238}$$

and

$$i_{TM} = \frac{E_{i0\theta}}{jk\eta\sin\theta_i}\,. \tag{239}$$

In the case where $\mathbf{k}_{i\perp} = \mathbf{0}$, or equivalently in the case in which $\theta_i = p\pi$, where p is an integer, φ_i is arbitrary. It follows from (96) that we can in this case assert $E_{i0\theta} = 0$ as well as define $E_{i0\varphi}$ and select φ_i in such a way that

$$\mathbf{e}_{TEM} = \mathbf{E}_{i0} = E_{i0\varphi}\mathbf{u}_{\varphi i}\,. \tag{240}$$

$E_{i0\theta}$, $E_{i0\varphi}$ and φ_i being now uniquely defined for any value of θ_i, we can use (80)–(83) and (237)–(240) to obtain

$$\mathbf{E}_{TE} = E_{i0\varphi}\mathbf{u}_{\varphi i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,, \tag{241}$$

$$\mathbf{H}_{TE} = \frac{1}{\eta}\, E_{i0\varphi}\mathbf{u}_{\theta i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,, \tag{242}$$

$$\mathbf{E}_{TM} = E_{i0\theta}\mathbf{u}_{\theta i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}} \tag{243}$$

and

$$\mathbf{H}_{TM} = -\frac{1}{\eta}\, E_{i0\theta}\mathbf{u}_{\varphi i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,. \tag{244}$$

Using (90)–(95) and (237)–(240) we obtain the following simple results, valid in the plane $z = 0$:

$$\mathbf{E}_A = E_{i0\varphi}\mathbf{u}_{\varphi i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,, \tag{245}$$

$$\mathbf{H}_A = -\frac{1}{\eta}\, E_{i0\varphi}\sin\theta_i\mathbf{u}_z\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,, \tag{246}$$

$$\mathbf{E}_B = E_{i0\theta}\cos\theta_i\mathbf{u}_{\rho i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,, \tag{247}$$

$$\mathbf{H}_C = \frac{1}{\eta} E_{i0\varphi}\cos\theta_i\mathbf{u}_{\rho i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,, \tag{248}$$

$$\mathbf{E}_D = -E_{i0\theta}\sin\theta_i\mathbf{u}_z\, e^{-j\mathbf{k}_i\cdot\mathbf{r}} \tag{249}$$

and

$$\mathbf{H}_D = -\frac{1}{\eta}\, E_{i0\theta}\mathbf{u}_{\varphi i}\, e^{-j\mathbf{k}_i\cdot\mathbf{r}}\,. \tag{250}$$

APPENDIX E

In this Appendix E, we want to show that:

- the boundary condition (152) for C_1 satisfies the compatibility condition (154) for C_1;
- the boundary condition (153) for C_2 satisfies the compatibility condition (155) for C_2;
- the boundary condition (162) for C_3 satisfies the compatibility condition (164) for C_3; and
- the boundary condition (163) for C_4 satisfies the compatibility condition (165) for C_4.

For any $\alpha \in \{1, \ldots, 4\}$, the compatibility condition for C_α, that is to say

$$\oint_{\partial\mathcal{T}} [\nabla_\perp C_\alpha] \cdot \mathbf{n}\, ds = \iint_{\mathcal{T}} \nabla_\perp^2 C_\alpha \, da \,, \tag{251}$$

is meant to determine whether the corresponding Newmann boundary condition is compatible with the corresponding Poisson equation, hence at a stage where the existence of a solution C_α of class C^2 in $\mathcal{T}$ and of class C^1 in $\overline{\mathcal{T}}$ is not yet established. Thus, we cannot assert that the compatibility condition for C_α directly follows from the 2D divergence theorem (Gauss-Ostrogradsky theorem) applied to $\nabla_\perp C_\alpha$.

According to these considerations, we can reformulate the compatibility condition as follows:

- by (123) and (152) the compatibility condition (154) for C_1 actually means

$$\frac{1}{2\eta}\oint_{\partial\mathcal{T}} [\mathbf{E}_{i\perp}(x,y,0) - \eta\, \mathbf{H}_{i\perp}(x,y,0) \times \mathbf{u}_z] \cdot \mathbf{n}\, ds = -\frac{j\omega\epsilon}{2}\iint_{\mathcal{T}} \left[E_{iz} + \frac{1}{jk}\frac{\partial E_{iz}}{\partial z}\right](x,y,0)\, da \,; \tag{252}$$

- by (124) and (153) the compatibility condition (155) for C_2 actually means

$$-\frac{1}{2\eta}\oint_{\partial\mathcal{T}} [\mathbf{E}_{i\perp}(x,y,0) + \eta\, \mathbf{H}_{i\perp}(x,y,0) \times \mathbf{u}_z] \cdot \mathbf{n}\, ds = -\frac{j\omega\epsilon}{2}\iint_{\mathcal{T}} \left[E_{iz} - \frac{1}{jk}\frac{\partial E_{iz}}{\partial z}\right](x,y,0)\, da \,; \tag{253}$$

- by (125) and (162) the compatibility condition (164) for C_3 actually means

$$\frac{1}{2}\oint_{\partial\mathcal{T}} [\mathbf{E}_{i\perp}(x,y,0) \times \mathbf{u}_z + \eta\, \mathbf{H}_{i\perp}(x,y,0)] \cdot \mathbf{n}\, ds = -\frac{j\omega\mu}{2}\iint_{\mathcal{T}} \left[H_{iz} + \frac{1}{jk}\frac{\partial H_{iz}}{\partial z}\right](x,y,0)\, da \,; \tag{254}$$

and

- by (126) and (163) the compatibility condition (165) for C_4 actually means

$$\frac{1}{2}\oint_{\partial\mathcal{T}} [\mathbf{E}_{i\perp}(x,y,0) \times \mathbf{u}_z - \eta\, \mathbf{H}_{i\perp}(x,y,0)] \cdot \mathbf{n}\, ds = -\frac{j\omega\mu}{2}\iint_{\mathcal{T}} \left[H_{iz} - \frac{1}{jk}\frac{\partial H_{iz}}{\partial z}\right](x,y,0)\, da \,. \tag{255}$$

Since $\mathbf{E}_i$ and $\mathbf{H}_i$ are of class C^1 in $\mathcal{O}$ (see Section VIII.A), we observe that we can apply the 2D divergence theorem (as it may be derived from the 3D divergence theorem set forth in [33, Sec. 10.7] or [34, Ch. 5]) to the left-hand sides of (252)–(255).

We can now prove that the compatibility conditions for C_1 and C_2 are met. It follows from $\nabla \cdot \mathbf{E}_i = 0$ in $\mathcal{D}$ that

$$\nabla_\perp \cdot \mathbf{E}_{i\perp} = -\frac{\partial E_{iz}}{\partial z} \,, \tag{256}$$

and it follows from $\nabla \times \mathbf{H}_i = j\omega\epsilon \mathbf{E}_i$ in $\mathcal{D}$ that

$$\begin{aligned}\nabla_\perp \cdot (\mathbf{H}_{i\perp} \times \mathbf{u}_z) &= \nabla \cdot (\mathbf{H}_i \times \mathbf{u}_z) \\ &= \mathbf{u}_z \cdot (\nabla \times \mathbf{H}_i) = j\omega\epsilon E_{iz} \,.\end{aligned} \tag{257}$$

Applying the 2D divergence theorem to the left-hand side of (252), and using (256) and (257), we obtain

$$\begin{aligned}&\frac{1}{2\eta}\oint_{\partial\mathcal{T}} [\mathbf{E}_{i\perp}(x,y,0) - \eta\, \mathbf{H}_{i\perp}(x,y,0) \times \mathbf{u}_z] \cdot \mathbf{n}\, ds \\ &= \frac{1}{2\eta}\iint_{\mathcal{T}} \nabla_\perp \cdot [\mathbf{E}_{i\perp}(x,y,0) - \eta\, (\mathbf{H}_{i\perp}(x,y,0) \times \mathbf{u}_z)]\, da \\ &= \frac{1}{2\eta}\iint_{\mathcal{T}} \left[-\frac{\partial E_{iz}}{\partial z} - \eta\, j\omega\epsilon E_{iz}\right](x,y,0)\, da \\ &= -\frac{j\omega\epsilon}{2}\iint_{\mathcal{T}} \left[E_{iz} + \frac{1}{jk}\frac{\partial E_{iz}}{\partial z}\right](x,y,0)\, da \,,\end{aligned} \tag{258}$$

which proves that (252) is fulfilled.

Applying the 2D divergence theorem to the left-hand side of (253), and using (256) and (257), we obtain

$$\begin{aligned}&-\frac{1}{2\eta}\oint_{\partial\mathcal{T}} [\mathbf{E}_{i\perp}(x,y,0) + \eta\, \mathbf{H}_{i\perp}(x,y,0) \times \mathbf{u}_z] \cdot \mathbf{n}\, ds \\ &= \frac{-1}{2\eta}\iint_{\mathcal{T}} \nabla_\perp \cdot [\mathbf{E}_{i\perp}(x,y,0) + \eta\, (\mathbf{H}_{i\perp}(x,y,0) \times \mathbf{u}_z)]\, da \\ &= -\frac{1}{2\eta}\iint_{\mathcal{T}} \left[-\frac{\partial E_{iz}}{\partial z} + \eta\, j\omega\epsilon E_{iz}\right](x,y,0)\, da \\ &= -\frac{j\omega\epsilon}{2}\iint_{\mathcal{T}} \left[E_{iz} - \frac{1}{jk}\frac{\partial E_{iz}}{\partial z}\right](x,y,0)\, da \,,\end{aligned} \tag{259}$$

which proves that (253) is fulfilled.

We can also prove that the compatibility conditions for C_3 and C_4 are met. It follows from $\nabla \cdot \mathbf{H}_i = 0$ in $\mathcal{D}$ that

$$\nabla_\perp \cdot \mathbf{H}_{i\perp} = -\frac{\partial H_{iz}}{\partial z} \,, \tag{260}$$

and it follows from $\nabla \times \mathbf{E}_i = -j\omega\mu \mathbf{H}_i$ in $\mathcal{D}$ that

$$\begin{aligned}\nabla_\perp \cdot (\mathbf{E}_{i\perp} \times \mathbf{u}_z) &= \nabla \cdot (\mathbf{E}_i \times \mathbf{u}_z) \\ &= \mathbf{u}_z \cdot (\nabla \times \mathbf{E}_i) = -j\omega\mu H_{iz} \,.\end{aligned} \tag{261}$$

Applying the 2D divergence theorem to the left-hand side of (254), and using (260) and (261), we obtain

$$\begin{aligned}
&\frac{1}{2}\oint_{\partial\mathcal{T}} \left[\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z+\eta\,\mathbf{H}_{i\perp}(x,y,0)\right]\cdot\mathbf{n}\,ds \\
&=\frac{1}{2}\iint_{\mathcal{T}} \nabla_\perp\cdot\left[\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z+\eta\,\mathbf{H}_{i\perp}(x,y,0)\right]\,da \\
&\qquad=\frac{1}{2}\iint_{\mathcal{T}}\left[-j\omega\mu H_{iz}-\eta\frac{\partial H_{iz}}{\partial z}\right](x,y,0)\,da \\
&\qquad=-\frac{j\omega\mu}{2}\iint_{\mathcal{T}}\left[H_{iz}+\frac{1}{jk}\frac{\partial H_{iz}}{\partial z}\right](x,y,0)\,da\,,
\end{aligned} \tag{262}$$

which proves that (254) is fulfilled.

Applying the 2D divergence theorem to the left-hand side of (255), and using (260) and (261), we obtain

$$\begin{aligned}
&\frac{1}{2}\oint_{\partial\mathcal{T}} \left[\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z-\eta\,\mathbf{H}_{i\perp}(x,y,0)\right]\cdot\mathbf{n}\,ds \\
&=\frac{1}{2}\iint_{\mathcal{T}} \nabla_\perp\cdot\left[\mathbf{E}_{i\perp}(x,y,0)\times\mathbf{u}_z-\eta\,\mathbf{H}_{i\perp}(x,y,0)\right]\,da \\
&\qquad=\frac{1}{2}\iint_{\mathcal{T}}\left[-j\omega\mu H_{iz}+\eta\frac{\partial H_{iz}}{\partial z}\right](x,y,0)\,da \\
&\qquad=-\frac{j\omega\mu}{2}\iint_{\mathcal{T}}\left[H_{iz}-\frac{1}{jk}\frac{\partial H_{iz}}{\partial z}\right](x,y,0)\,da\,,
\end{aligned} \tag{263}$$

which proves that (255) is fulfilled.

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FRÉDÉRIC BROYDÉ was born in France in 1960. He received the M.S. degree in physics engineering from the Ecole Nationale Supérieure d'Ingénieurs Electriciens de Grenoble (ENSIEG) and the Ph.D. in microwaves and microtechnologies from the Université des Sciences et Technologies de Lille (USTL).

He co-founded the Excem corporation in May 1988, a company providing engineering and research and development services. He is president of Excem since 1988. He is now also president and CTO of Eurexcem, a subsidiary of Excem. Most of his activity is allocated to engineering and research in electronics, radio, antennas, electromagnetic compatibility (EMC) and signal integrity.

Dr. Broydé is author or co-author of about 100 technical papers, and inventor or co-inventor of about 90 patent families, for which 71 patents of France and 49 patents of the USA have been granted. He is a Senior Member of the IEEE since 2001. He is a licensed radio amateur (F5OYE).

EVELYNE CLAVELIER was born in France in 1961. She received the M.S. degree in physics engineering from the Ecole Nationale Supérieure d'Ingénieurs Electriciens de Grenoble (ENSIEG).

She is co-founder of the Excem corporation, based in Maule, France, and she is CEO of Excem. She is also president of Tekcem, a company selling or licensing intellectual property rights to foster research. She is an active engineer and researcher. Her current research areas are radio communications, antennas, EMC and circuit theory.

Prior to starting Excem in 1988, she worked for Schneider Electrics (in Grenoble, France), STMicroelectronics (in Grenoble, France), and Signetics (in Mountain View, USA).

Ms. Clavelier is the author or a co-author of about 90 technical papers. She is co-inventor of about 90 patent families. She is a Senior Member of the IEEE since 2002. She is a licensed radio amateur (F1PHQ).

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