Some computations on a single-turn circular loop antenna

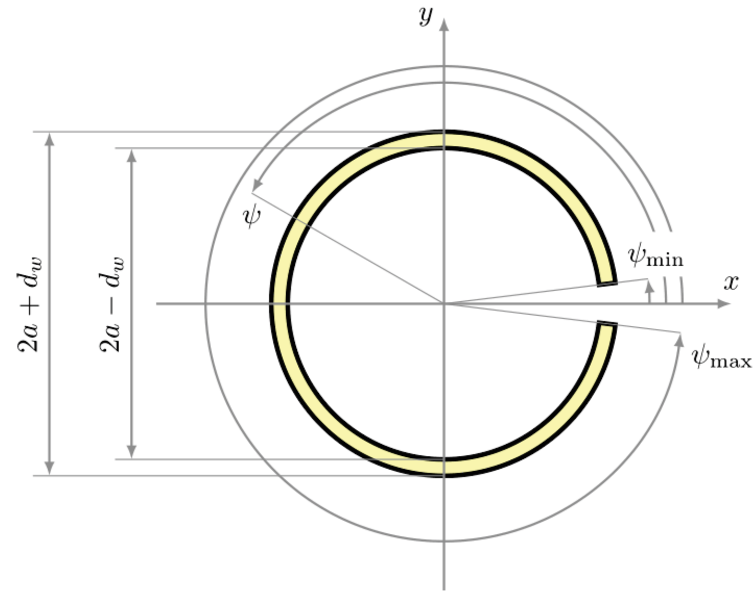

Fig. 1. The single-turn circular loop antenna.

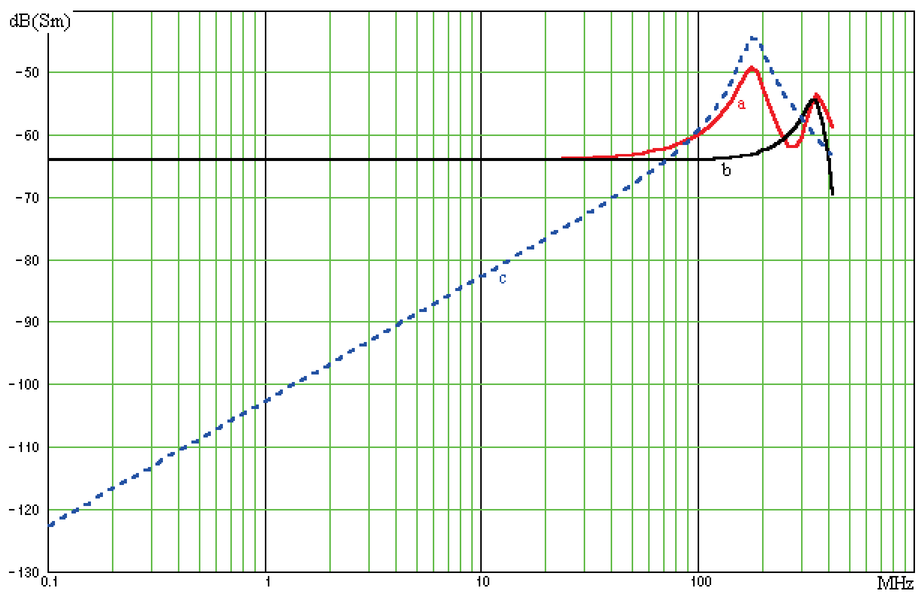

Fig. 2. Some loop antenna parameters versus frequency. $|h_{E\,i\,\varphi}/Z_{ant}|$ for $\theta_i = \pi/2$ and $\varphi_i = \pi$ is curve "a". $|h_{E\,i\,\varphi}/Z_{ant}|$ for $\theta_i = \pi/2$ and $\varphi_i = \pi/2$ is curve "b". $|h_{E\,i\,\theta}/Z_{ant}|$ for $\theta_i = 0$ and $\varphi_i = \pi/2$ is curve "c".

Table of contents

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This research note has 45 pages including annexes.

1. Scope

This research note provides most calculations that were performed to write our article "Some Mathematical Models of a Circular Wire Loop Antenna" [1]. This research note is intended to help those who wish to develop a program implementing the formulas shown in the article.

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2. Computations of Annex A

In the Mathcad worksheet *Loop Antenna 18B.mcd*, shown in Annex A below, we consider (see Fig. 1 in page 1 above) a loop antenna made of a PEC in vacuum, with a = 280 mm and d_w = 14 mm, which was defined in [1, Sec. II.D]. In Annex A, we assume C_T = 0 pF, L_T = 0 nH and N = 20. Values of $i_E(\psi)$ and Z_{ant} are computed in § 3 of Annex A and § 7 of Annex A, using equations (2)-(18) of [1].

Figure 3 of [1] is the plot shown in page A-5 of Annex A. Figure 4 of [1] is the plot shown in page A-4 of Annex A. According to the accurate computations of § 7 of Annex A, the first parallel resonance occurs near 79.3 MHz, the first series resonance near 179 MHz, the second parallel resonance near 256 MHz and the second series resonance near 352 MHz. Figures 5 to 7 of [1] are the plots shown in page A-7 of Annex A. Figures 9 to 11 of [1] are the plots shown in page A-10 of Annex A. Figures 12 to 14 of [1] are the plots shown in page A-11 of Annex A. Figures 15 and 16 of [1] are the plots shown in pages A-12 and A-13 of Annex A.

Figure 17 of [1] is the plot shown in page A-14 of Annex A. In page A-14 of Annex A, we used a maximum seeking algorithm to ascertain that, for this loop antenna:

- at about 31.6 MHz (for which $ka \approx 0.19$), the maximum gain is about 1.47, and occurs in the direction $\theta = \pi/2$ and $\varphi = 0$, which is consistent with two of the plots of page A-10;
- at about 79.4 MHz (near the first parallel resonance), the maximum gain is about 1.40, and occurs in the direction $\theta = \pi/2$ and $\varphi = 0$, which is consistent with two of the plots of page A-11; and

■ at about 178 MHz (near the first series resonance), the maximum gain is about 2.29, and occurs in the directions $\theta = 0$ or $\theta = \pi$, in line with one of the plots of page A-11.

Figure 18 of [1] is the plot shown in page A-17 of Annex A. It is also the plot of Fig. 2 shown in page 1 above.

3. Computations of Annex B

The Mathcad worksheet *Loop Antenna 19B.mcd*, shown in Annex B below, is practically identical to the Mathcad worksheet *Loop Antenna 18B.mcd*, shown in Annex A, except that equations (30)-(35) of [1] are used in Annex B to obtain the Wu-King factors, instead of equations (3)-(7), (9) and (11)-(12) of [1] in Annex A. In Annex B, for any nonnegative integer $n \leq N = 20$, we used $d_{B\,(n)} = 30$ in equation (32) of [1]. Consequently, the power series in the variable ka that is present in equation (33) of [1] was replaced with a polynomial of degree 30, and for any positive integer $n \leq N = 20$, the power series in the variable ka, that is present in equation (34) of [1] if $n = 1$, or equation (35) of [1] if $n \geq 2$, was replaced with a polynomial of degree 31.

The results obtained in Annex B are practically the same as those obtained in Annex A, though they use Wu-King factors that were computed differently. The computation of Annex B is much faster than the computation of Annex A.

4. Technical references

[1] F. Broydé, E. Clavelier, "Some Mathematical Models of a Circular Wire Loop Antenna", *Excem Research Papers in Electronics and Electromagnetics*, no. 9, doi: 10.5281/zenodo.11407457, Jun. 2024.

Annex A: Some models of an unshielded loop antenna

Authors: Frédéric Broydé and Evelyne Clavelier.
Prepared with Mathcad 2000 professional (Mathcad is a registered trademark of its owner).
date: 22 April 2024

File: Loop Antenna 18B.mcd

1) CONSTANTS AND SPECIAL FUNCTIONS

$ORIGIN := 0$ $TOL := 10^{-6}$ $toMEG := 10^{-6}$

$\varepsilon 0 := 8.854188 \cdot 10^{-12}$ F/m $\mu 0 := 4 \cdot \pi \cdot 10^{-7}$ H/m $c0 := \sqrt{\frac{1}{\mu 0 \cdot \varepsilon 0}}$ $c0 = 2.99792 \times 10^{8}$ m/s $\eta 0 := \sqrt{\frac{\mu 0}{\varepsilon 0}}$ $\eta 0 = 376.730$ Ω

$\rho Cu := 1.725 \cdot 10^{-8}$ Ωm Euler's constant $\gamma := 0.57721566490153286060651 2$

computation of the Bessel function Jν(z) for |z| less than or equal to 5 and for ν nonnegative integer less than or equal to 50, using Abramowitz and Stegun p. 360, eq. (9.1.10).

$kvals := 0..15$ $\nu vals := 0..50$ $CoeffJ_{\nu vals, kvals} := \frac{1}{kvals! \cdot (\nu vals + kvals)!}$

$J(z,\nu) := \left(\frac{z}{2}\right)^{\nu} \cdot \sum_{k=0}^{15} CoeffJ_{\nu,k} \cdot \left(\frac{-z^2}{4}\right)^{k}$

$J(5,0) = -0.177596771314351$ cf. Abramowitz and Stegun p. 390 $J0(5) = -0.177596771314338$

$J(5,50) = 2.29424761595254 \times 10^{-45}$ cf. Abramowitz and Stegun p. 407

computation of the Lommel-Weber function Eν(z) for |z| less than or equal to 5 and for ν nonnegative even integer less than or equal to 50, using Olver et al p. 296, eq. (11.10.9).

$kvals := 0..15$ $\nu vals := 0..50$ $CoeffE_{\nu vals, kvals} := if\left(mod(\nu vals, 2) \neq 1, \frac{1}{\Gamma\left(kvals + \frac{\nu vals}{2} + 1.5\right) \cdot \Gamma\left(kvals - \frac{\nu vals}{2} + 1.5\right)}, 0\right)$

$E(z,\nu) := \frac{-z}{2}(-1)^{\frac{\nu}{2}} \cdot \sum_{k=0}^{15} CoeffE_{\nu,k} \cdot \left(\frac{-z^2}{4}\right)^{k}$ $E(5,0) = 0.1852$ cf. Jahnke, Emde and Lösch, p. 256

2) IMPEDANCE OF A LOSSLESS SINGLE-TURN LOOP ANTENNA

In what follows, a is the radius of the loop (from the center of the loop to the center of the wire) and dw the wire diameter.

$$\mathrm{Zloop1}(a,dw,f) := \begin{array}{|l} L \leftarrow \mu0\cdot a\cdot\left(\ln\left(\frac{16\cdot a}{dw}\right)-2\right) \\ \omega \leftarrow 2\cdot\pi\cdot f \\ Rrad \leftarrow \eta0\cdot\frac{\pi}{6}\cdot\left(\frac{\omega}{c0}\cdot a\right)^4 \\ Z \leftarrow Rrad + j\cdot\omega\cdot L \end{array}$$

the function Zloop1 is a low-frequency approximation of the loop impedance, which disregards the internal inductance of the conductor.

$$C(n) := \ln(4\cdot n) + \gamma - 2\cdot\sum_{m=0}^{n-1}\frac{1}{2\cdot m+1}$$

$$B(z,\nu) := J(z,\nu) + j\cdot E(z,\nu)$$

$$\mathrm{WuCoeff}(a,dw,f,NMax) := \begin{array}{|l} k \leftarrow \frac{2\cdot\pi\cdot f}{c0} \\ twoka \leftarrow 2\cdot k\cdot a \\ dwOV2a \leftarrow \frac{dw}{2\cdot a} \\ \kappa a_0 \leftarrow \frac{1}{\pi}\cdot\ln\left(\frac{16\cdot a}{dw}\right) - \frac{j}{2}\cdot\int_0^{twoka} B(x,0)\,dx \\ \kappa a_1 \leftarrow \frac{K0(dwOV2a)\cdot I0(dwOV2a)+C(1)}{\pi} - \frac{j}{2}\cdot\int_0^{twoka} B(x,2)\,dx \\ A_0 \leftarrow k\cdot a\cdot\kappa a_1 \\ \text{for } n\in 1..NMax \\ \quad \begin{array}{|l} x \leftarrow (n+1)\cdot dwOV2a \\ \kappa a_{n+1} \leftarrow \frac{K0(x)\cdot I0(x)+C(n+1)}{\pi} - \frac{j}{2}\cdot\int_0^{twoka} B[x,2\cdot(n+1)]\,dx \\ A_n \leftarrow k\cdot a\cdot\frac{\kappa a_{n+1}+\kappa a_{n-1}}{2} - \frac{n^2}{k\cdot a}\cdot\kappa a_n \end{array} \\ A \end{array}$$

the function ZLOOP2 is the approximation of Wu and King, which disregards the internal inductance of the conductor, and which is valid up to a maximum frequency given by

$$\mathrm{fmax}(a) := \frac{2.5\cdot c0}{2\pi\cdot a}$$

$$\mathrm{ZLOOP2}(a,dw,f,NMax) := \begin{array}{|l} A \leftarrow \mathrm{WuCoeff}(a,dw,f,NMax) \\ Z \leftarrow \frac{j\cdot\pi\cdot\eta0}{\frac{1}{A_0}+2\cdot\sum_{n=1}^{NMax}\frac{1}{A_n}} \end{array}$$

$$nmax := 20$$

$$\mathrm{Zloop2}(a,dw,f) := \mathrm{ZLOOP2}(a,dw,f,nmax)$$

validation 1: comparison between Zloop1 and Zloop2

$aval := 1$ m $dwval := 0.01$ m

$Zloop1(aval, dwval, 10^4) = 3.805974 \times 10^{-13} + 0.424611j$ Ω

$Zloop2(aval, dwval, 10^4) = 3.805976 \times 10^{-13} + 0.424617j$ Ω

$Zloop1(aval, dwval, 10^5) = 3.805974 \times 10^{-9} + 4.246108j$ Ω

$Zloop2(aval, dwval, 10^5) = 3.806158 \times 10^{-9} + 4.246236j$ Ω

$Zloop1(aval, dwval, 10^6) = 3.805974 \times 10^{-5} + 42.461082j$ Ω

$Zloop2(aval, dwval, 10^6) = 3.824357 \times 10^{-5} + 42.530892j$ Ω

$Zloop1(aval, dwval, 10^7) = 0.380597 + 424.610824j$ Ω

$Zloop2(aval, dwval, 10^7) = 0.641461 + 509.718569j$ Ω

$Zloop1(aval, dwval, 10^8) = 3.805974 \times 10^3 + 4.246108j \times 10^3$ Ω

$Zloop2(aval, dwval, 10^8) = 190.133952 + 16.984443j$ Ω

$fmax(aval) = 1.193 \times 10^8$ Hz

O.K.

3) IMPEDANCE OF THE LOOP ANTENNA, AND CURRENT DISTRIBUTION IN THE LOOP ANTENNA

for the receiving antenna under study:

$a1val := 0.28$ m $2 \cdot \pi \cdot a1val = 1.759$ m $dw1val := 0.014$ m $fmax(a1val) = 4.260 \times 10^8$ Hz

$$Lloop := \mu 0 \cdot a1val \cdot \left(\ln\left(\frac{16 \cdot a1val}{dw1val} \right) - 2 \right) \qquad Lloop = 1.325915 \times 10^{-6} \text{ H}$$

$$\Omega := 2\ln\left(\frac{4 \cdot \pi \cdot a1val}{dw1val} \right) \qquad \Omega = 11.054$$

$$L0 := \mu 0 \cdot a1val \cdot \left(K0\left(\frac{dw1val}{2 \cdot a1val} \right) \cdot I0\left(\frac{dw1val}{2 \cdot a1val} \right) + C(1) \right) \qquad L0 = 1.326389 \times 10^{-6} \text{ H}$$

$NbDec := 3$ $NbPointPerDec := 40$ $fstart := 10^5$ $fstop := fstart \cdot 10^{NbDec}$ $fstop = 1.000 \times 10^8$ $Imax := NbDec \cdot NbPointPerDec$ $Imax = 120.000$

$supp := 25$ $i := 0..Imax + supp$

$$fL_i := fstart \cdot \exp\left(\frac{i}{Imax} \cdot \ln\left(\frac{fstop}{fstart} \right) \right)$$

$fL_0 = 1.000 \times 10^5$ Hz $fL_{Imax+supp} = 4.217 \times 10^8$ Hz

approximate loop impedance

$Zloopap_i := Zloop1(a1val, dw1val, fL_i)$

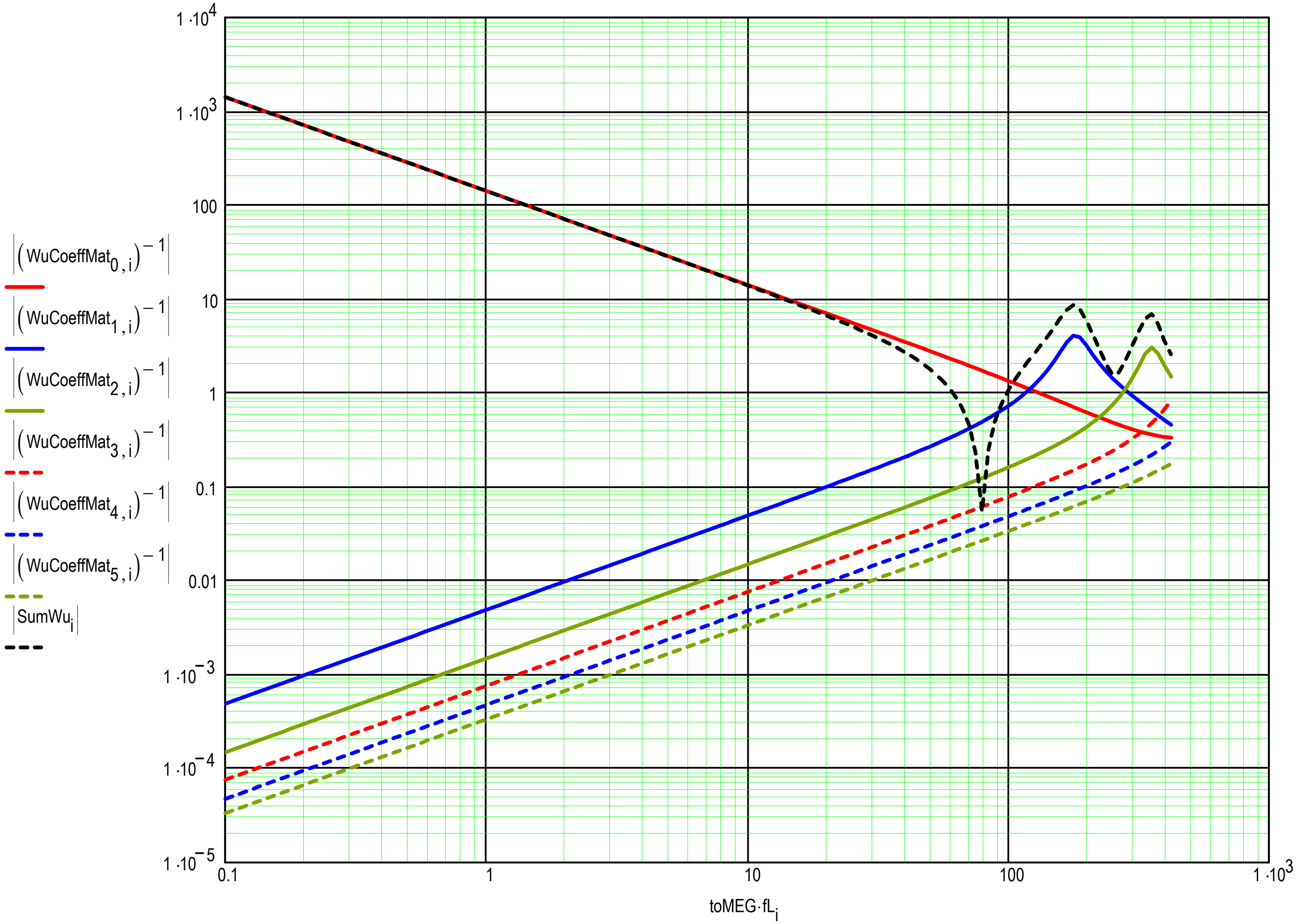
tabulation of the coefficients of Wu

$WuCoeffMat^{\langle i \rangle} := WuCoeff(a1val, dw1val, fL_i, nmax)$

exact loop impedance

$$Zloopex_i := \frac{j \cdot \pi \cdot \eta 0}{\dfrac{1}{WuCoeffMat_{0,i}} + 2 \cdot \displaystyle\sum_{n=1}^{nmax} \frac{1}{WuCoeffMat_{n,i}}}$$

$$SumWu_i := \frac{1}{WuCoeffMat_{0,i}} + 2 \cdot \sum_{n=1}^{nmax} \frac{1}{WuCoeffMat_{n,i}}$$



$fL_{40} = 1.000 \times 10^6$ Hz

$|Zloopex_{40}| = 8.335111$ Ω

$|Zloopap_{40}| = 8.330972$ Ω

$fL_{80} = 10.000 \times 10^6$ Hz

$|Zloopex_{80}| = 84.522882$ Ω

$|Zloopap_{80}| = 83.309716$ Ω

$fL_{100} = 31.623 \times 10^6$ Hz

$|Zloopex_{100}| = 307.267627$ Ω

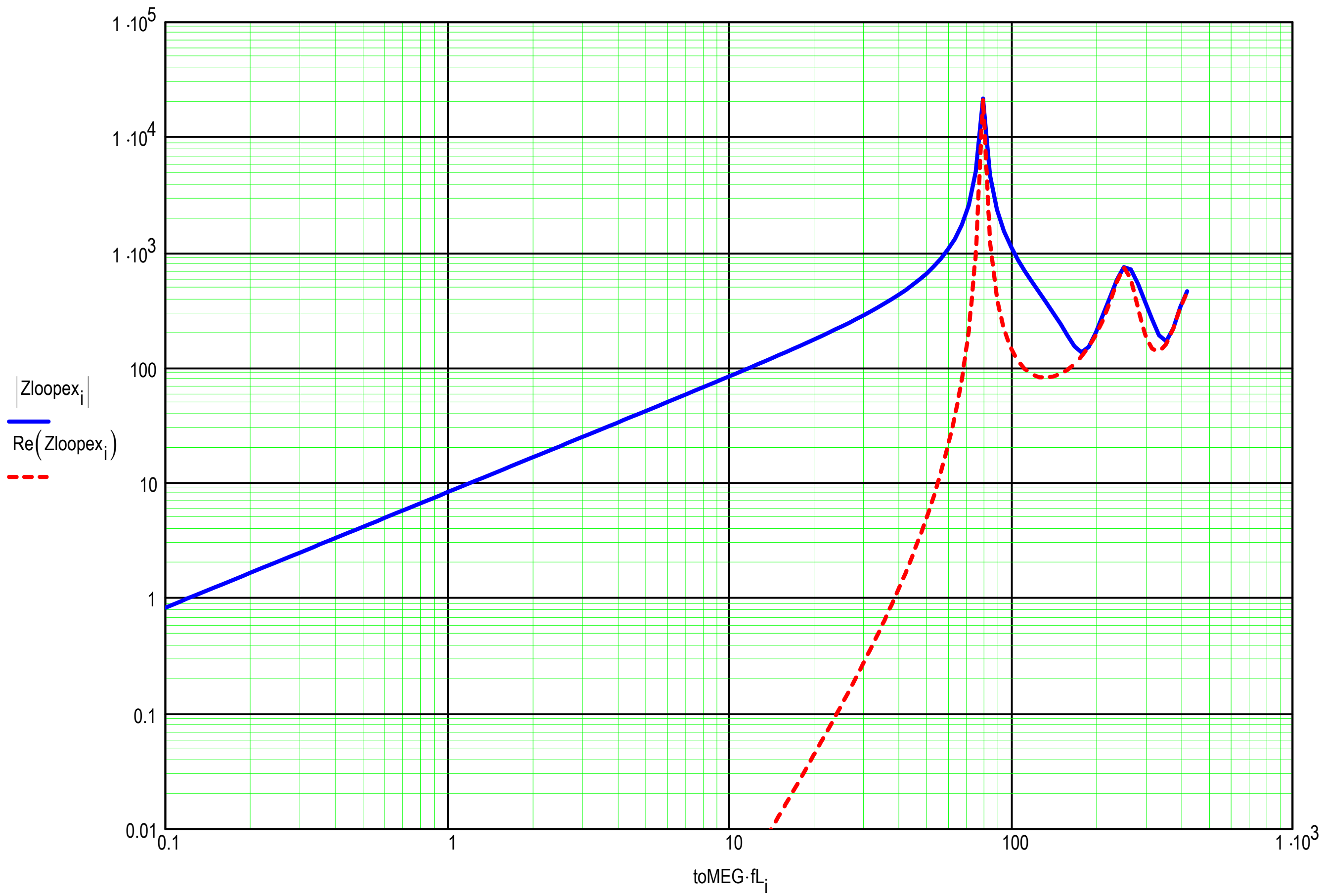
$|Zloopap_{100}| = 263.448558$ Ω

$fL_{120} = 100.000 \times 10^6$ Hz

$|Zloopex_{120}| = 1.119422 \times 10^3$ Ω

$fL_{133} = 211.349 \times 10^6$ Hz

$|Zloopex_{133}| = 284.018154$ Ω



First parallel resonance

$fL_{116} = 79.433 \times 10^6$ Hz $|Zloopex_{115}| = 5.054406 \times 10^3$ Ω $|Zloopex_{116}| = 2.166997 \times 10^4$ Ω $|Zloopex_{117}| = 4.704737 \times 10^3$ Ω

$Zloopex_{116} = 2.165 \times 10^4 - 832.794j$ Ω

Second parallel resonance

$fL_{136} = 251.189 \times 10^6$ Hz $|Zloopex_{135}| = 581.837601$ Ω $|Zloopex_{136}| = 748.903522$ Ω $|Zloopex_{137}| = 715.195733$ Ω

First series resonance

$fL_{130} = 177.828 \times 10^6$ Hz $(|Zloopex_{129}|)^{-1} = 6.498344 \times 10^{-3}$ S $(|Zloopex_{130}|)^{-1} = 7.299666 \times 10^{-3}$ S $(|Zloopex_{131}|)^{-1} = 6.588160 \times 10^{-3}$ S

$|Zloopex_{130}| = 136.992573$ Ω $Zloopex_{130} = 125.868 - 54.075j$ Ω

Second series resonance

$fL_{142} = 354.813 \times 10^6$ Hz $|(Zloopex_{141})^{-1}| = 5.203157 \times 10^{-3}$ S $(|Zloopex_{142}|)^{-1} = 5.872626 \times 10^{-3}$ S $(|Zloopex_{143}|)^{-1} = 4.562293 \times 10^{-3}$ S

$fL_{40} = 1.000 \times 10^6$ Hz

$\frac{1}{|Zloopex_{40}|} = 0.119974$ S

$fL_{80} = 10.000 \times 10^6$ Hz

$\frac{1}{|Zloopex_{80}|} = 0.011831$ S

$fL_{100} = 31.623 \times 10^6$ Hz

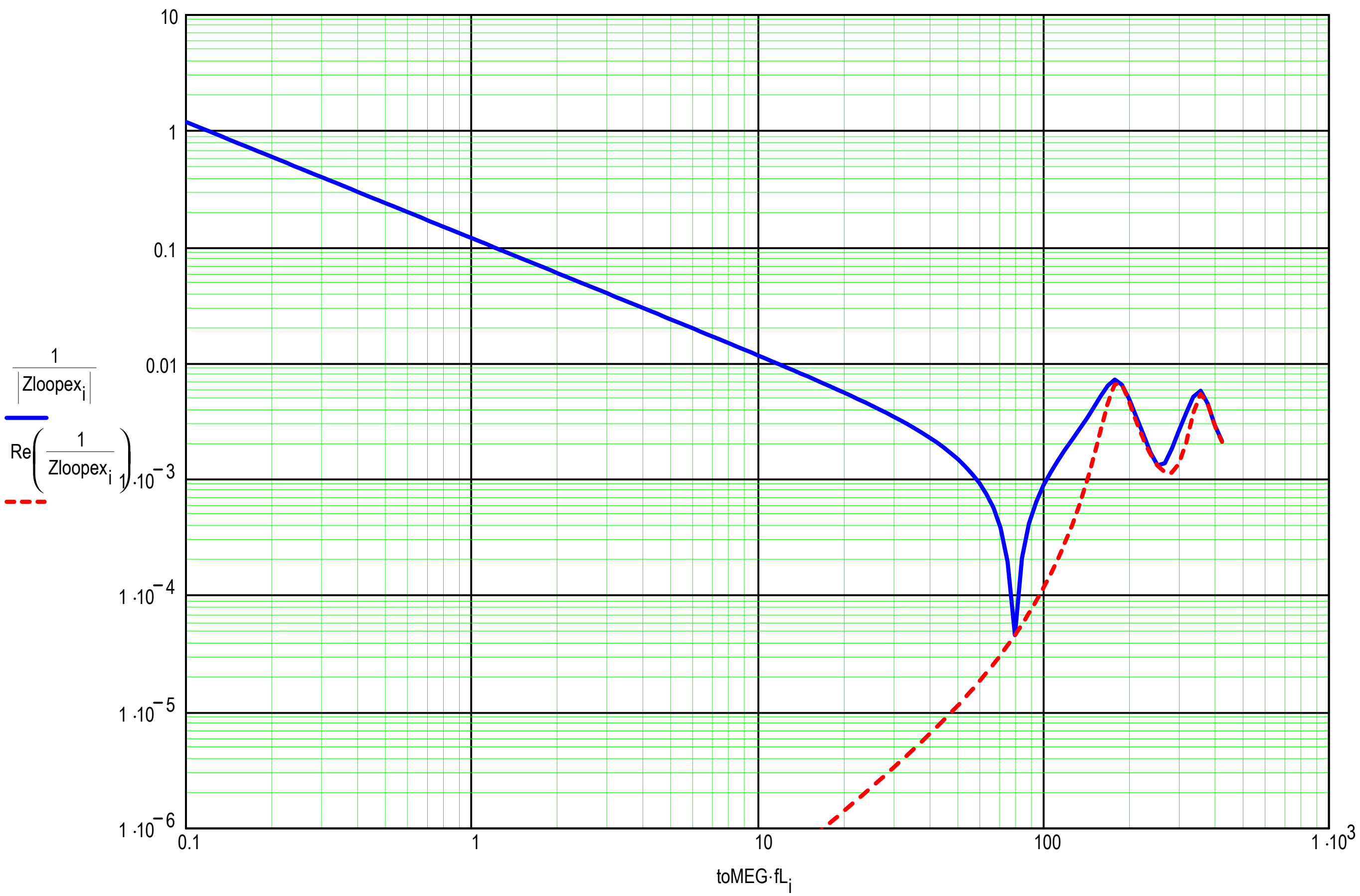
$\frac{1}{|Zloopex_{100}|} = 3.254492 \times 10^{-3}$ S

$fL_{120} = 100.000 \times 10^6$ Hz

$\frac{1}{|Zloopex_{120}|} = 8.933180 \times 10^{-4}$ S

$fL_{133} = 211.349 \times 10^6$ Hz

$\frac{1}{|Zloopex_{133}|} = 3.520902 \times 10^{-3}$ S



normalized current distribution according to the approximation of Wu and King

$Nangle := 120$

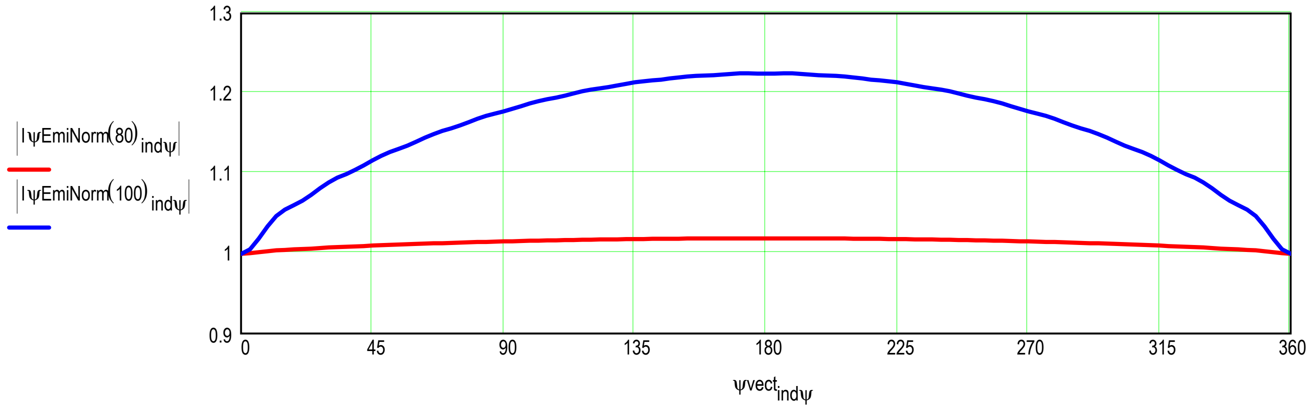
$$I\psi EmiNorm(i) := \begin{array}{l} \text{for } p \in 0..Nangle \\ \quad \psi \leftarrow \frac{2 \cdot \pi \cdot p}{Nangle} \\ \quad res\psi_p \leftarrow \frac{1}{WuCoeffMat_{0,i}} + 2 \cdot \sum_{n=1}^{nmax} \frac{\cos(n \cdot \psi)}{WuCoeffMat_{n,i}} \\ I\psi norm \leftarrow \frac{res\psi}{\frac{1}{WuCoeffMat_{0,i}} + 2 \cdot \sum_{n=1}^{nmax} \frac{1}{WuCoeffMat_{n,i}}} \end{array}$$

$ind\psi := 0..Nangle$

$\psi vect_{ind\psi} := \frac{360 \cdot ind\psi}{Nangle}$

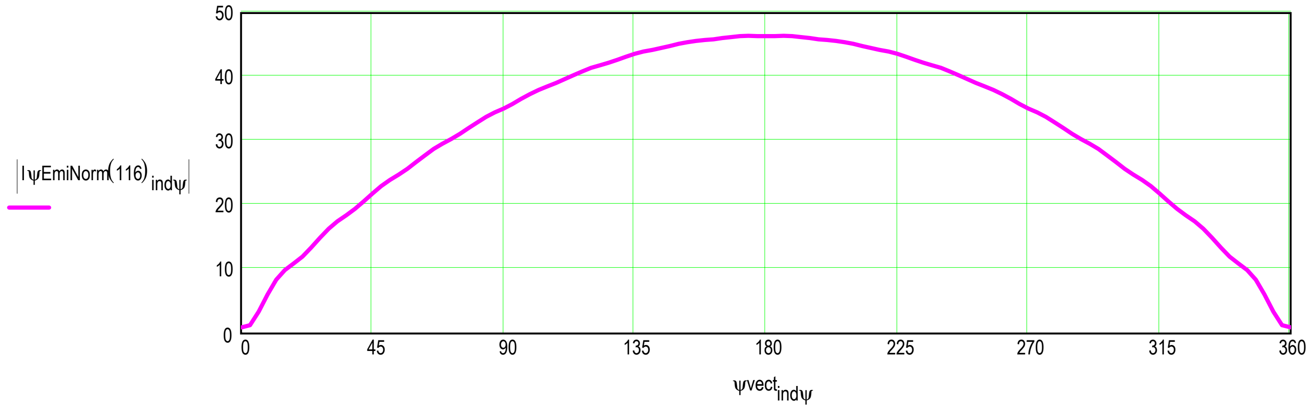
$fL_{80} = 10.000 \times 10^6$ Hz

$fL_{100} = 31.623 \times 10^6$ Hz



$fL_{116} = 79.433 \times 10^6$ Hz

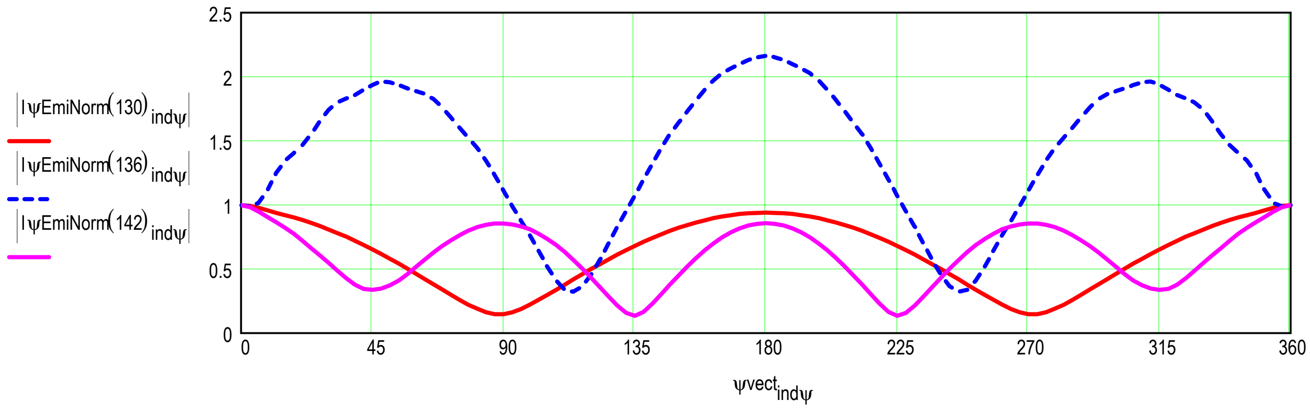
$fL_{136} = 251.189 \times 10^6$ Hz



$fL_{130} = 177.828 \times 10^6$ Hz

$fL_{136} = 251.189 \times 10^6$ Hz

$fL_{142} = 354.813 \times 10^6$ Hz



4) EFFECTIVE LENGTH OF THE LOOP ANTENNA

n being a nonnegative integer

$$Jprimen(n,x) := \text{if}\left(n > 0.5, \frac{Jn(n-1,x) - Jn(n+1,x)}{2}, -Jn(1,x)\right)$$

direct computation of the exact entries of the vector effective length

$$Dirhe\theta(j,\theta,\phi) := \left| \begin{array}{l} k \leftarrow \dfrac{2\cdot\pi\cdot fL_j}{c0} \\ kas \leftarrow k\cdot a1val\cdot\sin(\theta) \\ \left[\dfrac{j\cdot\dfrac{4\cdot\pi}{k\cdot\tan(\theta)}}{\dfrac{1}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}}\cdot\left[\displaystyle\sum_{n=1}^{nmax}\dfrac{n\cdot(j)^n\cdot\sin(n\cdot\phi)\cdot Jn(n,kas)}{WuCoeffMat_{n,j}}\right]\right] \text{ if } \tan(\theta) > 10^{-10} \\ \left(\dfrac{-2\cdot\pi\cdot a1val}{WuCoeffMat_{1,j}}\cdot\dfrac{\cos(\theta)\cdot\sin(\phi)}{\dfrac{1}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}}\right) \text{ otherwise} \end{array}\right.$$

$$Dirhe\phi(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \dfrac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \dfrac{j\cdot 2\cdot\pi\cdot a1val}{\dfrac{1}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}}\cdot\left[\dfrac{Jprimen(0,kas)}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{(j)^n\cdot\cos(n\cdot\phi)\cdot Jprimen(n,kas)}{WuCoeffMat_{n,j}}\right] \end{array}\right.$$

numerical computation of the vector effective length entries

$$Numhe\phi(j,\theta,\phi) := \frac{-a1val}{\dfrac{1}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}}\cdot\left[\int_0^{2\cdot\pi}\left(\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{\cos(n\cdot\psi)}{WuCoeffMat_{n,j}}\right)\cdot\cos(\phi-\psi)\cdot\exp\left(j\cdot\frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta)\cdot\cos(\phi-\psi)\right)d\psi\right]$$

$$Numhe\theta(j,\theta,\phi) := \frac{-a1val}{\dfrac{1}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}}\cdot\left[\int_0^{2\cdot\pi}\left(\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{\cos(n\cdot\psi)}{WuCoeffMat_{n,j}}\right)\cdot\cos(\theta)\cdot\sin(\phi-\psi)\cdot\exp\left(j\cdot\frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta)\cdot\cos(\phi-\psi)\right)d\psi\right]$$

$fL_{120} = 100.000\times 10^{6}$ Hz

$\text{Dirhe}\phi\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -0.86817860 + 0.40271921j$ m

$\text{Numhe}\phi\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -0.86817860 + 0.40271921j$ m

$\text{Dirhe}\phi\left(120,0,\frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

$\text{Numhe}\phi\left(120,0,\frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

$\text{Dirhe}\theta\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -0.58151060 - 0.07285994j$ m

$\text{Numhe}\theta\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -0.58151060 - 0.07285994j$ m

$\text{Dirhe}\theta\left(120,0,\frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

$\text{Numhe}\theta\left(120,0,\frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

approximate entries of the vector effective length

$$\text{Apphe}\theta(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ j\cdot 2\cdot\pi\cdot a1val\cdot\cos(\theta)\cdot\dfrac{j\cdot\dfrac{\sin(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\dfrac{kas^2}{8}\right) - \dfrac{\sin(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas - j\cdot\dfrac{\sin(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2}{\dfrac{1}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}} \end{array} \right.$$

$$\text{Apphe}\phi(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ j\cdot 2\cdot\pi\cdot a1val\cdot\dfrac{\dfrac{-1}{2\cdot WuCoeffMat_{0,j}}\cdot kas\cdot\left(1-\dfrac{kas^2}{8}\right) + j\cdot\dfrac{\cos(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\dfrac{3\cdot kas^2}{8}\right) - \dfrac{\cos(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas - j\cdot\dfrac{\cos(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2}{\dfrac{1}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}} \end{array} \right.$$

Extreme approximation

$$\text{APPhe}\phi(j,\theta) := -j\cdot\pi\cdot\frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val^2\cdot\sin(\theta)$$

$Nangle2 := 180$

$indangle := 0..Nangle2$

$\text{Dirhe}\phi(j,\theta,\phi)$

$\theta vect_{indangle} := \frac{180\cdot indangle}{Nangle2}$

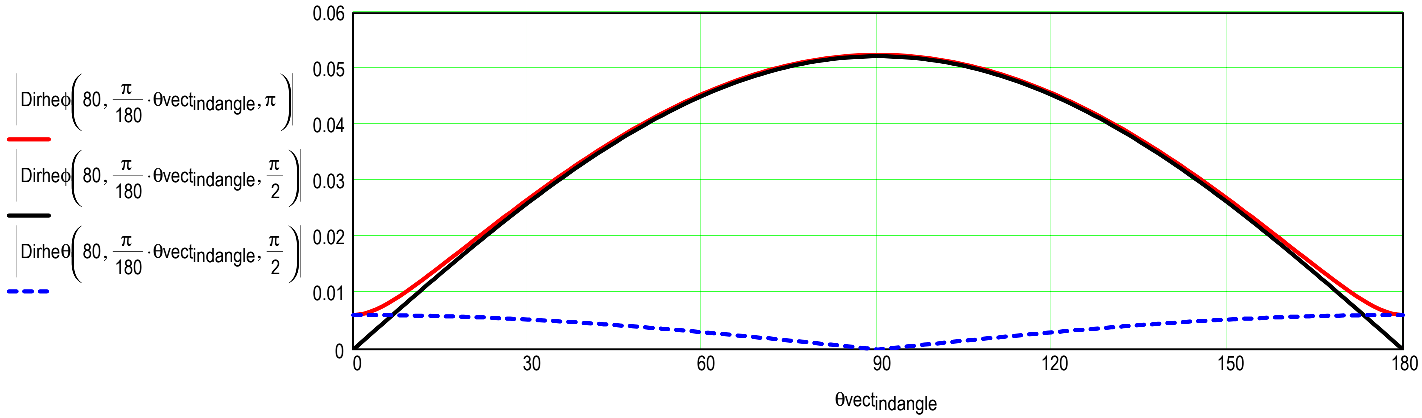
$\phi vect_{indangle} := \frac{360\cdot indangle}{Nangle2}$

for $\phi = \pi$ and $\phi = \pi/2$

$fL_{80} = 10.000 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{80}}{c0} \cdot a1val = 0.059$

(dirheθ=0 for ϕ=π)

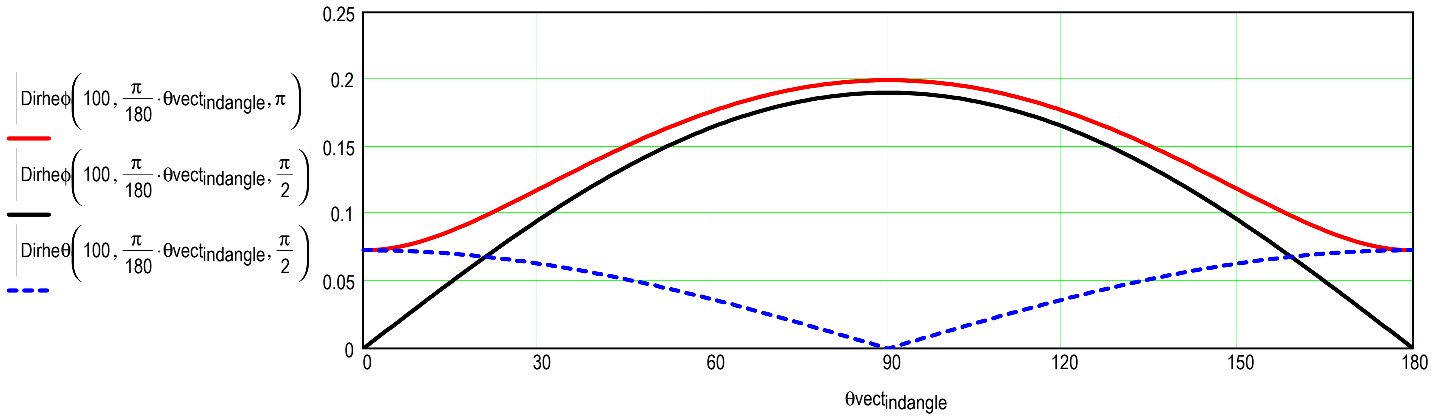


for $\phi = \pi$ and $\phi = \pi/2$

$fL_{100} = 31.623 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{100}}{c0} \cdot a1val = 0.186$

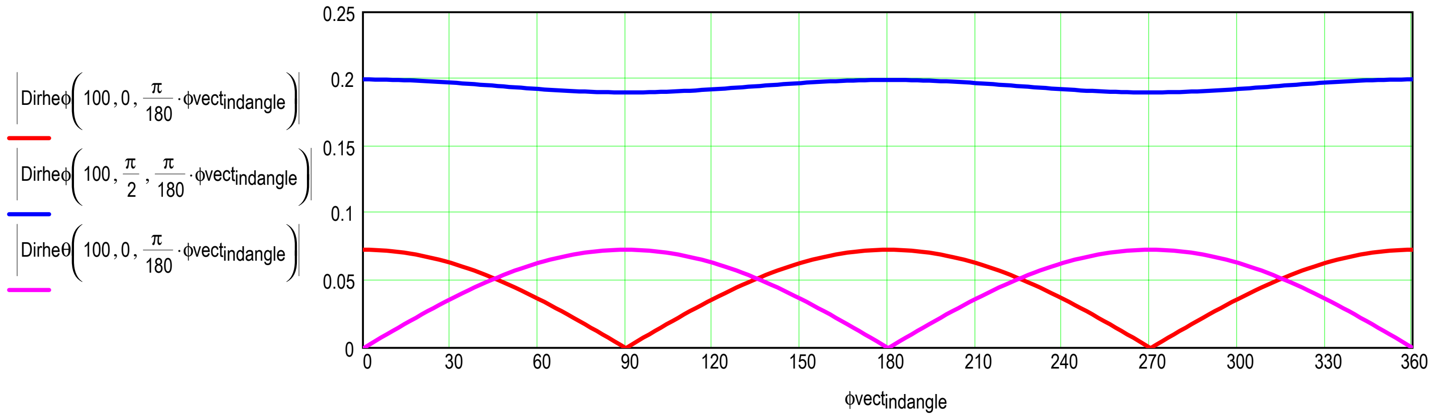
(dirheθ=0 for ϕ=π)



for $\theta = 0$ and $\theta = \pi/2$

$fL_{100} = 31.623 \times 10^{6}$ Hz

(dirheθ=0 for θ=π/2)

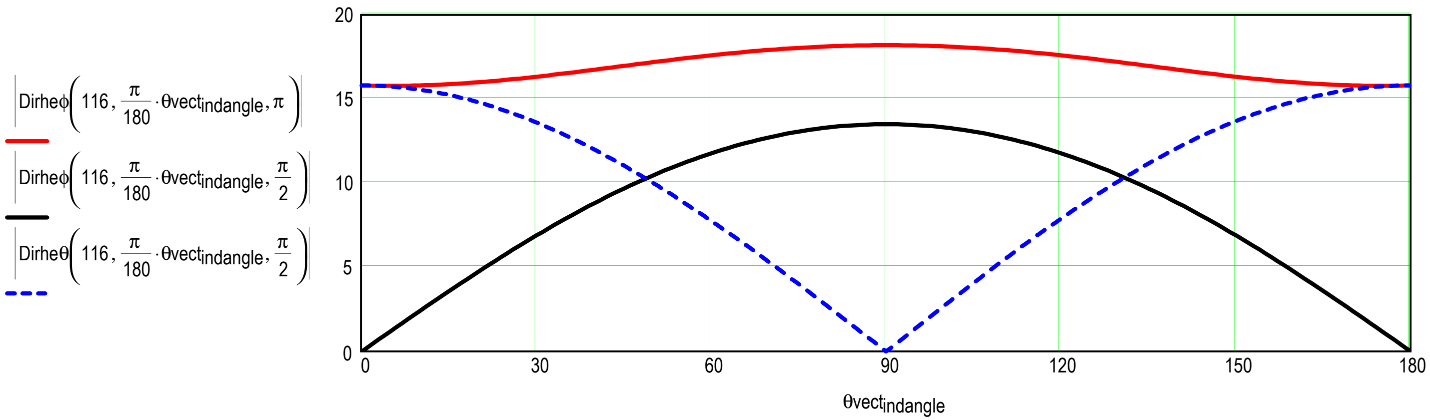


for $\phi = \pi$ and $\phi = \pi/2$

$fL_{116} = 79.433 \times 10^{6}$ Hz

$$\frac{2 \cdot \pi \cdot fL_{116}}{c0} \cdot a1val = 0.466$$

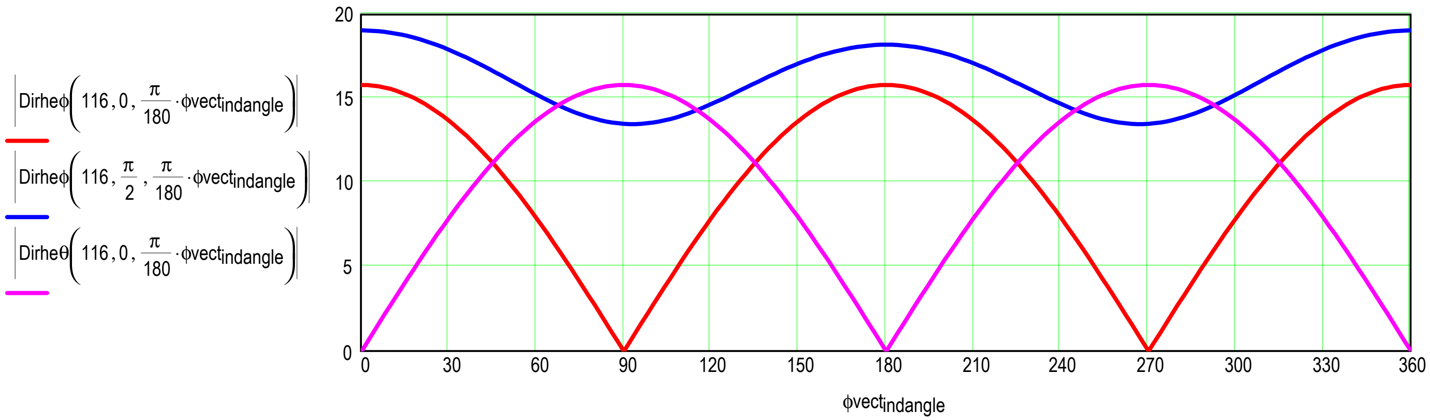
(dirheθ=0 for $\phi=\pi$)



for $\theta = 0$ and $\theta = \pi/2$

$fL_{116} = 79.433 \times 10^{6}$ Hz

(dirheθ=0 for $\theta=\pi/2$)

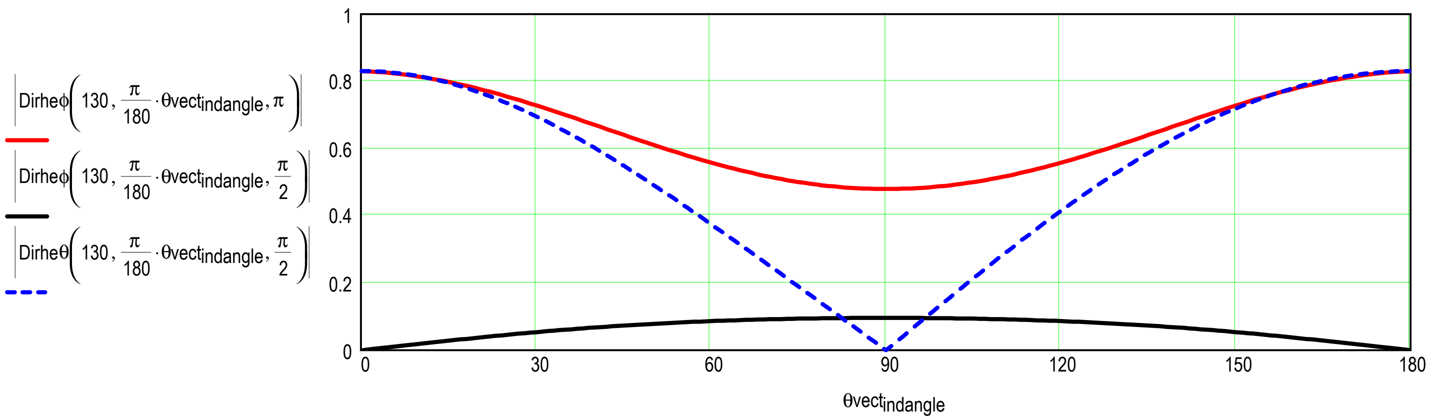


for $\phi = \pi$ and $\phi = \pi/2$

$fL_{130} = 177.828 \times 10^{6}$ Hz

$$\frac{2 \cdot \pi \cdot fL_{130}}{c0} \cdot a1val = 1.044$$

(dirheθ=0 for $\phi=\pi$)



effective length entries versus frequency

$He\phi 1dBmeter_i := 20 \cdot \log\left(\left|Dirhe\phi\left(i, \frac{\pi}{2}, \pi\right)\right|\right)$

$He\phi 2dBmeter_i := 20 \cdot \log\left(\left|Dirhe\phi\left(i, \frac{\pi}{2}, \frac{\pi}{2}\right)\right|\right)$

$He\theta 3dBmeter_i := 20 \cdot \log\left(\left|Dirhe\theta\left(i, 0, \frac{\pi}{2}\right)\right|\right)$

$He\phi APPdBmeter_i := 20 \cdot \log\left(\left|APPhe\phi\left(i, \frac{\pi}{2}\right)\right|\right)$

$fL_{40} = 1.000 \times 10^6$ Hz

$He\phi 1dBmeter_{40} = -45.742$ dB(m)

$He\phi 2dBmeter_{40} = -45.742$ dB(m)

$He\theta 3dBmeter_{40} = -84.351$ dB(m)

$He\phi APPdBmeter_{40} = -45.743$ dB(m)

$fL_{80} = 10.000 \times 10^6$ Hz

$He\phi 1dBmeter_{80} = -25.579$ dB(m)

$He\phi 2dBmeter_{80} = -25.621$ dB(m)

$He\theta 3dBmeter_{80} = -44.200$ dB(m)

$He\phi APPdBmeter_{80} = -25.743$ dB(m)

$fL_{100} = 31.623 \times 10^6$ Hz

$He\phi 1dBmeter_{100} = -13.993$ dB(m)

$He\phi 2dBmeter_{100} = -14.408$ dB(m)

$He\theta 3dBmeter_{100} = -22.717$ dB(m)

$He\phi APPdBmeter_{100} = -15.743$ dB(m)

$fL_{120} = 100.000 \times 10^6$ Hz

$He\phi 1dBmeter_{120} = 1.098$ dB(m)

$He\phi 2dBmeter_{120} = -3.039$ dB(m)

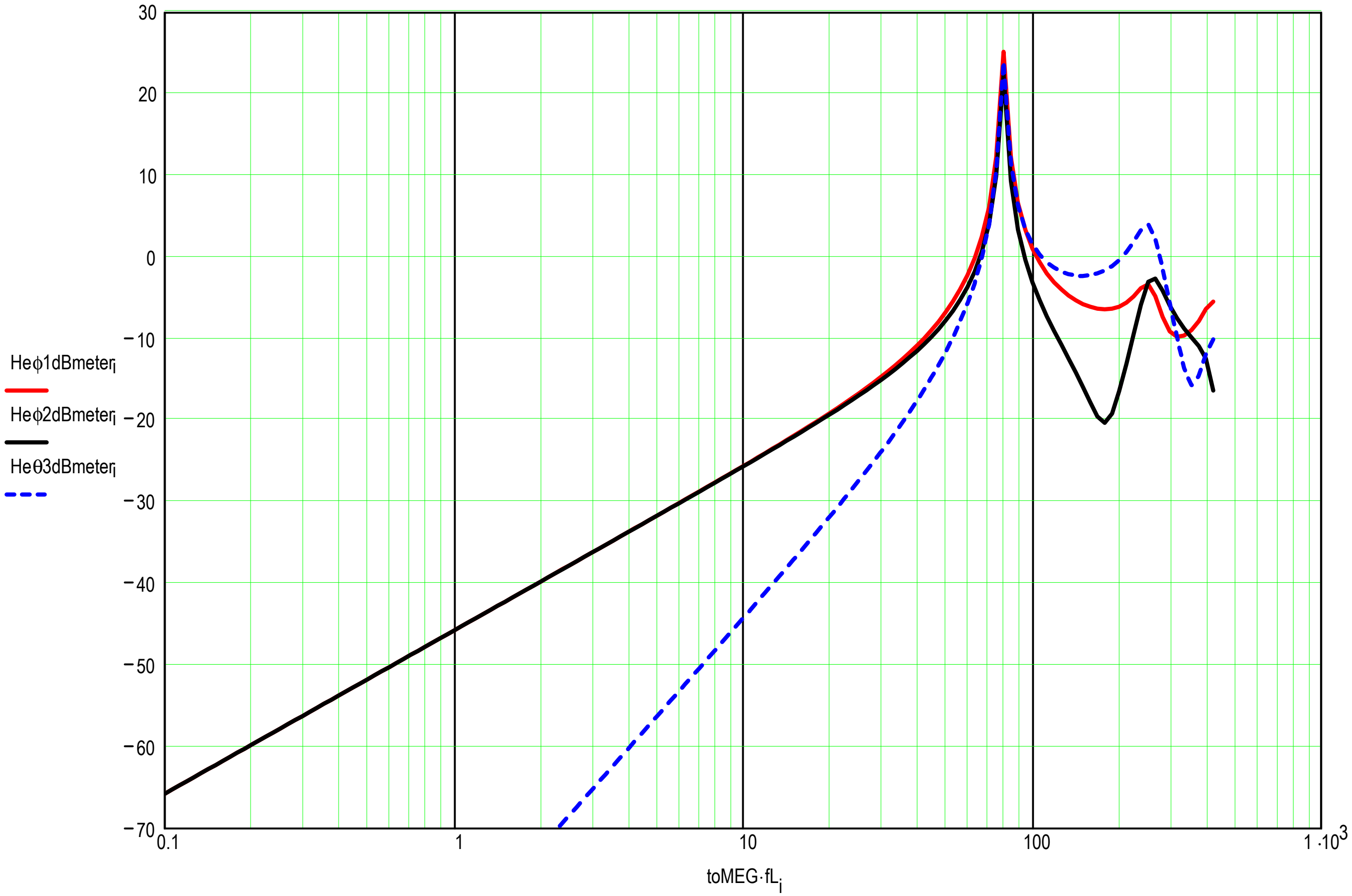
$He\theta 3dBmeter_{120} = 1.610$ dB(m)

$fL_{133} = 211.349 \times 10^6$ Hz

$He\phi 1dBmeter_{133} = -5.567$ dB(m)

$He\phi 2dBmeter_{133} = -13.048$ dB(m)

$He\theta 3dBmeter_{133} = 0.634$ dB(m)



First parallel resonance

$fL_{116} = 79.433 \times 10^{6}$ Hz

$He\phi 1dBmeter_{116} = 25.209$ dB(m)

$He\phi 2dBmeter_{116} = 22.616$ dB(m)

$He\theta 3dBmeter_{116} = 23.985$ dB(m)

First series resonance

$fL_{130} = 177.828 \times 10^{6}$ Hz

$He\phi 1dBmeter_{130} = -6.371$ dB(m)

$He\phi 2dBmeter_{130} = -20.295$ dB(m)

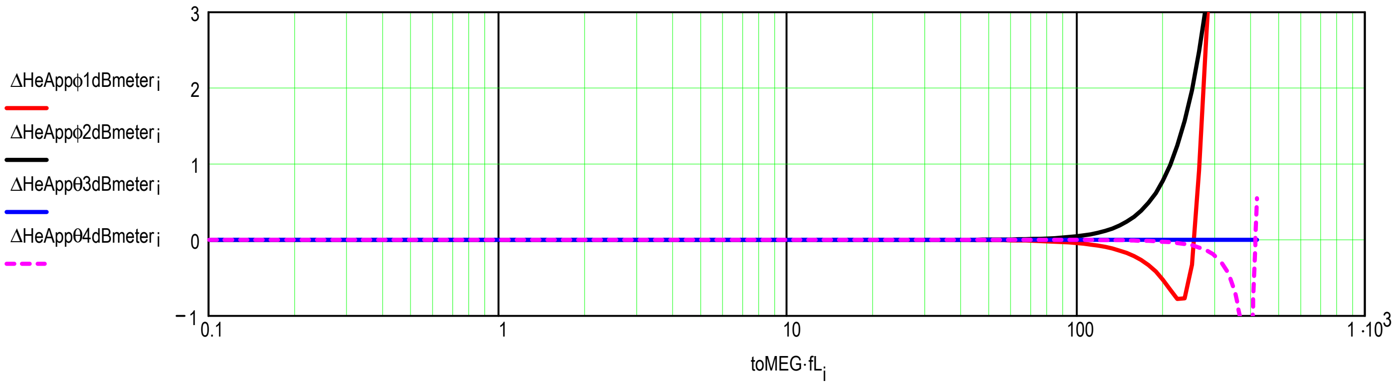
$He\theta 3dBmeter_{130} = -1.606$ dB(m)

$$\Delta HeApp\phi 1dBmeter_i := 20 \cdot \log\left(\frac{\left|Apphe\phi\left(i, \frac{\pi}{2}, \pi\right)\right|}{\left|Dirhe\phi\left(i, \frac{\pi}{2}, \pi\right)\right|}\right)$$

$$\Delta HeApp\phi 2dBmeter_i := 20 \cdot \log\left(\frac{\left|Apphe\phi\left(i, \frac{\pi}{2}, \frac{\pi}{2}\right)\right|}{\left|Dirhe\phi\left(i, \frac{\pi}{2}, \frac{\pi}{2}\right)\right|}\right)$$

$$\Delta HeApp\theta 3dBmeter_i := 20 \cdot \log\left(\frac{\left|Apphe\theta\left(i, 0, \frac{\pi}{2}\right)\right|}{\left|Dirhe\theta\left(i, 0, \frac{\pi}{2}\right)\right|}\right)$$

$$\Delta HeApp\theta 4dBmeter_i := 20 \cdot \log\left(\frac{\left|Apphe\theta\left(i, \frac{\pi}{4}, \frac{\pi}{2}\right)\right|}{\left|Dirhe\theta\left(i, \frac{\pi}{4}, \frac{\pi}{2}\right)\right|}\right)$$



$fL_{132} = 199.526 \times 10^{6}$ Hz

$fL_{133} = 211.349 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{132}}{c0} \cdot a1val = 1.171$

$\frac{2 \cdot \pi \cdot fL_{133}}{c0} \cdot a1val = 1.240$

$\Delta HeApp\phi 1dBmeter_{133} = -0.657$ dB

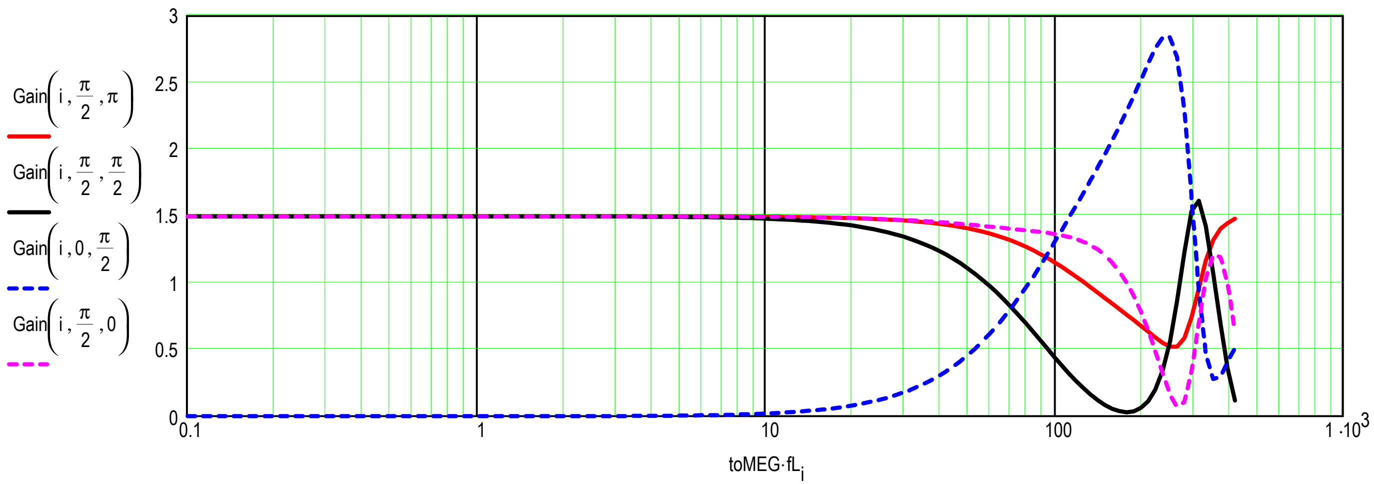
$\Delta HeApp\phi 2dBmeter_{133} = 0.984$ dB

$\Delta HeApp\theta 3dBmeter_{133} = 0.000$ dB

$\Delta HeApp\theta 4dBmeter_{133} = -0.031$ dB

5) GAIN OF THE LOOP ANTENNA

$$\mathrm{Gain}(j,\theta,\phi) := \frac{\eta 0\cdot\left(\frac{2\cdot\pi\cdot fL_j}{c0}\right)^2\cdot\left[\left(\left|\mathrm{Dirhe}\theta(j,\theta,\phi)\right|\right)^2+\left(\left|\mathrm{Dirhe}\phi(j,\theta,\phi)\right|\right)^2\right]}{4\cdot\pi\cdot\mathrm{Re}\left(\mathrm{Zloopex}_j\right)}$$



$fL_{80} = 10.000\times10^6$ Hz

$fL_{92} = 19.953\times10^6$ Hz

$\mathrm{Gain}\left(92,\frac{\pi}{2},\frac{\pi}{2}\right) = 1.431$

$10\cdot\log\left(\frac{\mathrm{Gain}\left(92,\frac{\pi}{2},\frac{\pi}{2}\right)}{1.5}\right) = -0.205$ dB

$fL_{100} = 31.623\times10^6$ Hz

$\mathrm{Gain}\left(100,\frac{\pi}{2},\frac{\pi}{2}\right) = 1.332$

$10\cdot\log\left(\frac{\mathrm{Gain}\left(100,\frac{\pi}{2},\frac{\pi}{2}\right)}{1.5}\right) = -0.517$ dB

$fL_{110} = 56.234\times10^6$ Hz

$fL_{116} = 79.433\times10^6$ Hz

$fL_{130} = 177.828\times10^6$ Hz

the last row of this matrix (row 360) contains the maximum gain, and then the angles θ and ϕ in degrees corresponding to this maximum gain.

```
GainMaxMat(j) := | M ← 0
                 | for iθ ∈ 0..18
                 |   for iϕ ∈ 0..18
                 |     | θdeg ← 10·iθ
                 |     | ϕdeg ← 10·iϕ
                 |     | row ← 19·iθ + iϕ
                 |     | M_row,0 ← Gain(j, θdeg·π/180, ϕdeg·π/180)
                 |     | M_row,1 ← θdeg
                 |     | M_row,2 ← ϕdeg
                 | csort(M,0)
```

$$\mathrm{submatrix}(\mathrm{GainMaxMat}(80),359,360,0,2) = \begin{pmatrix}1.496594 & 90.000000 & 180.000000\\ 1.496653 & 90.000000 & 0.000000\end{pmatrix}$$

$$\mathrm{submatrix}(\mathrm{GainMaxMat}(100),359,360,0,2) = \begin{pmatrix}1.466183 & 90.000000 & 10.000000\\ 1.470318 & 90.000000 & 0.000000\end{pmatrix}$$

$$\mathrm{submatrix}(\mathrm{GainMaxMat}(110),359,360,0,2) = \begin{pmatrix}1.416428 & 90.000000 & 10.000000\\ 1.428108 & 90.000000 & 0.000000\end{pmatrix}$$

$$\mathrm{submatrix}(\mathrm{GainMaxMat}(116),359,360,0,2) = \begin{pmatrix}1.383600 & 80.000000 & 0.000000\\ 1.395027 & 90.000000 & 0.000000\end{pmatrix}$$

$$\mathrm{submatrix}(\mathrm{GainMaxMat}(130),357,360,0,2) = \begin{pmatrix}2.285764 & 180.000000 & 30.000000\\ 2.285764 & 0.000000 & 150.000000\\ 2.285764 & 0.000000 & 70.000000\\ 2.285764 & 0.000000 & 30.000000\end{pmatrix}$$

6) SHORT-CIRCUIT CURRENT OF THE LOOP ANTENNA

numerical computation of the entries of the vector effective length divided by the loop impedance

$$NumW\theta(j,\theta,\phi) := \frac{Numhe\theta(j,\theta,\phi)}{Zloopex_j} \qquad NumW\phi(j,\theta,\phi) := \frac{Numhe\phi(j,\theta,\phi)}{Zloopex_j}$$

direct computation of the exact entries of the vector effective length divided by the loop impedance

$$DirW\theta(j,\theta,\phi) := \left| \begin{array}{l} k \leftarrow \dfrac{2\cdot\pi\cdot fL_j}{c0} \\ kas \leftarrow k\cdot a1val\cdot\sin(\theta) \\ \left[\dfrac{4}{\eta 0\cdot k\cdot\tan(\theta)}\cdot\left[\displaystyle\sum_{n=1}^{nmax}\frac{n\cdot(j)^n\cdot\sin(n\cdot\phi)\cdot Jn(n,kas)}{WuCoeffMat_{n,j}}\right]\right] \text{ if } \tan(\theta) > 10^{-10} \\ \left(\dfrac{j\cdot 2\cdot a1val}{\eta 0\cdot WuCoeffMat_{1,j}}\cdot\cos(\theta)\cdot\sin(\phi)\right) \text{ otherwise} \end{array}\right.$$

$$DirW\phi(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \dfrac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \dfrac{2\cdot a1val}{\eta 0}\cdot\left[\dfrac{Jprimen(0,kas)}{WuCoeffMat_{0,j}} + 2\cdot\displaystyle\sum_{n=1}^{nmax}\frac{(j)^n\cdot\cos(n\cdot\phi)\cdot Jprimen(n,kas)}{WuCoeffMat_{n,j}}\right] \end{array}\right.$$

$fL_{120} = 100.000\times 10^{6}$ Hz

$DirW\phi\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.58235829\times 10^{-4} - 7.21759732j\times 10^{-4}$ mS

$NumW\phi\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.58235829\times 10^{-4} - 7.21759732j\times 10^{-4}$ mS

$DirW\phi\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

$NumW\phi\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

$DirW\theta\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -3.51143841\times 10^{-6} - 5.23523725j\times 10^{-4}$ mS

$NumW\theta\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -3.51143841\times 10^{-6} - 5.23523725j\times 10^{-4}$ mS

$DirW\theta\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

$NumW\theta\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

approximate entries
of the vector
effective length
divided
by the loop impedance

$$AppW\theta(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \frac{2\cdot a1val\cdot\cos(\theta)}{\eta 0}\cdot\left[j\cdot\frac{\sin(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\frac{kas^2}{8}\right)-\frac{\sin(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas-j\cdot\frac{\sin(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2\right] \end{array}\right.$$

$$AppW\phi(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \frac{2\cdot a1val}{\eta 0}\cdot\left[\frac{-1}{2\cdot WuCoeffMat_{0,j}}\cdot kas\cdot\left(1-\frac{kas^2}{8}\right)+j\cdot\frac{\cos(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\frac{3\cdot kas^2}{8}\right)-\frac{\cos(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas-j\cdot\frac{\cos(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2\right] \end{array}\right.$$

Extreme approximation

$$APPW\phi(j,\theta) := \frac{-\pi\cdot a1val^2}{c0\cdot L0}\cdot\sin(\theta)$$

Comparisons:

$fL_{80} = 10.000\times 10^6$ Hz

$DirW\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.37627652\times 10^{-4} - 5.15633974j\times 10^{-5}$ mS

$AppW\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.37627656\times 10^{-4} - 5.15633938j\times 10^{-5}$ mS

$APPW\phi\left(80,\frac{\pi}{4}\right) = -4.379852\times 10^{-4}$ mS

$Dirhe\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 4.35722134\times 10^{-3} - 0.03698968j$ m

$Apphe\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 4.35722103\times 10^{-3} - 0.03698968j$ m

$APPhe\phi\left(80,\frac{\pi}{4}\right) = -0.036501j$ m

$DirW\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.21735999\times 10^{-7} - 3.64654337j\times 10^{-5}$ mS

$AppW\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.21782464\times 10^{-7} - 3.64654330j\times 10^{-5}$ mS

$Dirhe\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.08216435\times 10^{-3} + 2.71052597j\times 10^{-5}$ m

$Apphe\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.08216429\times 10^{-3} + 2.71091870j\times 10^{-5}$ m

versus frequency

$$W\phi 1dBmeter_i := 20\cdot\log\left(\left|DirW\phi\left(i,\frac{\pi}{2},\pi\right)\right|\right)$$

$$W\phi 2dBmeter_i := 20\cdot\log\left(\left|DirW\phi\left(i,\frac{\pi}{2},\frac{\pi}{2}\right)\right|\right)$$

$$W\theta 3dBmeter_i := 20\cdot\log\left(\left|DirW\theta\left(i,0,\frac{\pi}{2}\right)\right|\right)$$

$fL_{40} = 1.000\times 10^6$ Hz

$W\phi 1dBmeter_{40} = -64.160$ dB(mS)

$W\phi 2dBmeter_{40} = -64.161$ dB(mS)

$W\theta 3dBmeter_{40} = -102.769$ dB(mS)

$fL_{80} = 10.000\times 10^6$ Hz

$W\phi 1dBmeter_{80} = -64.119$ dB(mS)

$W\phi 2dBmeter_{80} = -64.160$ dB(mS)

$W\theta 3dBmeter_{80} = -82.740$ dB(mS)

$fL_{100} = 31.623\times 10^6$ Hz

$W\phi 1dBmeter_{100} = -63.744$ dB(mS)

$W\phi 2dBmeter_{100} = -64.159$ dB(mS)

$W\theta 3dBmeter_{100} = -72.467$ dB(mS)

$fL_{120} = 100.000\times 10^6$ Hz

$W\phi 1dBmeter_{120} = -59.882$ dB(mS)

$W\phi 2dBmeter_{120} = -64.019$ dB(mS)

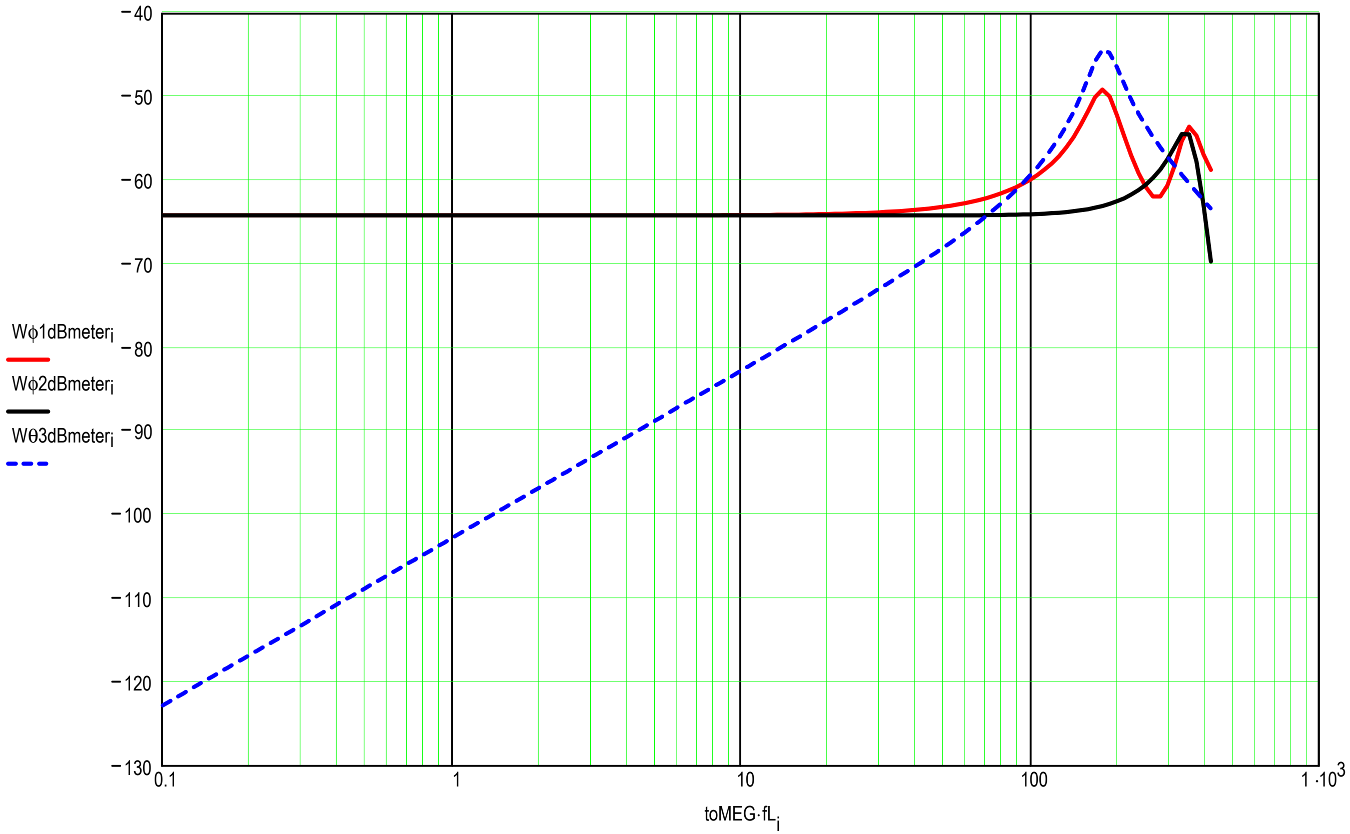
$W\theta 3dBmeter_{120} = -59.370$ dB(mS)

$fL_{133} = 211.349\times 10^6$ Hz

$W\phi 1dBmeter_{133} = -54.634$ dB(mS)

$W\phi 2dBmeter_{133} = -62.115$ dB(mS)

$W\theta 3dBmeter_{133} = -48.433$ dB(mS)



difference of about 20dB between second and third curves at what frequency?

$fL_{76} = 7.943 \times 10^6$ Hz $W\phi 2dBmeter_{76} = -64.160$ dB(mS) $W\theta 3dBmeter_{76} = -84.751$ dB(mS)

$fL_{77} = 8.414 \times 10^6$ Hz $W\phi 2dBmeter_{77} = -64.160$ dB(mS) $W\theta 3dBmeter_{77} = -84.248$ dB(mS)

$fL_{78} = 8.913 \times 10^6$ Hz $W\phi 2dBmeter_{78} = -64.160$ dB(mS) $W\theta 3dBmeter_{78} = -83.746$ dB(mS)

at $fL_{77} = 8.414 \times 10^6$ Hz k a1val is equal to $\frac{2 \cdot \pi \cdot fL_{77}}{c0} \cdot a1val = 0.049376$ and 2*a1val/λ is equal to $\frac{2 \cdot a1val}{\left(\frac{c0}{fL_{77}}\right)} = 0.016$

the second curve deviates by about 1dB from its low frequency plateau at what frequency?

$fL_{0} = 100.000 \times 10^3$ Hz $W\phi 2dBmeter_{0} = -64.161$ dB(mS)

$fL_{40} = 1.000 \times 10^6$ Hz $W\phi 2dBmeter_{40} = -64.161$ dB(mS)

$fL_{128} = 158.489 \times 10^6$ Hz $W\phi 2dBmeter_{128} = -63.404$ dB(mS)

$fL_{129} = 167.880 \times 10^6$ Hz $W\phi 2dBmeter_{129} = -63.235$ dB(mS)

$fL_{130} = 177.828 \times 10^6$ Hz $W\phi 2dBmeter_{130} = -63.029$ dB(mS)

at $fL_{129} = 167.880 \times 10^6$ Hz k a1val is equal to $\frac{2 \cdot \pi \cdot fL_{129}}{c0} \cdot a1val = 0.985184$ and 2*a1val/λ is equal to $\frac{2 \cdot a1val}{\left(\frac{c0}{fL_{129}}\right)} = 0.314$

First parallel resonance

$fL_{116} = 79.433 \times 10^{6}$ Hz

$W\phi1dBmeter_{116} = -61.509$ dB(m)

$W\phi2dBmeter_{116} = -64.101$ dB(m)

$W\theta3dBmeter_{116} = -62.732$ dB(m)

$\frac{2\cdot\pi\cdot fL_{116}}{c0}\cdot a1val = 0.466141$

First series resonance

$fL_{130} = 177.828 \times 10^{6}$ Hz

$W\phi1dBmeter_{130} = -49.105$ dB(m)

$W\phi2dBmeter_{130} = -63.029$ dB(m)

$W\theta3dBmeter_{130} = -44.340$ dB(m)

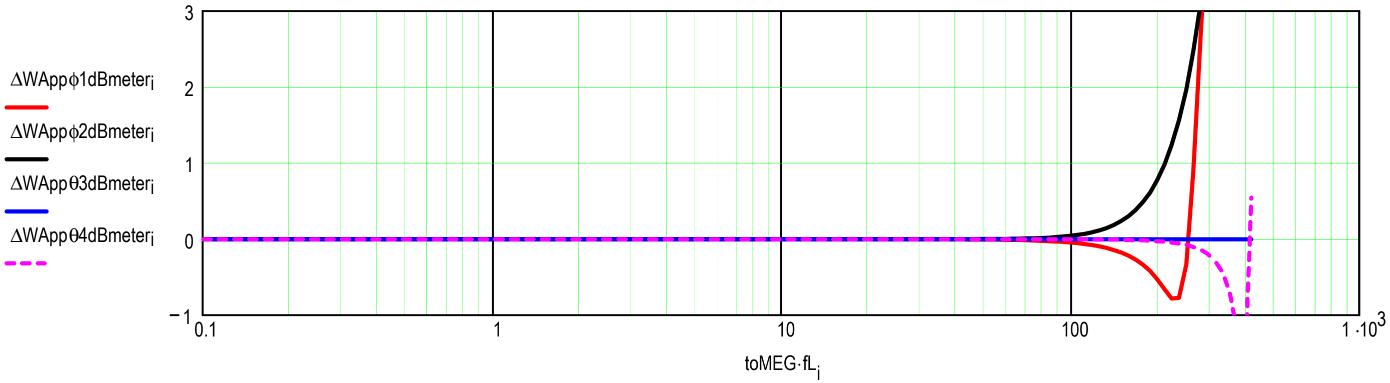
$\frac{2\cdot\pi\cdot fL_{130}}{c0}\cdot a1val = 1.043559$

$$\Delta WApp\phi1dBmeter_i := 20\cdot\log\left(\left|\frac{\left|AppW\phi\left(i,\frac{\pi}{2},\pi\right)\right|}{\left|DirW\phi\left(i,\frac{\pi}{2},\pi\right)\right|}\right|\right)$$

$$\Delta WApp\phi2dBmeter_i := 20\cdot\log\left(\left|\frac{\left|AppW\phi\left(i,\frac{\pi}{2},\frac{\pi}{2}\right)\right|}{\left|DirW\phi\left(i,\frac{\pi}{2},\frac{\pi}{2}\right)\right|}\right|\right)$$

$$\Delta WApp\theta3dBmeter_i := 20\cdot\log\left(\left|\frac{\left|AppW\theta\left(i,0,\frac{\pi}{2}\right)\right|}{\left|DirW\theta\left(i,0,\frac{\pi}{2}\right)\right|}\right|\right)$$

$$\Delta WApp\theta4dBmeter_i := 20\cdot\log\left(\left|\frac{\left|AppW\theta\left(i,\frac{\pi}{4},\frac{\pi}{2}\right)\right|}{\left|DirW\theta\left(i,\frac{\pi}{4},\frac{\pi}{2}\right)\right|}\right|\right)$$



$fL_{132} = 199.526 \times 10^{6}$ Hz

$\frac{2\cdot\pi\cdot fL_{132}}{c0}\cdot a1val = 1.171$

$fL_{133} = 211.349 \times 10^{6}$ Hz

$\frac{2\cdot\pi\cdot fL_{133}}{c0}\cdot a1val = 1.240$

$\Delta WApp\phi1dBmeter_{133} = -0.657$ dB

$\Delta WApp\phi2dBmeter_{133} = 0.984$ dB

$\Delta WApp\theta3dBmeter_{133} = 1.929 \times 10^{-15}$ dB

$\Delta WApp\theta4dBmeter_{133} = -0.031$ dB

$\frac{2\cdot\pi\cdot 200\cdot 10^{6}}{c0}\cdot a1val = 1.174$

7) MORE ACCURATE COMPUTATION OF SOME RESULTS

$$\text{VectConst}\kappa\text{a} := \left| \begin{array}{l} \text{dwOV2a} \leftarrow \dfrac{\text{dw1val}}{2\cdot\text{a1val}} \\ \text{Const}\kappa\text{a}_0 \leftarrow \dfrac{1}{\pi}\cdot\ln\left(\dfrac{16\cdot\text{a1val}}{\text{dw1val}}\right) \\ \text{Const}\kappa\text{a}_1 \leftarrow \dfrac{\text{K0(dwOV2a)}\cdot\text{I0(dwOV2a)} + \text{C}(1)}{\pi} \\ \text{for } n \in 1..\text{nmax} \\ \quad \left| \begin{array}{l} x \leftarrow (n+1)\cdot\text{dwOV2a} \\ \text{Const}\kappa\text{a}_{n+1} \leftarrow \dfrac{\text{K0}(x)\cdot\text{I0}(x) + \text{C}(n+1)}{\pi} \end{array} \right. \\ \text{Const}\kappa\text{a} \end{array} \right.$$

$\text{a1val} = 0.280$

$\text{dw1val} = 0.014$

$\text{nmax} = 20.000$

$$\text{FastZloop2}(f) := \left| \begin{array}{l} \text{ka} \leftarrow \dfrac{2\cdot\pi\cdot f}{c0}\cdot\text{a1val} \\ \text{twoka} \leftarrow 2\cdot\text{ka} \\ \kappa\text{a}_0 \leftarrow \text{VectConst}\kappa\text{a}_0 - \dfrac{j}{2}\cdot\displaystyle\int_0^{\text{twoka}} B(x,0)\,dx \\ \kappa\text{a}_1 \leftarrow \text{VectConst}\kappa\text{a}_1 - \dfrac{j}{2}\cdot\displaystyle\int_0^{\text{twoka}} B(x,2)\,dx \\ A_0 \leftarrow \text{ka}\cdot\kappa\text{a}_1 \\ \text{for } n \in 1..\text{nmax} \\ \quad \left| \begin{array}{l} \kappa\text{a}_{n+1} \leftarrow \text{VectConst}\kappa\text{a}_{n+1} - \dfrac{j}{2}\cdot\displaystyle\int_0^{\text{twoka}} B[x, 2\cdot(n+1)]\,dx \\ A_n \leftarrow \text{ka}\cdot\dfrac{\kappa\text{a}_{n+1} + \kappa\text{a}_{n-1}}{2} - \dfrac{n^2}{\text{ka}}\cdot\kappa\text{a}_n \end{array} \right. \\ Z \leftarrow \dfrac{j\cdot\pi\cdot\eta 0}{\dfrac{1}{A_0} + 2\cdot\displaystyle\sum_{n=1}^{\text{nmax}} \dfrac{1}{A_n}} \end{array} \right.$$

$fL_{80} = 10.000\times10^6$ Hz

$|\text{Zloopex}_{80}| = 84.522882$ Ω

$|\text{FastZloop2}(fL_{80})| = 84.522882$ Ω

$fL_{100} = 31.623\times10^6$ Hz

$|\text{Zloopex}_{100}| = 307.267627$ Ω

$|\text{FastZloop2}(fL_{100})| = 307.267627$ Ω

$fL_{120} = 100.000\times10^6$ Hz

$|\text{Zloopex}_{120}| = 1.119422\times10^3$ Ω

$|\text{FastZloop2}(fL_{120})| = 1.119422\times10^3$ Ω

$fL_{133} = 211.349\times10^6$ Hz

$|\text{Zloopex}_{133}| = 284.018154$ Ω

$|\text{FastZloop2}(fL_{133})| = 284.018154$ Ω

$\text{ModFastZloop2}(f) := |\text{FastZloop2}(f)|$

$\text{FastYloop2}(f) := \dfrac{1}{\text{FastZloop2}(f)}$

$\text{ModFastYloop2}(f) := |\text{FastYloop2}(f)|$

Parallel resonances:

first

$fres1ph := root(Im(FastZloop2(fx)), fx, 70\cdot 10^6, 90\cdot 10^6)$ | $fres1ph = 79.392\times 10^6$ Hz | $FastZloop2(fres1ph) = 2.173\times 10^4 + 2.657j\times 10^{-10}$ Ω

$f := fres1ph$ | $|FastZloop2(fres1ph)| = 2.173\times 10^4$ Ω

Given $f > 75\cdot 10^6$ $f < 85\cdot 10^6$

$fres1mag := Maximize(ModFastZloop2, f)$ | $fres1mag = 79.342\times 10^6$ Hz | $FastZloop2(fres1mag) = 2.173\times 10^4 + 1.034j\times 10^3$ Ω

$\frac{2\cdot\pi\cdot fres1mag}{c0}\cdot a1val = 0.466$ | $|FastZloop2(fres1mag)| = 2.175\times 10^4$ Ω

second

$fres3ph := root(Im(FastZloop2(fx)), fx, 230\cdot 10^6, 270\cdot 10^6)$ | $fres3ph = 247.592\times 10^6$ Hz | $FastZloop2(fres3ph) = 717.988 - 9.430j\times 10^{-13}$ Ω

$f := fres3ph$ | $|FastZloop2(fres3ph)| = 717.988$ Ω

Given $f > 230\cdot 10^6$ $f < 270\cdot 10^6$

$fres3mag := Maximize(ModFastZloop2, f)$ | $fres3mag = 256.492\times 10^6$ Hz | $FastZloop2(fres3mag) = 734.427 - 221.068j$ Ω

$\frac{2\cdot\pi\cdot fres3mag}{c0}\cdot a1val = 1.505$ | $|FastZloop2(fres3mag)| = 766.978$ Ω

Series resonances:

first

$fres2ph := root(Im(FastYloop2(fx)), fx, 160\cdot 10^6, 200\cdot 10^6)$ | $fres2ph = 187.982\times 10^6$ Hz | $FastYloop2(fres2ph) = 6.637\times 10^{-3} + 1.351j\times 10^{-17}$ S

$f := fres2ph$ | $|FastYloop2(fres2ph)| = 6.637\times 10^{-3}$ S

Given $f > 160\cdot 10^6$ $f < 200\cdot 10^6$ | $|FastZloop2(fres2ph)| = 150.663$ Ω

$fres2mag := Maximize(ModFastYloop2, f)$ | $fres2mag = 178.612\times 10^6$ Hz | $FastYloop2(fres2mag) = 6.803\times 10^{-3} + 2.660j\times 10^{-3}$ S

$\frac{2\cdot\pi\cdot fres2mag}{c0}\cdot a1val = 1.048$ | $|FastYloop2(fres2mag)| = 7.305\times 10^{-3}$ S

$|FastZloop2(fres2mag)| = 136.896$ Ω

second

$fres4ph := root(Im(FastYloop2(fx)), fx, 300\cdot 10^6, 400\cdot 10^6)$ | $fres4ph = 372.329\times 10^6$ Hz | $FastYloop2(fres4ph) = 4.849\times 10^{-3} - 1.055j\times 10^{-17}$ S

$f := fres4ph$ | $|FastYloop2(fres4ph)| = 4.849\times 10^{-3}$ S

Given $f > 300\cdot 10^6$ $f < 400\cdot 10^6$ | $|FastZloop2(fres4ph)| = 206.243$ Ω

$fres4mag := Maximize(ModFastYloop2, f)$ | $fres4mag = 351.585\times 10^6$ Hz | $FastYloop2(fres4mag) = 5.411\times 10^{-3} + 2.363j\times 10^{-3}$ S

$\frac{2\cdot\pi\cdot fres4mag}{c0}\cdot a1val = 2.063$ | $|FastYloop2(fres4mag)| = 5.904\times 10^{-3}$ S

$|FastZloop2(fres4mag)| = 169.366$ Ω

Annex B: Some models of an unshielded loop antenna

Authors: Frédéric Broydé and Evelyne Clavelier.
Prepared with Mathcad 2000 professional (Mathcad is a registered trademark of its owner).

date: 24 April 2024

File: Loop Antenna 19B.mcd

1) CONSTANTS AND SPECIAL FUNCTIONS

$ORIGIN := 0$ $TOL := 10^{-6}$ $toMEG := 10^{-6}$

$\varepsilon 0 := 8.854188 \cdot 10^{-12}$ F/m $\mu 0 := 4 \cdot \pi \cdot 10^{-7}$ H/m $c0 := \sqrt{\frac{1}{\mu 0 \cdot \varepsilon 0}}$ $c0 = 2.99792 \times 10^{8}$ m/s $\eta 0 := \sqrt{\frac{\mu 0}{\varepsilon 0}}$ $\eta 0 = 376.730$ Ω

$\rho Cu := 1.725 \cdot 10^{-8}$ Ωm Euler's constant $\gamma := 0.577215664901532860606512$

$$\chi func(q,n) := \left| \begin{array}{l} qov2 \leftarrow \frac{q}{2} \\ \text{if } |qov2 - round(qov2)| < 10^{-8} \\ \quad \left| \begin{array}{l} p \leftarrow round(qov2 - 1) \\ coeff \leftarrow if\left[p \geq -0.001, \frac{(-1)^{(n+p+1)}}{2 \cdot \Gamma(p+n+1.5) \cdot \Gamma(p-n+1.5) \cdot (p+1)}, 0\right] \end{array} \right. \\ \text{otherwise} \\ \quad \left| \begin{array}{l} p \leftarrow round(qov2 - n - 0.5) \\ coeff \leftarrow if\left[p \geq -0.001, -j \cdot \frac{(-1)^{p}}{p! \cdot (2 \cdot n + p)! \cdot (2 \cdot p + 2 \cdot n + 1)}, 0\right] \end{array} \right. \end{array} \right.$$

$nind := 0..21$

$qind := 0..31$

$\chi_{qind, nind} := \chi func(qind, nind)$

$\chi =$

	0	1	2	3	4	5	6
0	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$
1	$-1.000000j \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$
2	$-6.366198 \cdot 10^{-1}$	$2.122066 \cdot 10^{-1}$	$4.244132 \cdot 10^{-2}$	$1.818914 \cdot 10^{-2}$	$1.010508 \cdot 10^{-2}$	$6.430503 \cdot 10^{-3}$	$4.451887 \cdot 10^{-3}$
3	$3.333333j \cdot 10^{-1}$	$-1.666667j \cdot 10^{-1}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$
4	$1.414711 \cdot 10^{-1}$	$-8.488264 \cdot 10^{-2}$	$1.212609 \cdot 10^{-2}$	$1.347343 \cdot 10^{-3}$	$3.674573 \cdot 10^{-4}$	$1.413297 \cdot 10^{-4}$	$6.595387 \cdot 10^{-5}$
5	$-5.000000j \cdot 10^{-2}$	$3.333333j \cdot 10^{-2}$	$-8.333333j \cdot 10^{-3}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$	$0.000000 \cdot 10^{0}$

$$C(n) := \ln(4 \cdot n) + \gamma - 2 \cdot \sum_{m=0}^{n-1} \frac{1}{2 \cdot m + 1}$$

2) IMPEDANCE OF A LOSSLESS SINGLE-TURN LOOP ANTENNA

In what follows, a is the radius of the loop (from the center of the loop to the center of the wire) and dw the wire diameter.

$$Zloop1(a,dw,f) := \left| \begin{array}{l} L \leftarrow \mu 0 \cdot a \cdot \left(\ln\left(\frac{16 \cdot a}{dw} \right) - 2 \right) \\ \omega \leftarrow 2 \cdot \pi \cdot f \\ Rrad \leftarrow \eta 0 \cdot \frac{\pi}{6} \cdot \left(\frac{\omega}{c0} \cdot a \right)^4 \\ Z \leftarrow Rrad + j \cdot \omega \cdot L \end{array} \right.$$

the function Zloop1 is a low-frequency approximation of the loop impedance, which disregards the internal inductance of the conductor.

$$PolyIntB(a,f,\nu) := \left| \begin{array}{l} ka \leftarrow \frac{2 \cdot \pi \cdot f}{c0} \cdot a \\ \sum_{q=0}^{30} \chi_{q, \frac{\nu}{2}} \cdot ka^q \end{array} \right.$$

$$WuCoeff(a,dw,f,NMax) := \left| \begin{array}{l} ka \leftarrow \frac{2 \cdot \pi \cdot f}{c0} \cdot a \\ dwOV2a \leftarrow \frac{dw}{2 \cdot a} \\ \kappa a_0 \leftarrow \frac{1}{\pi} \cdot \ln\left(\frac{16 \cdot a}{dw} \right) + \sum_{q=0}^{30} \chi_{q,0} \cdot ka^q \\ \kappa a_1 \leftarrow \frac{K0(dwOV2a) \cdot I0(dwOV2a) + C(1)}{\pi} + \sum_{q=0}^{30} \chi_{q,1} \cdot ka^q \\ A_0 \leftarrow ka \cdot \kappa a_1 \\ \text{for } n \in 1..NMax \\ \quad \left| \begin{array}{l} x \leftarrow (n+1) \cdot dwOV2a \\ \kappa a_{n+1} \leftarrow \frac{K0(x) \cdot I0(x) + C(n+1)}{\pi} + \sum_{q=0}^{30} \chi_{q,n+1} \cdot ka^q \\ A_n \leftarrow ka \cdot \frac{\kappa a_{n+1} + \kappa a_{n-1}}{2} - \frac{n^2}{ka} \cdot \kappa a_n \end{array} \right. \\ A \end{array} \right.$$

the function ZLOOP2 is the approximation of Wu and King, which disregards the internal inductance of the conductor, and which is valid up to a maximum frequency given by

$$fmax(a) := \frac{2.5 \cdot c0}{2\pi \cdot a}$$

$$ZLOOP2(a,dw,f,NMax) := \left| \begin{array}{l} A \leftarrow WuCoeff(a,dw,f,NMax) \\ Z \leftarrow \frac{j \cdot \pi \cdot \eta 0}{\frac{1}{A_0} + 2 \cdot \sum_{n=1}^{NMax} \frac{1}{A_n}} \end{array} \right.$$

$$nmax := 20$$

$$Zloop2(a,dw,f) := ZLOOP2(a,dw,f,nmax)$$

validation 1: comparison between Zloop1 and Zloop2

$aval := 1$ m $dwval := 0.01$ m

$Zloop1(aval, dwval, 10^4) = 3.805974 \times 10^{-13} + 0.424611j$ Ω

$Zloop2(aval, dwval, 10^4) = 3.805976 \times 10^{-13} + 0.424617j$ Ω

$Zloop1(aval, dwval, 10^5) = 3.805974 \times 10^{-9} + 4.246108j$ Ω

$Zloop2(aval, dwval, 10^5) = 3.806158 \times 10^{-9} + 4.246236j$ Ω

$Zloop1(aval, dwval, 10^6) = 3.805974 \times 10^{-5} + 42.461082j$ Ω

$Zloop2(aval, dwval, 10^6) = 3.824357 \times 10^{-5} + 42.530892j$ Ω

$Zloop1(aval, dwval, 10^7) = 0.380597 + 424.610824j$ Ω

$Zloop2(aval, dwval, 10^7) = 0.641461 + 509.718569j$ Ω

$Zloop1(aval, dwval, 10^8) = 3.805974 \times 10^3 + 4.246108j \times 10^3$ Ω

$Zloop2(aval, dwval, 10^8) = 190.133952 + 16.984443j$ Ω

$fmax(aval) = 1.193 \times 10^8$ Hz

O.K.

3) IMPEDANCE OF THE LOOP ANTENNA, AND CURRENT DISTRIBUTION IN THE LOOP ANTENNA

for the receiving antenna under study:

$a1val := 0.28$ m $2 \cdot \pi \cdot a1val = 1.759$ m $dw1val := 0.014$ m $fmax(a1val) = 4.260 \times 10^8$ Hz

$$Lloop := \mu 0 \cdot a1val \cdot \left(\ln\left(\frac{16 \cdot a1val}{dw1val} \right) - 2 \right)$$

$Lloop = 1.325915 \times 10^{-6}$ H

$$\Omega := 2\ln\left(\frac{4 \cdot \pi \cdot a1val}{dw1val} \right)$$

$\Omega = 11.054$

$$L0 := \mu 0 \cdot a1val \cdot \left(K0\left(\frac{dw1val}{2 \cdot a1val} \right) \cdot I0\left(\frac{dw1val}{2 \cdot a1val} \right) + C(1) \right)$$

$L0 = 1.326389 \times 10^{-6}$ H

$NbDec := 3$ $NbPointPerDec := 40$ $fstart := 10^5$ $fstop := fstart \cdot 10^{NbDec}$ $fstop = 1.000 \times 10^8$ $Imax := NbDec \cdot NbPointPerDec$ $Imax = 120.000$

$supp := 25$ $i := 0 .. Imax + supp$

$$fL_i := fstart \cdot \exp\left(\frac{i}{Imax} \cdot \ln\left(\frac{fstop}{fstart} \right) \right)$$

$fL_0 = 1.000 \times 10^5$ Hz $fL_{Imax+supp} = 4.217 \times 10^8$ Hz

approximate loop impedance

$Zloopap_i := Zloop1(a1val, dw1val, fL_i)$

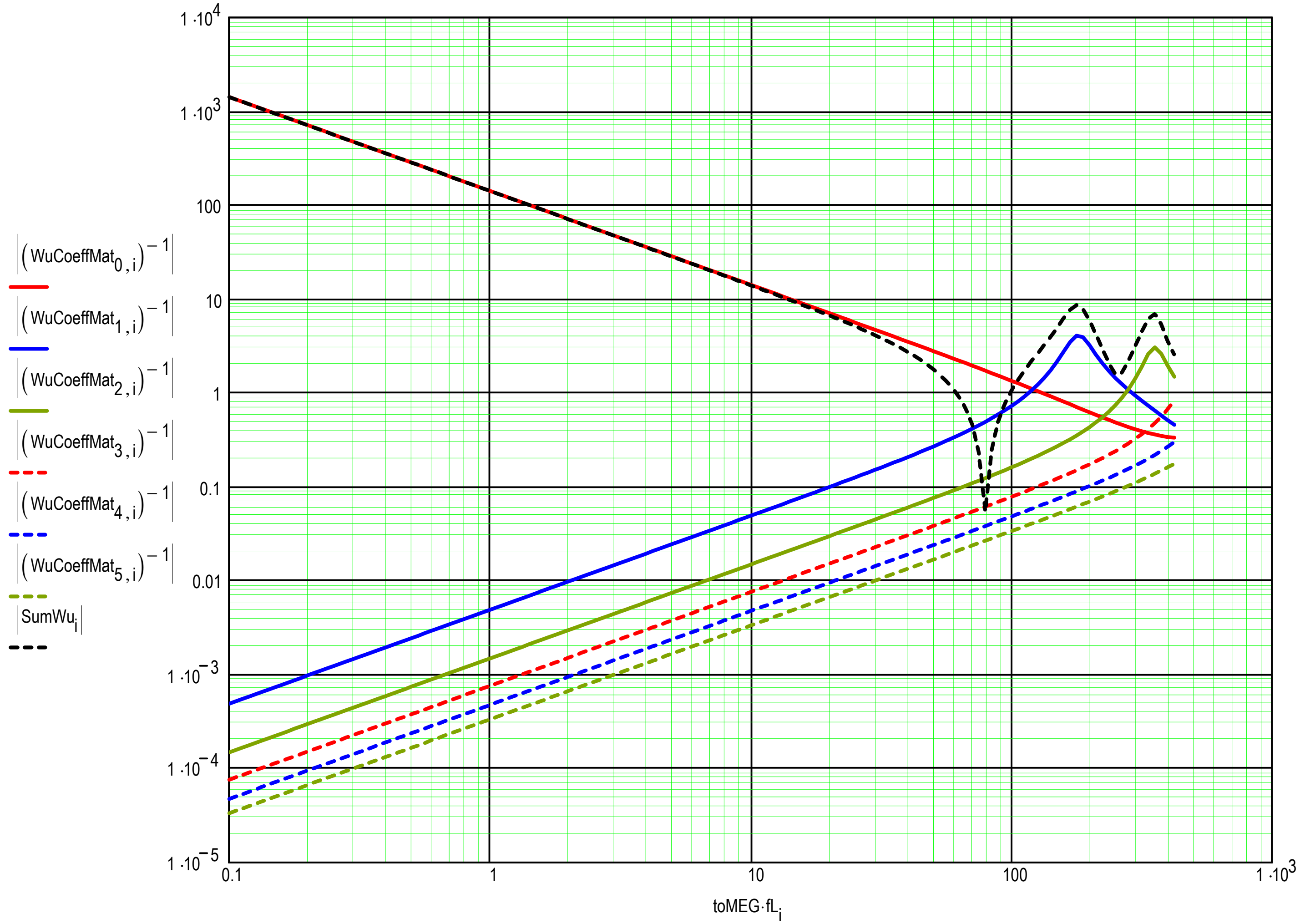
tabulation of the coefficients of Wu

$WuCoeffMat^{\langle i \rangle} := WuCoeff(a1val, dw1val, fL_i, nmax)$

exact loop impedance

$$Zloopex_i := \frac{j \cdot \pi \cdot \eta 0}{\frac{1}{WuCoeffMat_{0,i}} + 2 \cdot \sum_{n=1}^{nmax} \frac{1}{WuCoeffMat_{n,i}}}$$

$$SumWu_i := \frac{1}{WuCoeffMat_{0,i}} + 2 \cdot \sum_{n=1}^{nmax} \frac{1}{WuCoeffMat_{n,i}}$$



$fL_{40} = 1.000 \times 10^6$ Hz

$|Zloopex_{40}| = 8.335111$ Ω

$|Zloopap_{40}| = 8.330972$ Ω

$fL_{80} = 10.000 \times 10^6$ Hz

$|Zloopex_{80}| = 84.522882$ Ω

$|Zloopap_{80}| = 83.309716$ Ω

$fL_{100} = 31.623 \times 10^6$ Hz

$|Zloopex_{100}| = 307.267627$ Ω

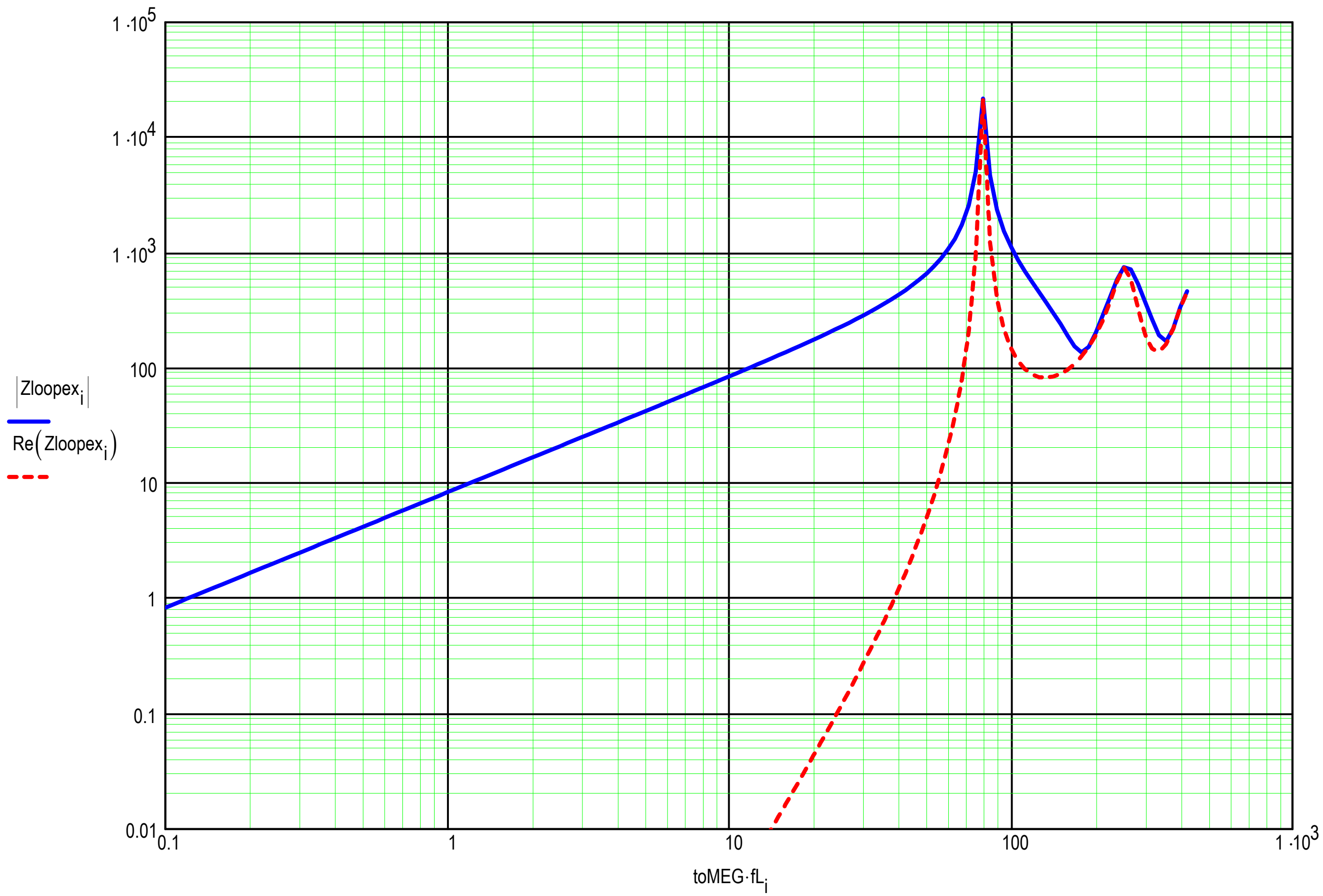
$|Zloopap_{100}| = 263.448558$ Ω

$fL_{120} = 100.000 \times 10^6$ Hz

$|Zloopex_{120}| = 1.119422 \times 10^3$ Ω

$fL_{133} = 211.349 \times 10^6$ Hz

$|Zloopex_{133}| = 284.018154$ Ω



First parallel resonance

$fL_{116} = 79.433 \times 10^6$ Hz $|Zloopex_{115}| = 5.054406 \times 10^3$ Ω $|Zloopex_{116}| = 2.166997 \times 10^4$ Ω $|Zloopex_{117}| = 4.704737 \times 10^3$ Ω

$Zloopex_{116} = 2.165 \times 10^4 - 832.794j$ Ω

Second parallel resonance

$fL_{136} = 251.189 \times 10^6$ Hz $|Zloopex_{135}| = 581.837601$ Ω $|Zloopex_{136}| = 748.903522$ Ω $|Zloopex_{137}| = 715.195733$ Ω

First series resonance

$fL_{130} = 177.828 \times 10^6$ Hz $(|Zloopex_{129}|)^{-1} = 6.498344 \times 10^{-3}$ S $(|Zloopex_{130}|)^{-1} = 7.299666 \times 10^{-3}$ S $(|Zloopex_{131}|)^{-1} = 6.588160 \times 10^{-3}$ S

$|Zloopex_{130}| = 136.992573$ Ω $Zloopex_{130} = 125.868 - 54.075j$ Ω

Second series resonance

$fL_{142} = 354.813 \times 10^6$ Hz $|(Zloopex_{141})^{-1}| = 5.203157 \times 10^{-3}$ S $(|Zloopex_{142}|)^{-1} = 5.872626 \times 10^{-3}$ S $(|Zloopex_{143}|)^{-1} = 4.562293 \times 10^{-3}$ S

$fL_{40} = 1.000 \times 10^{6}$ Hz

$\frac{1}{|Zloopex_{40}|} = 0.119974$ S

$fL_{80} = 10.000 \times 10^{6}$ Hz

$\frac{1}{|Zloopex_{80}|} = 0.011831$ S

$fL_{100} = 31.623 \times 10^{6}$ Hz

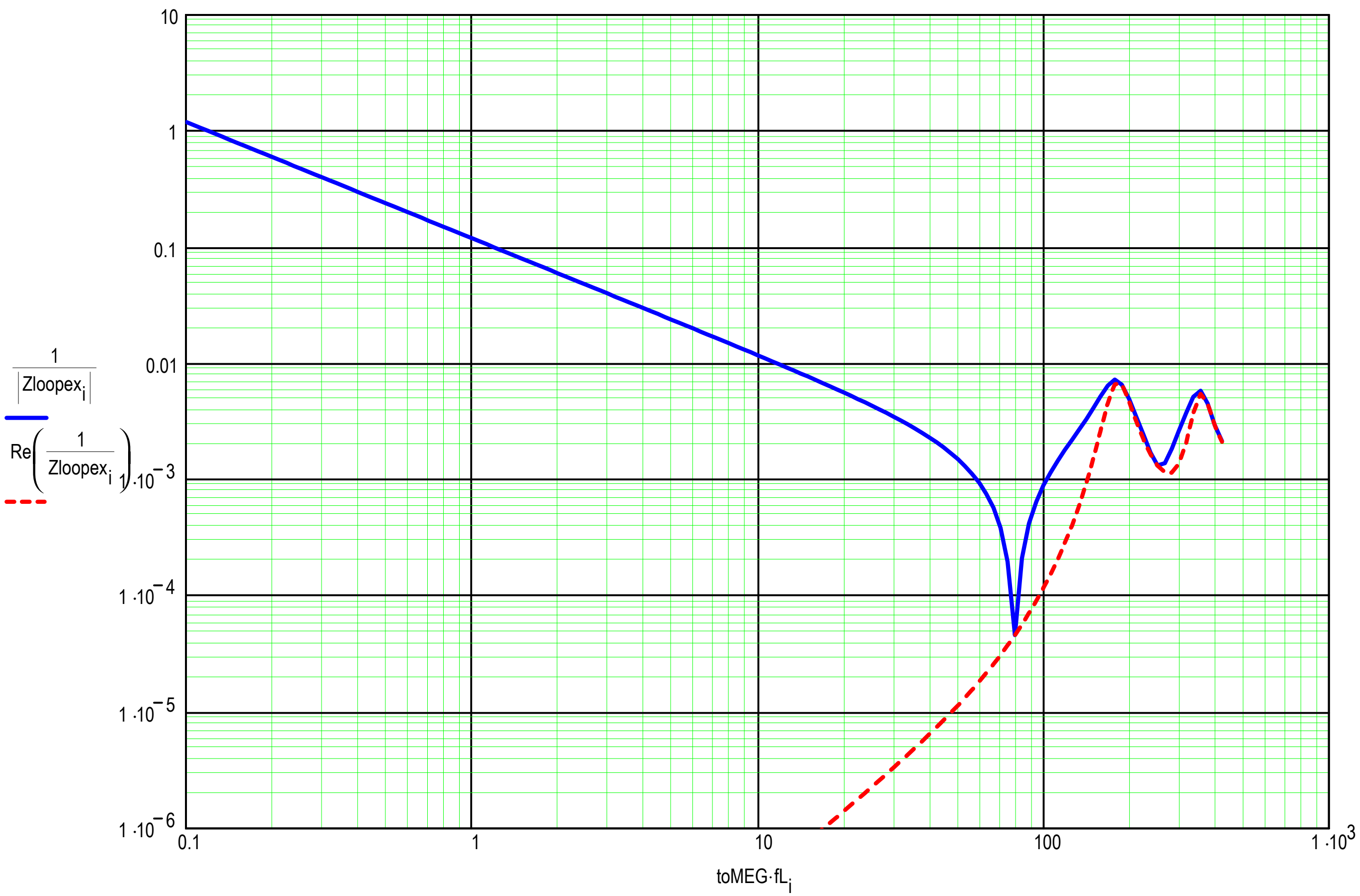
$\frac{1}{|Zloopex_{100}|} = 3.254492 \times 10^{-3}$ S

$fL_{120} = 100.000 \times 10^{6}$ Hz

$\frac{1}{|Zloopex_{120}|} = 8.933180 \times 10^{-4}$ S

$fL_{133} = 211.349 \times 10^{6}$ Hz

$\frac{1}{|Zloopex_{133}|} = 3.520902 \times 10^{-3}$ S



normalized current distribution according to the approximation of Wu and King

$Nangle := 120$

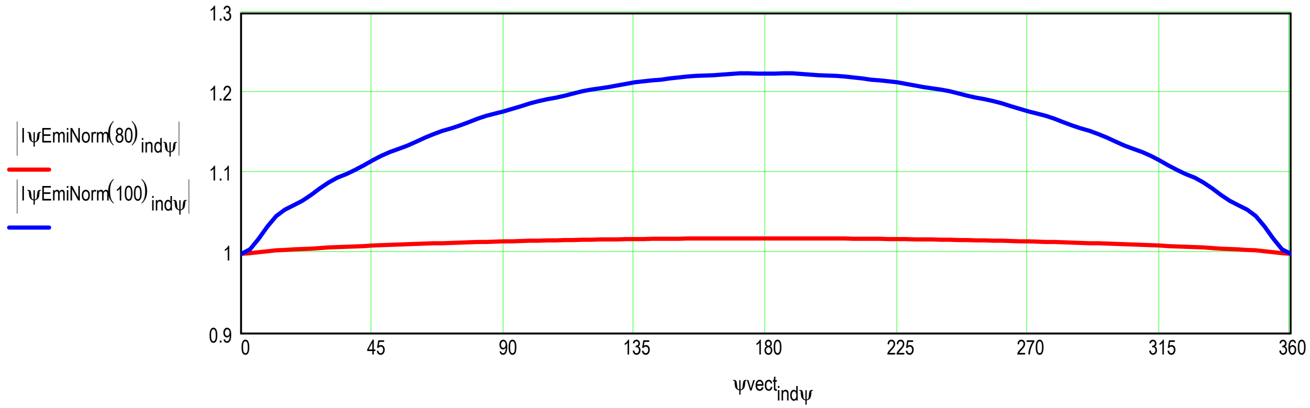
$$I\psi EmiNorm(i) := \left| \begin{array}{l} \text{for } p \in 0..Nangle \\ \quad \left| \begin{array}{l} \psi \leftarrow \frac{2\cdot\pi\cdot p}{Nangle} \\ res\psi_p \leftarrow \frac{1}{WuCoeffMat_{0,i}} + 2\cdot\sum_{n=1}^{nmax}\frac{\cos(n\cdot\psi)}{WuCoeffMat_{n,i}} \end{array}\right. \\ I\psi norm \leftarrow \frac{res\psi}{\frac{1}{WuCoeffMat_{0,i}} + 2\cdot\sum_{n=1}^{nmax}\frac{1}{WuCoeffMat_{n,i}}} \end{array}\right.$$

$ind\psi := 0..Nangle$

$\psi vect_{ind\psi} := \frac{360\cdot ind\psi}{Nangle}$

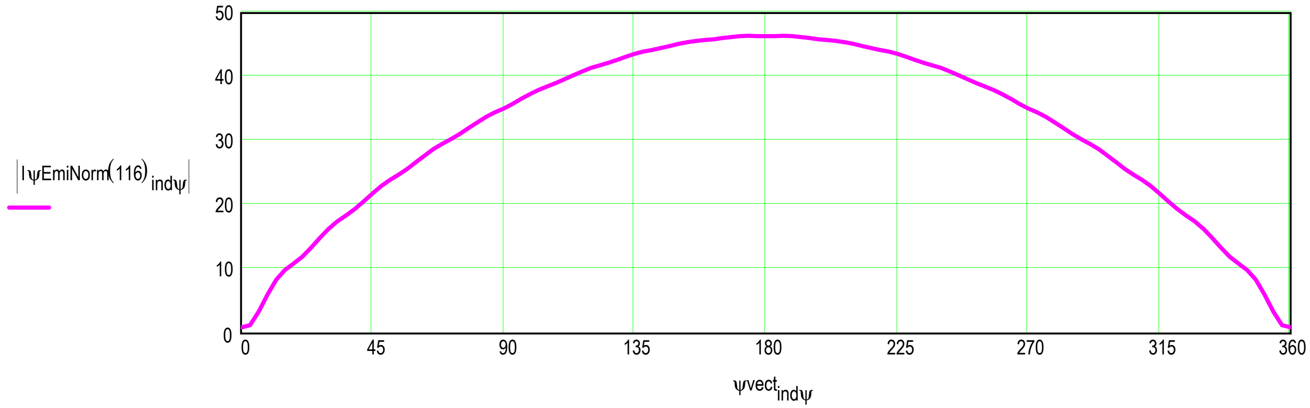
$fL_{80} = 10.000 \times 10^{6}$ Hz

$fL_{100} = 31.623 \times 10^{6}$ Hz



$fL_{116} = 79.433 \times 10^{6}$ Hz

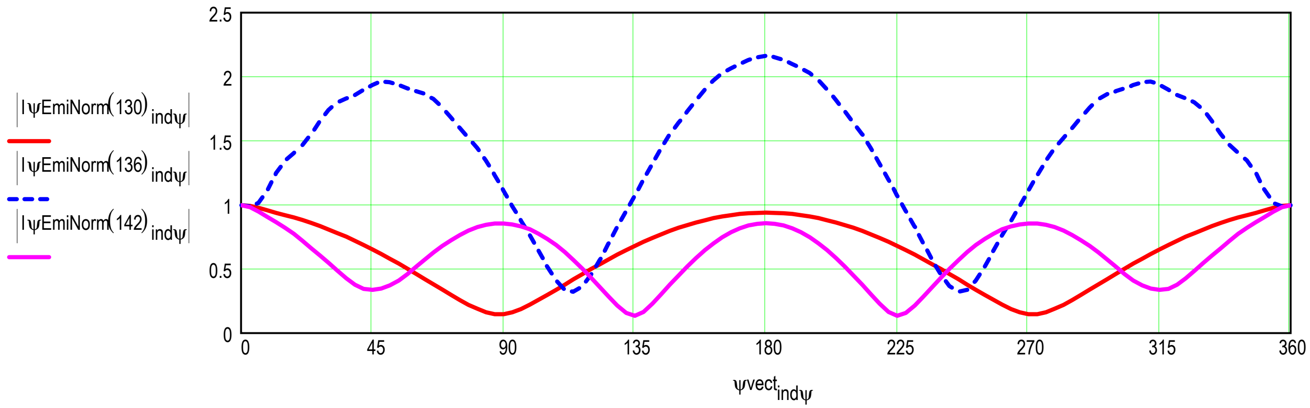
$fL_{136} = 251.189 \times 10^{6}$ Hz



$fL_{130} = 177.828 \times 10^{6}$ Hz

$fL_{136} = 251.189 \times 10^{6}$ Hz

$fL_{142} = 354.813 \times 10^{6}$ Hz



4) EFFECTIVE LENGTH OF THE LOOP ANTENNA

n being a nonnegative integer

$$Jprimen(n,x) := if\left(n > 0.5, \frac{Jn(n-1,x) - Jn(n+1,x)}{2}, -Jn(1,x)\right)$$

direct computation of the exact entries of the vector effective length

$$Dirhe\theta(j,\theta,\phi) := \left| \begin{array}{l} k \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0} \\ kas \leftarrow k\cdot a1val\cdot\sin(\theta) \\ \left[\frac{j\cdot\frac{4\cdot\pi}{k\cdot\tan(\theta)}}{\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{1}{WuCoeffMat_{n,j}}} \cdot \left[\sum_{n=1}^{nmax} \frac{n\cdot(j)^n\cdot\sin(n\cdot\phi)\cdot Jn(n,kas)}{WuCoeffMat_{n,j}}\right]\right] \text{ if } \tan(\theta) > 10^{-10} \\ \left(\frac{-2\cdot\pi\cdot a1val}{WuCoeffMat_{1,j}} \cdot \frac{\cos(\theta)\cdot\sin(\phi)}{\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{1}{WuCoeffMat_{n,j}}}\right) \text{ otherwise} \end{array}\right.$$

$$Dirhe\phi(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \frac{j\cdot 2\cdot\pi\cdot a1val}{\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{1}{WuCoeffMat_{n,j}}} \cdot \left[\frac{Jprimen(0,kas)}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{(j)^n\cdot\cos(n\cdot\phi)\cdot Jprimen(n,kas)}{WuCoeffMat_{n,j}}\right] \end{array}\right.$$

numerical computation of the vector effective length entries

$$Numhe\phi(j,\theta,\phi) := \frac{-a1val}{\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{1}{WuCoeffMat_{n,j}}} \cdot \left[\int_0^{2\cdot\pi} \left(\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{\cos(n\cdot\psi)}{WuCoeffMat_{n,j}}\right)\cdot\cos(\phi-\psi)\cdot\exp\left(j\cdot\frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta)\cdot\cos(\phi-\psi)\right)d\psi\right]$$

$$Numhe\theta(j,\theta,\phi) := \frac{-a1val}{\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{1}{WuCoeffMat_{n,j}}} \cdot \left[\int_0^{2\cdot\pi} \left(\frac{1}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{\cos(n\cdot\psi)}{WuCoeffMat_{n,j}}\right)\cdot\cos(\theta)\cdot\sin(\phi-\psi)\cdot\exp\left(j\cdot\frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta)\cdot\cos(\phi-\psi)\right)d\psi\right]$$

$fL_{120} = 100.000 \times 10^{6}$ Hz

$\text{Dirhe}\phi\left(120, \frac{\pi}{4}, \frac{\pi}{4}\right) = -0.86817860 + 0.40271921j$ m

$\text{Numhe}\phi\left(120, \frac{\pi}{4}, \frac{\pi}{4}\right) = -0.86817860 + 0.40271921j$ m

$\text{Dirhe}\phi\left(120, 0, \frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

$\text{Numhe}\phi\left(120, 0, \frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

$\text{Dirhe}\theta\left(120, \frac{\pi}{4}, \frac{\pi}{4}\right) = -0.58151060 - 0.07285994j$ m

$\text{Numhe}\theta\left(120, \frac{\pi}{4}, \frac{\pi}{4}\right) = -0.58151060 - 0.07285994j$ m

$\text{Dirhe}\theta\left(120, 0, \frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

$\text{Numhe}\theta\left(120, 0, \frac{\pi}{4}\right) = -0.84959391 - 0.05013768j$ m

approximate entries of the vector effective length

$$\text{Apphe}\theta(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ j\cdot 2\cdot\pi\cdot a1val\cdot\cos(\theta)\cdot\dfrac{j\cdot\dfrac{\sin(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\dfrac{kas^2}{8}\right)-\dfrac{\sin(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas - j\cdot\dfrac{\sin(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2}{\dfrac{1}{WuCoeffMat_{0,j}}+2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}} \end{array} \right.$$

$$\text{Apphe}\phi(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ j\cdot 2\cdot\pi\cdot a1val\cdot\dfrac{\dfrac{-1}{2\cdot WuCoeffMat_{0,j}}\cdot kas\cdot\left(1-\dfrac{kas^2}{8}\right)+j\cdot\dfrac{\cos(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\dfrac{3\cdot kas^2}{8}\right)-\dfrac{\cos(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas - j\cdot\dfrac{\cos(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2}{\dfrac{1}{WuCoeffMat_{0,j}}+2\cdot\displaystyle\sum_{n=1}^{nmax}\dfrac{1}{WuCoeffMat_{n,j}}} \end{array} \right.$$

Extreme approximation

$$\text{APPhe}\phi(j,\theta) := -j\cdot\pi\cdot\frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val^2\cdot\sin(\theta)$$

$Nangle2 := 180$ $indangle := 0..Nangle2$ $\text{Dirhe}\phi(j,\theta,\phi)$

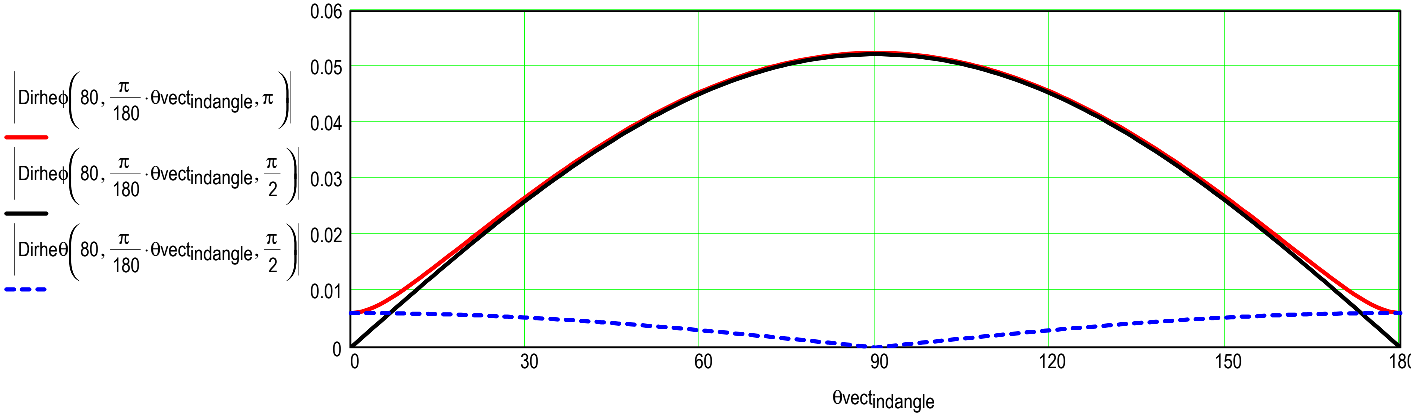
$\theta vect_{indangle} := \frac{180\cdot indangle}{Nangle2}$ $\phi vect_{indangle} := \frac{360\cdot indangle}{Nangle2}$

for $\phi = \pi$ and $\phi = \pi/2$

$fL_{80} = 10.000 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{80}}{c0} \cdot a1val = 0.059$

(dirheθ=0 for ϕ=π)

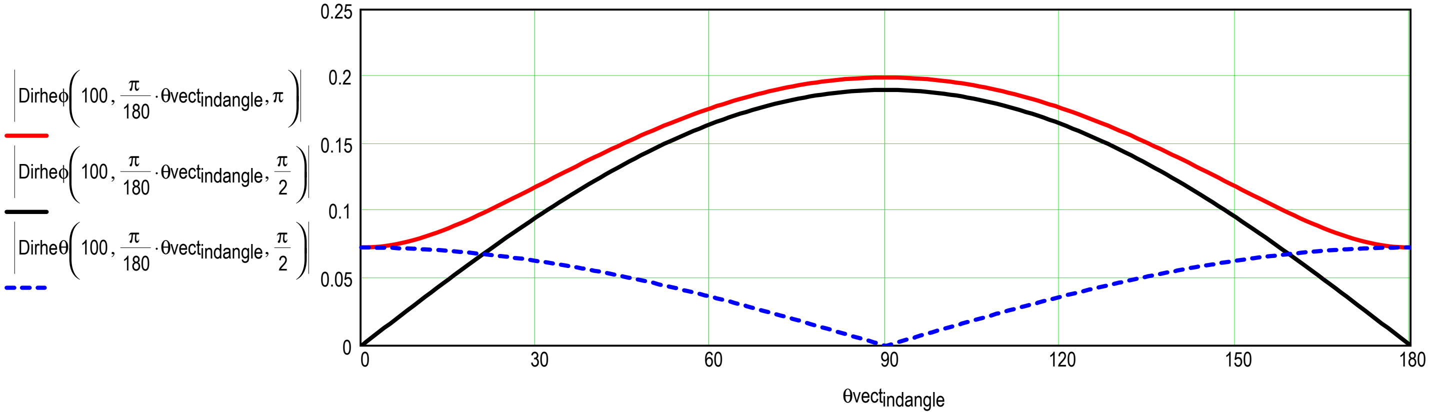


for $\phi = \pi$ and $\phi = \pi/2$

$fL_{100} = 31.623 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{100}}{c0} \cdot a1val = 0.186$

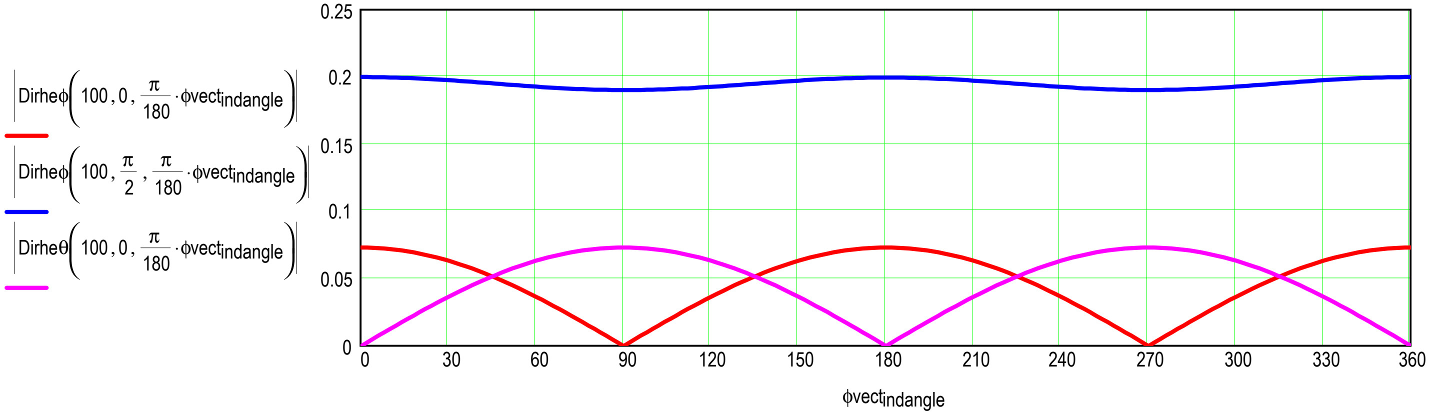
(dirheθ=0 for ϕ=π)



for $\theta = 0$ and $\theta = \pi/2$

$fL_{100} = 31.623 \times 10^{6}$ Hz

(dirheθ=0 for θ=π/2)

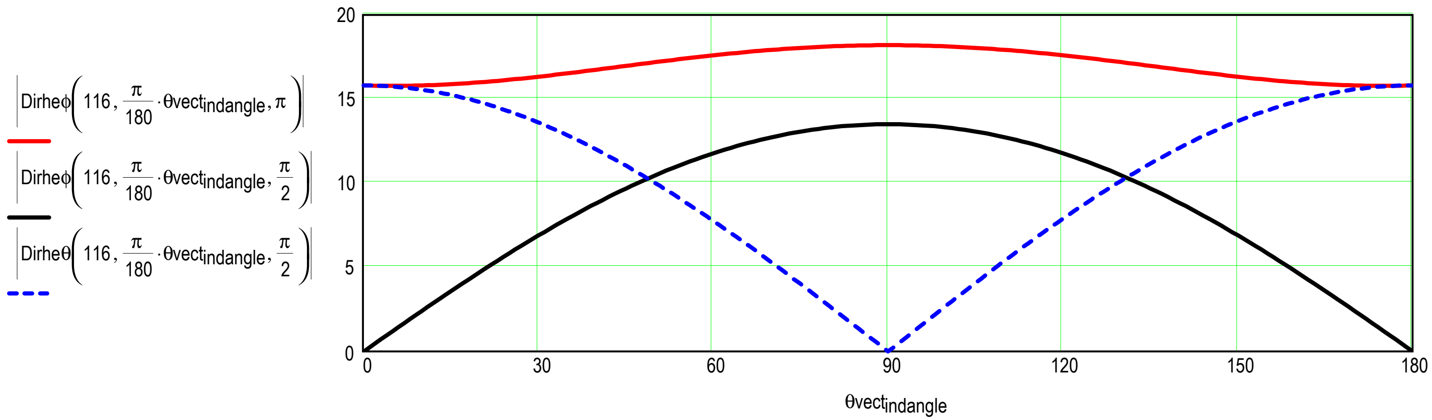


for $\phi = \pi$ and $\phi = \pi/2$

$fL_{116} = 79.433 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{116}}{c0} \cdot a1val = 0.466$

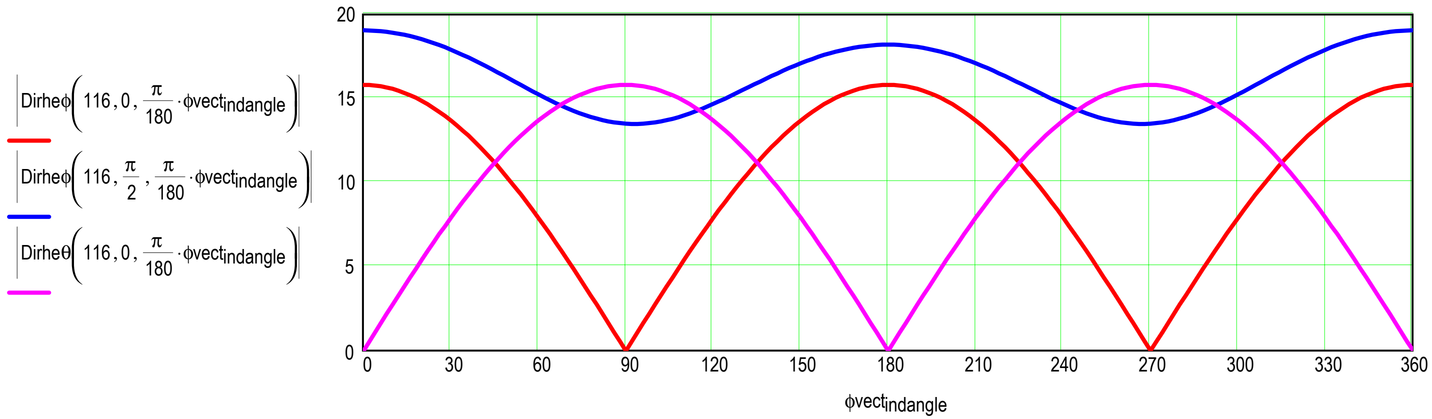
(dirheθ=0 for ϕ=π)



for $\theta = 0$ and $\theta = \pi/2$

$fL_{116} = 79.433 \times 10^{6}$ Hz

(dirheθ=0 for θ=π/2)

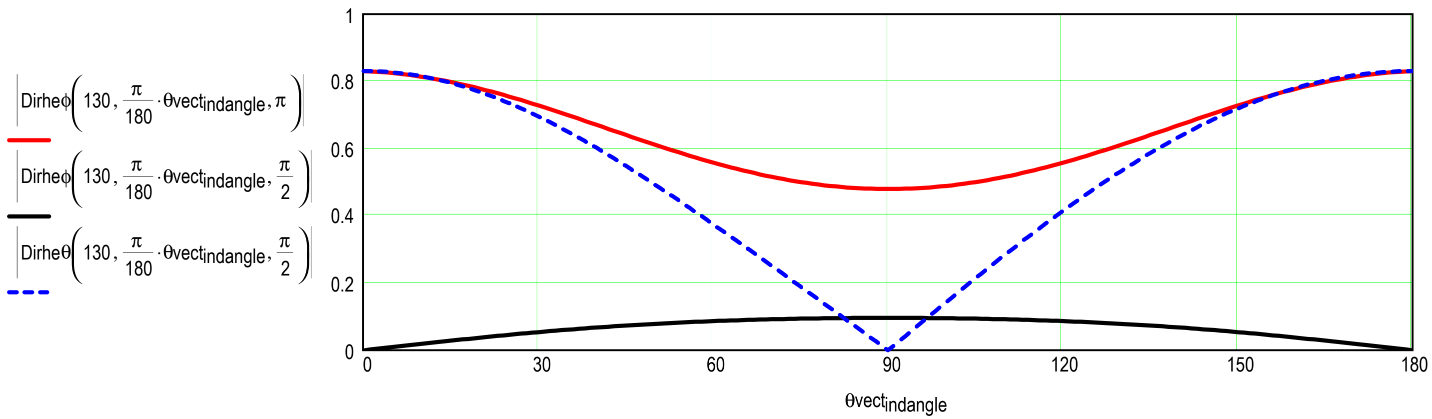


for $\phi = \pi$ and $\phi = \pi/2$

$fL_{130} = 177.828 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{130}}{c0} \cdot a1val = 1.044$

(dirheθ=0 for ϕ=π)



effective length entries versus frequency

$$He\phi 1dBmeter_i := 20\cdot\log\left(\left|Dirhe\phi\left(i,\frac{\pi}{2},\pi\right)\right|\right)$$

$$He\phi 2dBmeter_i := 20\cdot\log\left(\left|Dirhe\phi\left(i,\frac{\pi}{2},\frac{\pi}{2}\right)\right|\right)$$

$$He\theta 3dBmeter_i := 20\cdot\log\left(\left|Dirhe\theta\left(i,0,\frac{\pi}{2}\right)\right|\right)$$

$$He\phi APPdBmeter_i := 20\cdot\log\left(\left|APPhe\phi\left(i,\frac{\pi}{2}\right)\right|\right)$$

$fL_{40} = 1.000\times 10^6$ Hz

$He\phi 1dBmeter_{40} = -45.742$ dB(m)

$He\phi 2dBmeter_{40} = -45.742$ dB(m)

$He\theta 3dBmeter_{40} = -84.351$ dB(m)

$He\phi APPdBmeter_{40} = -45.743$ dB(m)

$fL_{80} = 10.000\times 10^6$ Hz

$He\phi 1dBmeter_{80} = -25.579$ dB(m)

$He\phi 2dBmeter_{80} = -25.621$ dB(m)

$He\theta 3dBmeter_{80} = -44.200$ dB(m)

$He\phi APPdBmeter_{80} = -25.743$ dB(m)

$fL_{100} = 31.623\times 10^6$ Hz

$He\phi 1dBmeter_{100} = -13.993$ dB(m)

$He\phi 2dBmeter_{100} = -14.408$ dB(m)

$He\theta 3dBmeter_{100} = -22.717$ dB(m)

$He\phi APPdBmeter_{100} = -15.743$ dB(m)

$fL_{120} = 100.000\times 10^6$ Hz

$He\phi 1dBmeter_{120} = 1.098$ dB(m)

$He\phi 2dBmeter_{120} = -3.039$ dB(m)

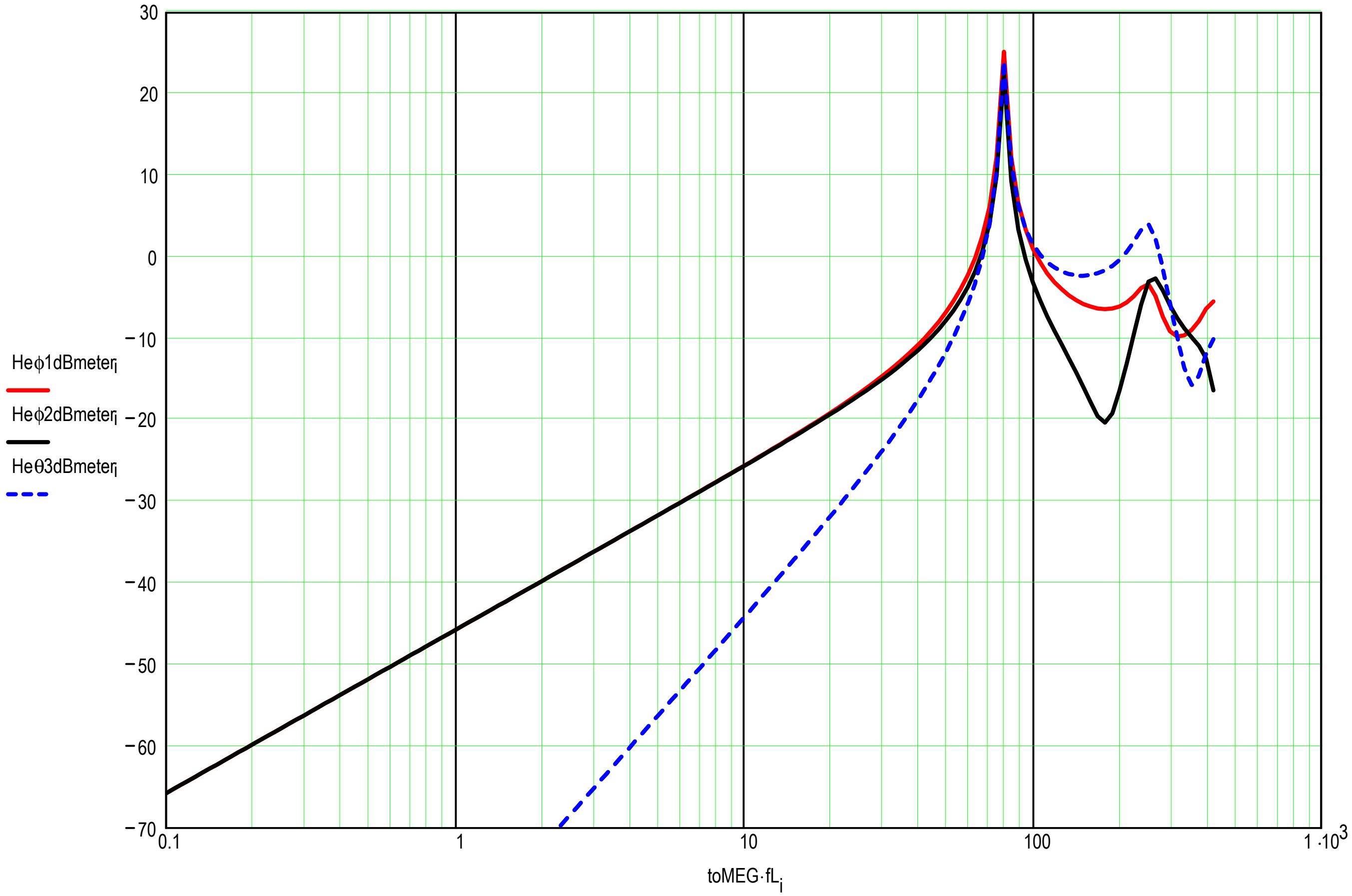
$He\theta 3dBmeter_{120} = 1.610$ dB(m)

$fL_{133} = 211.349\times 10^6$ Hz

$He\phi 1dBmeter_{133} = -5.567$ dB(m)

$He\phi 2dBmeter_{133} = -13.048$ dB(m)

$He\theta 3dBmeter_{133} = 0.634$ dB(m)



First parallel resonance $fL_{116} = 79.433 \times 10^{6}$ Hz

$He\phi1dBmeter_{116} = 25.209$ dB(m)

$He\phi2dBmeter_{116} = 22.616$ dB(m)

$He\theta3dBmeter_{116} = 23.985$ dB(m)

First series resonance $fL_{130} = 177.828 \times 10^{6}$ Hz

$He\phi1dBmeter_{130} = -6.371$ dB(m)

$He\phi2dBmeter_{130} = -20.295$ dB(m)

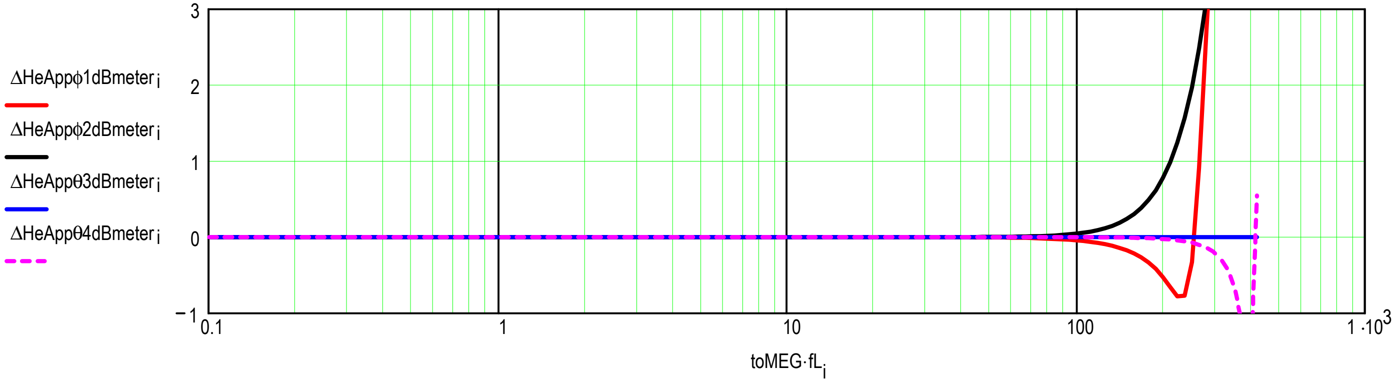
$He\theta3dBmeter_{130} = -1.606$ dB(m)

$$\Delta HeApp\phi1dBmeter_i := 20\cdot\log\left(\frac{\left|Apphe\phi\left(i,\frac{\pi}{2},\pi\right)\right|}{\left|Dirhe\phi\left(i,\frac{\pi}{2},\pi\right)\right|}\right)$$

$$\Delta HeApp\phi2dBmeter_i := 20\cdot\log\left(\frac{\left|Apphe\phi\left(i,\frac{\pi}{2},\frac{\pi}{2}\right)\right|}{\left|Dirhe\phi\left(i,\frac{\pi}{2},\frac{\pi}{2}\right)\right|}\right)$$

$$\Delta HeApp\theta3dBmeter_i := 20\cdot\log\left(\frac{\left|Apphe\theta\left(i,0,\frac{\pi}{2}\right)\right|}{\left|Dirhe\theta\left(i,0,\frac{\pi}{2}\right)\right|}\right)$$

$$\Delta HeApp\theta4dBmeter_i := 20\cdot\log\left(\frac{\left|Apphe\theta\left(i,\frac{\pi}{4},\frac{\pi}{2}\right)\right|}{\left|Dirhe\theta\left(i,\frac{\pi}{4},\frac{\pi}{2}\right)\right|}\right)$$



$fL_{132} = 199.526 \times 10^{6}$ Hz

$\frac{2\cdot\pi\cdot fL_{132}}{c0}\cdot a1val = 1.171$

$fL_{133} = 211.349 \times 10^{6}$ Hz

$\frac{2\cdot\pi\cdot fL_{133}}{c0}\cdot a1val = 1.240$

$\Delta HeApp\phi1dBmeter_{133} = -0.657$ dB

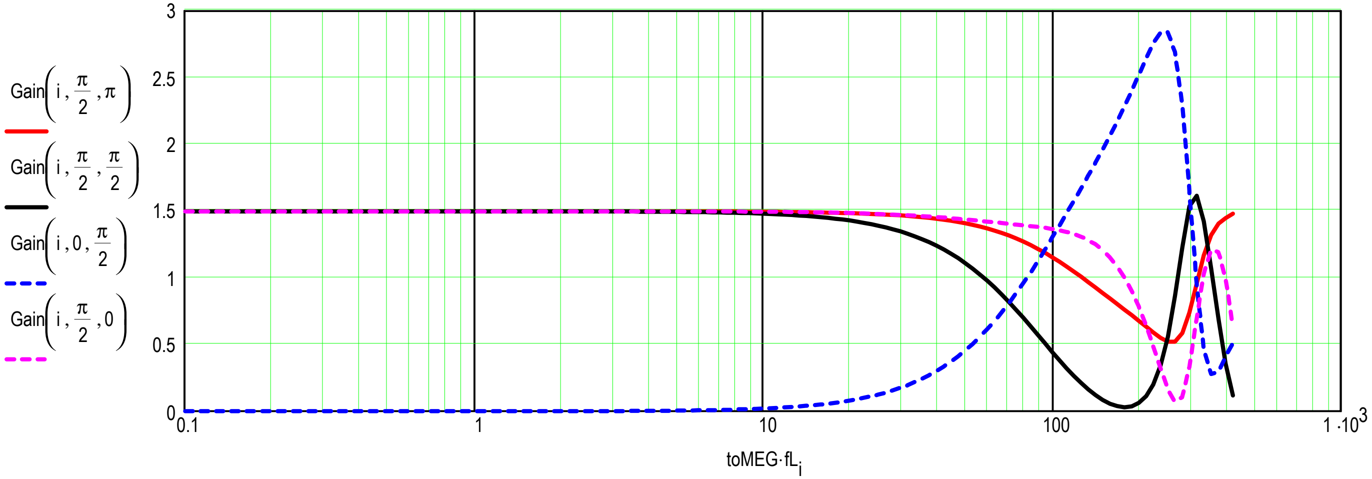
$\Delta HeApp\phi2dBmeter_{133} = 0.984$ dB

$\Delta HeApp\theta3dBmeter_{133} = 0.000$ dB

$\Delta HeApp\theta4dBmeter_{133} = -0.031$ dB

5) GAIN OF THE LOOP ANTENNA

$$Gain(j,\theta,\phi) := \frac{\eta0\cdot\left(\frac{2\cdot\pi\cdot fL_j}{c0}\right)^2\cdot\left[\left(\left|Dirhe\theta(j,\theta,\phi)\right|\right)^2+\left(\left|Dirhe\phi(j,\theta,\phi)\right|\right)^2\right]}{4\cdot\pi\cdot Re(Zloopex_j)}$$



$fL_{80} = 10.000\times10^6$ Hz

$fL_{92} = 19.953\times10^6$ Hz

$Gain\left(92,\frac{\pi}{2},\frac{\pi}{2}\right) = 1.431$

$10\cdot\log\left(\frac{Gain\left(92,\frac{\pi}{2},\frac{\pi}{2}\right)}{1.5}\right) = -0.205$ dB

$fL_{100} = 31.623\times10^6$ Hz

$Gain\left(100,\frac{\pi}{2},\frac{\pi}{2}\right) = 1.332$

$10\cdot\log\left(\frac{Gain\left(100,\frac{\pi}{2},\frac{\pi}{2}\right)}{1.5}\right) = -0.517$ dB

$fL_{110} = 56.234\times10^6$ Hz

$fL_{116} = 79.433\times10^6$ Hz

$fL_{130} = 177.828\times10^6$ Hz

the last row of this matrix (row 360) contains the maximum gain, and then the angles θ and ϕ in degrees corresponding to this maximum gain.

```
GainMaxMat(j) := | M ← 0
                 | for iθ ∈ 0..18
                 |   for iϕ ∈ 0..18
                 |     | θdeg ← 10·iθ
                 |     | ϕdeg ← 10·iϕ
                 |     | row ← 19·iθ + iϕ
                 |     | M_row,0 ← Gain(j, θdeg·π/180, ϕdeg·π/180)
                 |     | M_row,1 ← θdeg
                 |     | M_row,2 ← ϕdeg
                 | csort(M,0)
```

$submatrix(GainMaxMat(80),359,360,0,2) = \begin{pmatrix} 1.496594 & 90.000000 & 180.000000 \\ 1.496653 & 90.000000 & 0.000000 \end{pmatrix}$

$submatrix(GainMaxMat(100),359,360,0,2) = \begin{pmatrix} 1.466183 & 90.000000 & 10.000000 \\ 1.470318 & 90.000000 & 0.000000 \end{pmatrix}$

$submatrix(GainMaxMat(110),359,360,0,2) = \begin{pmatrix} 1.416428 & 90.000000 & 10.000000 \\ 1.428108 & 90.000000 & 0.000000 \end{pmatrix}$

$submatrix(GainMaxMat(116),359,360,0,2) = \begin{pmatrix} 1.383600 & 80.000000 & 0.000000 \\ 1.395027 & 90.000000 & 0.000000 \end{pmatrix}$

$submatrix(GainMaxMat(130),357,360,0,2) = \begin{pmatrix} 2.285764 & 180.000000 & 20.000000 \\ 2.285764 & 0.000000 & 80.000000 \\ 2.285764 & 0.000000 & 150.000000 \\ 2.285764 & 0.000000 & 30.000000 \end{pmatrix}$

6) SHORT-CIRCUIT CURRENT OF THE LOOP ANTENNA

numerical computation of the entries of the vector effective length divided by the loop impedance

$$NumW\theta(j,\theta,\phi) := \frac{Numhe\theta(j,\theta,\phi)}{Zloopex_j} \qquad NumW\phi(j,\theta,\phi) := \frac{Numhe\phi(j,\theta,\phi)}{Zloopex_j}$$

direct computation of the exact entries of the vector effective length divided by the loop impedance

$$DirW\theta(j,\theta,\phi) := \left| \begin{array}{l} k \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0} \\ kas \leftarrow k\cdot a1val\cdot\sin(\theta) \\ \left[\frac{4}{\eta 0\cdot k\cdot\tan(\theta)}\cdot\left[\sum_{n=1}^{nmax}\frac{n\cdot(j)^n\cdot\sin(n\cdot\phi)\cdot Jn(n,kas)}{WuCoeffMat_{n,j}}\right]\right] \text{ if } \tan(\theta) > 10^{-10} \\ \left(\frac{j\cdot 2\cdot a1val}{\eta 0\cdot WuCoeffMat_{1,j}}\cdot\cos(\theta)\cdot\sin(\phi)\right) \text{ otherwise} \end{array}\right.$$

$$DirW\phi(j,\theta,\phi) := \left| \begin{array}{l} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \frac{2\cdot a1val}{\eta 0}\cdot\left[\frac{Jprimen(0,kas)}{WuCoeffMat_{0,j}} + 2\cdot\sum_{n=1}^{nmax}\frac{(j)^n\cdot\cos(n\cdot\phi)\cdot Jprimen(n,kas)}{WuCoeffMat_{n,j}}\right] \end{array}\right.$$

$fL_{120} = 100.000\times 10^6$ Hz

$DirW\phi\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.58235829\times 10^{-4} - 7.21759732j\times 10^{-4}$ mS

$NumW\phi\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.58235829\times 10^{-4} - 7.21759732j\times 10^{-4}$ mS

$DirW\phi\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

$NumW\phi\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

$DirW\theta\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -3.51143841\times 10^{-6} - 5.23523725j\times 10^{-4}$ mS

$NumW\theta\left(120,\frac{\pi}{4},\frac{\pi}{4}\right) = -3.51143841\times 10^{-6} - 5.23523725j\times 10^{-4}$ mS

$DirW\theta\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

$NumW\theta\left(120,0,\frac{\pi}{4}\right) = -5.50010540\times 10^{-5} - 7.58285838j\times 10^{-4}$ mS

approximate entries
of the vector
effective length
divided
by the loop impedance

$$AppW\theta(j,\theta,\phi) := \begin{vmatrix} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \frac{2\cdot a1val\cdot\cos(\theta)}{\eta 0}\cdot\left[j\cdot\frac{\sin(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\frac{kas^2}{8}\right)-\frac{\sin(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas - j\cdot\frac{\sin(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2\right]\end{vmatrix}$$

$$AppW\phi(j,\theta,\phi) := \begin{vmatrix} kas \leftarrow \frac{2\cdot\pi\cdot fL_j}{c0}\cdot a1val\cdot\sin(\theta) \\ \frac{2\cdot a1val}{\eta 0}\cdot\left[\frac{-1}{2\cdot WuCoeffMat_{0,j}}\cdot kas\cdot\left(1-\frac{kas^2}{8}\right)+j\cdot\frac{\cos(\phi)}{WuCoeffMat_{1,j}}\cdot\left(1-\frac{3\cdot kas^2}{8}\right)-\frac{\cos(2\cdot\phi)}{2\cdot WuCoeffMat_{2,j}}\cdot kas - j\cdot\frac{\cos(3\cdot\phi)}{8\cdot WuCoeffMat_{3,j}}\cdot kas^2\right]\end{vmatrix}$$

Extreme approximation

$$APPW\phi(j,\theta) := \frac{-\pi\cdot a1val^2}{c0\cdot L0}\cdot\sin(\theta)$$

Comparisons:

$fL_{80} = 10.000\times 10^6$ Hz

$DirW\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.37627652\times 10^{-4} - 5.15633974j\times 10^{-5}$ mS

$AppW\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = -4.37627656\times 10^{-4} - 5.15633938j\times 10^{-5}$ mS

$APPW\phi\left(80,\frac{\pi}{4}\right) = -4.379852\times 10^{-4}$ mS

$Dirhe\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 4.35722134\times 10^{-3} - 0.03698968j$ m

$Apphe\phi\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 4.35722103\times 10^{-3} - 0.03698968j$ m

$APPhe\phi\left(80,\frac{\pi}{4}\right) = -0.036501j$ m

$DirW\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.21735999\times 10^{-7} - 3.64654337j\times 10^{-5}$ mS

$AppW\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.21782464\times 10^{-7} - 3.64654330j\times 10^{-5}$ mS

$Dirhe\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.08216435\times 10^{-3} + 2.71052597j\times 10^{-5}$ m

$Apphe\theta\left(80,\frac{\pi}{4},\frac{\pi}{4}\right) = 3.08216429\times 10^{-3} + 2.71091870j\times 10^{-5}$ m

versus frequency

$W\phi1dBmeter_i := 20 \cdot \log\left(\left|DirW\phi\left(i, \frac{\pi}{2}, \pi\right)\right|\right)$

$W\phi2dBmeter_i := 20 \cdot \log\left(\left|DirW\phi\left(i, \frac{\pi}{2}, \frac{\pi}{2}\right)\right|\right)$

$W\theta3dBmeter_i := 20 \cdot \log\left(\left|DirW\theta\left(i, 0, \frac{\pi}{2}\right)\right|\right)$

$fL_{40} = 1.000 \times 10^6$ Hz

$W\phi1dBmeter_{40} = -64.160$ dB(mS)

$W\phi2dBmeter_{40} = -64.161$ dB(mS)

$W\theta3dBmeter_{40} = -102.769$ dB(mS)

$fL_{80} = 10.000 \times 10^6$ Hz

$W\phi1dBmeter_{80} = -64.119$ dB(mS)

$W\phi2dBmeter_{80} = -64.160$ dB(mS)

$W\theta3dBmeter_{80} = -82.740$ dB(mS)

$fL_{100} = 31.623 \times 10^6$ Hz

$W\phi1dBmeter_{100} = -63.744$ dB(mS)

$W\phi2dBmeter_{100} = -64.159$ dB(mS)

$W\theta3dBmeter_{100} = -72.467$ dB(mS)

$fL_{120} = 100.000 \times 10^6$ Hz

$W\phi1dBmeter_{120} = -59.882$ dB(mS)

$W\phi2dBmeter_{120} = -64.019$ dB(mS)

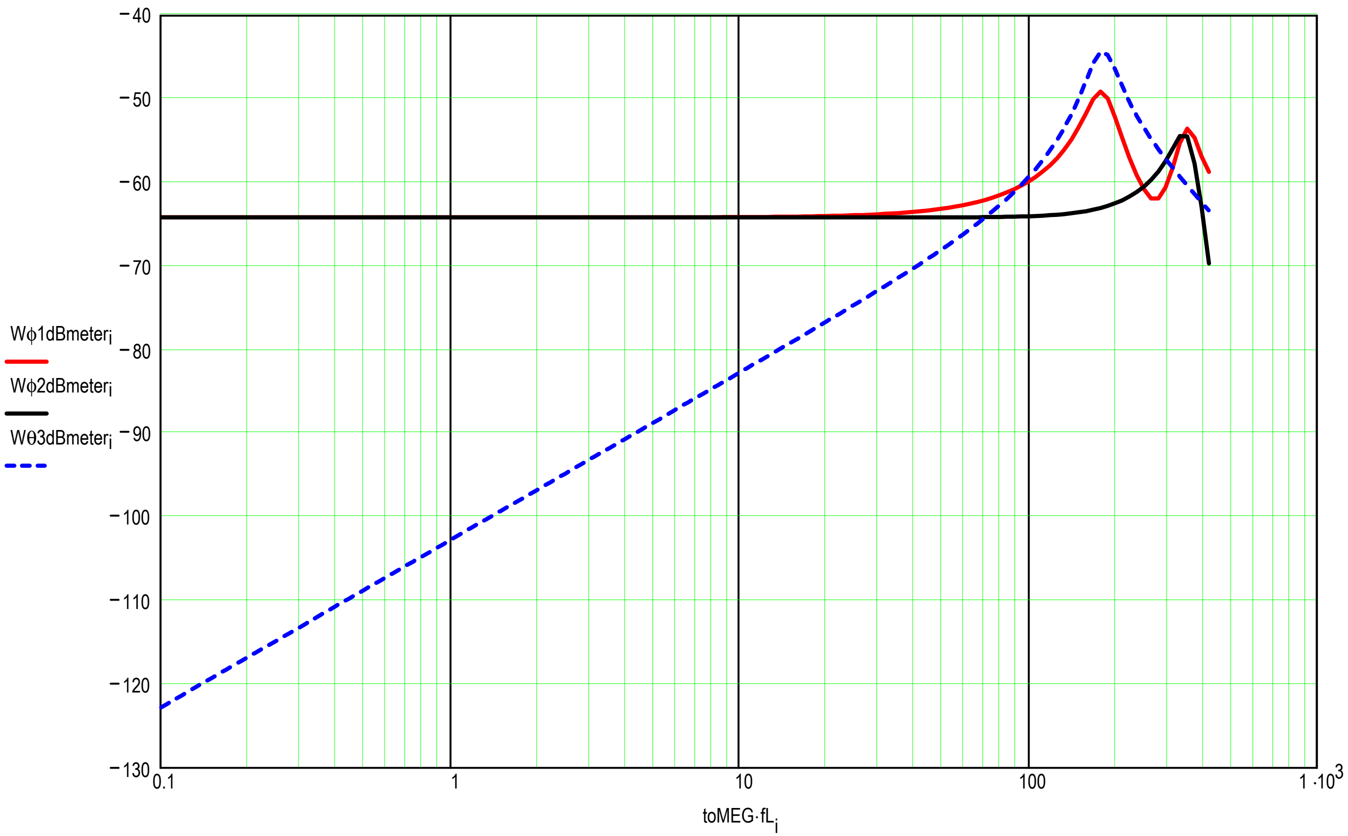
$W\theta3dBmeter_{120} = -59.370$ dB(mS)

$fL_{133} = 211.349 \times 10^6$ Hz

$W\phi1dBmeter_{133} = -54.634$ dB(mS)

$W\phi2dBmeter_{133} = -62.115$ dB(mS)

$W\theta3dBmeter_{133} = -48.433$ dB(mS)



difference of about 20dB between second and third curves at what frequency?

$fL_{76} = 7.943\times 10^{6}$ Hz $W\phi2dBmeter_{76} = -64.160$ dB(mS) $W\theta3dBmeter_{76} = -84.751$ dB(mS)

$fL_{77} = 8.414\times 10^{6}$ Hz $W\phi2dBmeter_{77} = -64.160$ dB(mS) $W\theta3dBmeter_{77} = -84.248$ dB(mS)

$fL_{78} = 8.913\times 10^{6}$ Hz $W\phi2dBmeter_{78} = -64.160$ dB(mS) $W\theta3dBmeter_{78} = -83.746$ dB(mS)

at $fL_{77} = 8.414\times 10^{6}$ Hz k a1val is equal to $\frac{2\cdot\pi\cdot fL_{77}}{c0}\cdot a1val = 0.049376$ and 2*a1val/λ is equal to $\frac{2\cdot a1val}{\left(\frac{c0}{fL_{77}}\right)} = 0.016$

the second curve deviates by about 1dB from its low frequency plateau at what frequency?

$fL_{0} = 100.000\times 10^{3}$ Hz $W\phi2dBmeter_{0} = -64.161$ dB(mS)

$fL_{40} = 1.000\times 10^{6}$ Hz $W\phi2dBmeter_{40} = -64.161$ dB(mS)

$fL_{128} = 158.489\times 10^{6}$ Hz $W\phi2dBmeter_{128} = -63.404$ dB(mS)

$fL_{129} = 167.880\times 10^{6}$ Hz $W\phi2dBmeter_{129} = -63.235$ dB(mS)

$fL_{130} = 177.828\times 10^{6}$ Hz $W\phi2dBmeter_{130} = -63.029$ dB(mS)

at $fL_{129} = 167.880\times 10^{6}$ Hz k a1val is equal to $\frac{2\cdot\pi\cdot fL_{129}}{c0}\cdot a1val = 0.985184$ and 2*a1val/λ is equal to $\frac{2\cdot a1val}{\left(\frac{c0}{fL_{129}}\right)} = 0.314$

First parallel resonance

$fL_{116} = 79.433 \times 10^{6}$ Hz

$W\phi 1dBmeter_{116} = -61.509$ dB(m)

$W\phi 2dBmeter_{116} = -64.101$ dB(m)

$W\theta 3dBmeter_{116} = -62.732$ dB(m)

$\frac{2 \cdot \pi \cdot fL_{116}}{c0} \cdot a1val = 0.466141$

First series resonance

$fL_{130} = 177.828 \times 10^{6}$ Hz

$W\phi 1dBmeter_{130} = -49.105$ dB(m)

$W\phi 2dBmeter_{130} = -63.029$ dB(m)

$W\theta 3dBmeter_{130} = -44.340$ dB(m)

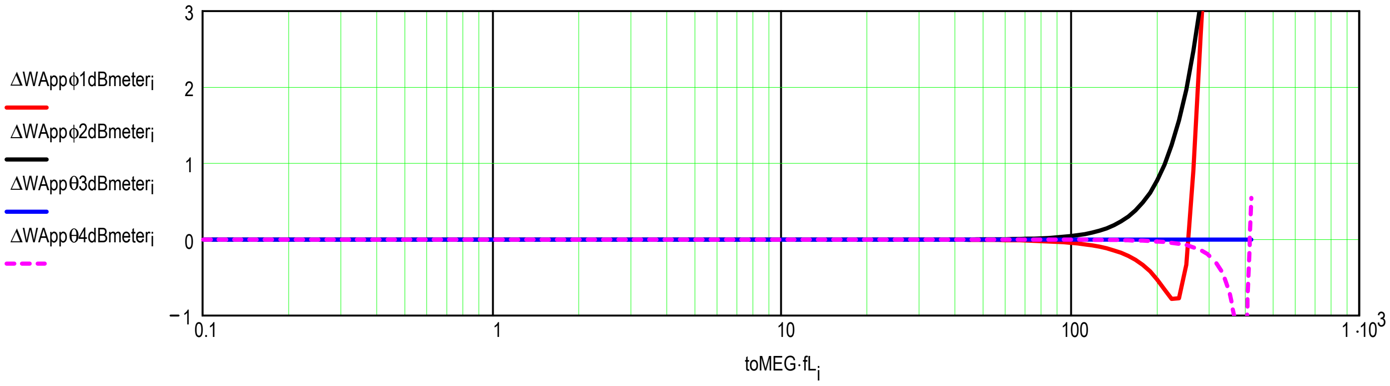
$\frac{2 \cdot \pi \cdot fL_{130}}{c0} \cdot a1val = 1.043559$

$$\Delta WApp\phi 1dBmeter_i := 20 \cdot \log\left(\frac{\left|AppW\phi\left(i, \frac{\pi}{2}, \pi\right)\right|}{\left|DirW\phi\left(i, \frac{\pi}{2}, \pi\right)\right|}\right)$$

$$\Delta WApp\phi 2dBmeter_i := 20 \cdot \log\left(\frac{\left|AppW\phi\left(i, \frac{\pi}{2}, \frac{\pi}{2}\right)\right|}{\left|DirW\phi\left(i, \frac{\pi}{2}, \frac{\pi}{2}\right)\right|}\right)$$

$$\Delta WApp\theta 3dBmeter_i := 20 \cdot \log\left(\frac{\left|AppW\theta\left(i, 0, \frac{\pi}{2}\right)\right|}{\left|DirW\theta\left(i, 0, \frac{\pi}{2}\right)\right|}\right)$$

$$\Delta WApp\theta 4dBmeter_i := 20 \cdot \log\left(\frac{\left|AppW\theta\left(i, \frac{\pi}{4}, \frac{\pi}{2}\right)\right|}{\left|DirW\theta\left(i, \frac{\pi}{4}, \frac{\pi}{2}\right)\right|}\right)$$



$fL_{132} = 199.526 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{132}}{c0} \cdot a1val = 1.171$

$fL_{133} = 211.349 \times 10^{6}$ Hz

$\frac{2 \cdot \pi \cdot fL_{133}}{c0} \cdot a1val = 1.240$

$\Delta WApp\phi 1dBmeter_{133} = -0.657$ dB

$\Delta WApp\phi 2dBmeter_{133} = 0.984$ dB

$\Delta WApp\theta 3dBmeter_{133} = 0.000$ dB

$\Delta WApp\theta 4dBmeter_{133} = -0.031$ dB

$\frac{2 \cdot \pi \cdot 200 \cdot 10^{6}}{c0} \cdot a1val = 1.174$

7) FAST AND MORE ACCURATE COMPUTATION OF SOME RESULTS

$$\mathrm{VectConst\kappa a} := \left| \begin{array}{l} \mathrm{dwOV2a} \leftarrow \dfrac{\mathrm{dw1val}}{2\cdot \mathrm{a1val}} \\ \mathrm{Const\kappa a}_0 \leftarrow \dfrac{1}{\pi}\cdot \ln\left(\dfrac{16\cdot \mathrm{a1val}}{\mathrm{dw1val}}\right) \\ \mathrm{Const\kappa a}_1 \leftarrow \dfrac{\mathrm{K0(dwOV2a)}\cdot \mathrm{I0(dwOV2a)} + \mathrm{C}(1)}{\pi} \\ \text{for } n \in 1..\mathrm{nmax} \\ \quad \left| \begin{array}{l} x \leftarrow (n+1)\cdot \mathrm{dwOV2a} \\ \mathrm{Const\kappa a}_{n+1} \leftarrow \dfrac{\mathrm{K0}(x)\cdot \mathrm{I0}(x) + \mathrm{C}(n+1)}{\pi} \end{array} \right. \\ \mathrm{Const\kappa a} \end{array} \right.$$

$\mathrm{a1val} = 0.280$

$\mathrm{dw1val} = 0.014$

$\mathrm{nmax} = 20.000$

$$\mathrm{FastZloop2(f)} := \left| \begin{array}{l} \mathrm{ka} \leftarrow \dfrac{2\cdot\pi\cdot f}{\mathrm{c0}}\cdot \mathrm{a1val} \\ \kappa a_0 \leftarrow \mathrm{VectConst\kappa a}_0 + \displaystyle\sum_{q=0}^{30} \chi_{q,0}\cdot \mathrm{ka}^q \\ \kappa a_1 \leftarrow \mathrm{VectConst\kappa a}_1 + \displaystyle\sum_{q=0}^{30} \chi_{q,1}\cdot \mathrm{ka}^q \\ A_0 \leftarrow \mathrm{ka}\cdot \kappa a_1 \\ \text{for } n \in 1..\mathrm{nmax} \\ \quad \left| \begin{array}{l} \kappa a_{n+1} \leftarrow \mathrm{VectConst\kappa a}_{n+1} + \displaystyle\sum_{q=0}^{30} \chi_{q,n+1}\cdot \mathrm{ka}^q \\ A_n \leftarrow \mathrm{ka}\cdot \dfrac{\kappa a_{n+1} + \kappa a_{n-1}}{2} - \dfrac{n^2}{\mathrm{ka}}\cdot \kappa a_n \end{array} \right. \\ Z \leftarrow \dfrac{j\cdot\pi\cdot\eta 0}{\dfrac{1}{A_0} + 2\cdot \displaystyle\sum_{n=1}^{\mathrm{nmax}} \dfrac{1}{A_n}} \end{array} \right.$$

$fL_{80} = 10.000\times 10^6$ Hz

$|\mathrm{Zloopex}_{80}| = 84.522882$ Ω

$|\mathrm{FastZloop2}(fL_{80})| = 84.522882$ Ω

$fL_{100} = 31.623\times 10^6$ Hz

$|\mathrm{Zloopex}_{100}| = 307.267627$ Ω

$|\mathrm{FastZloop2}(fL_{100})| = 307.267627$ Ω

$fL_{120} = 100.000\times 10^6$ Hz

$|\mathrm{Zloopex}_{120}| = 1.119422\times 10^3$ Ω

$|\mathrm{FastZloop2}(fL_{120})| = 1.119422\times 10^3$ Ω

$fL_{133} = 211.349\times 10^6$ Hz

$|\mathrm{Zloopex}_{133}| = 284.018154$ Ω

$|\mathrm{FastZloop2}(fL_{133})| = 284.018154$ Ω

$\mathrm{ModFastZloop2(f)} := |\mathrm{FastZloop2(f)}|$

$\mathrm{FastYloop2(f)} := \dfrac{1}{\mathrm{FastZloop2(f)}}$

$\mathrm{ModFastYloop2(f)} := |\mathrm{FastYloop2(f)}|$

Parallel resonances:

first

$fres1ph := root(Im(FastZloop2(fx)), fx, 70\cdot10^6, 90\cdot10^6)$ — $fres1ph = 79.392\times10^6$ Hz — $FastZloop2(fres1ph) = 2.173\times10^4 + 2.657j\times10^{-10}$ Ω

$f := fres1ph$ — $|FastZloop2(fres1ph)| = 2.173\times10^4$ Ω

Given $f > 75\cdot10^6$ $f < 85\cdot10^6$

$fres1mag := Maximize(ModFastZloop2, f)$ — $fres1mag = 79.342\times10^6$ Hz — $FastZloop2(fres1mag) = 2.173\times10^4 + 1.034j\times10^3$ Ω

$\frac{2\cdot\pi\cdot fres1mag}{c0}\cdot a1val = 0.466$ — $|FastZloop2(fres1mag)| = 2.175\times10^4$ Ω

second

$fres3ph := root(Im(FastZloop2(fx)), fx, 230\cdot10^6, 270\cdot10^6)$ — $fres3ph = 247.592\times10^6$ Hz — $FastZloop2(fres3ph) = 717.988 - 1.475j\times10^{-12}$ Ω

$f := fres3ph$ — $|FastZloop2(fres3ph)| = 717.988$ Ω

Given $f > 230\cdot10^6$ $f < 270\cdot10^6$

$fres3mag := Maximize(ModFastZloop2, f)$ — $fres3mag = 256.492\times10^6$ Hz — $FastZloop2(fres3mag) = 734.427 - 221.068j$ Ω

$\frac{2\cdot\pi\cdot fres3mag}{c0}\cdot a1val = 1.505$ — $|FastZloop2(fres3mag)| = 766.978$ Ω

Series resonances:

first

$fres2ph := root(Im(FastYloop2(fx)), fx, 160\cdot10^6, 200\cdot10^6)$ — $fres2ph = 187.982\times10^6$ Hz — $FastYloop2(fres2ph) = 6.637\times10^{-3} + 1.895j\times10^{-17}$ S

$f := fres2ph$ — $|FastYloop2(fres2ph)| = 6.637\times10^{-3}$ S

Given $f > 160\cdot10^6$ $f < 200\cdot10^6$ — $|FastZloop2(fres2ph)| = 150.663$ Ω

$fres2mag := Maximize(ModFastYloop2, f)$ — $fres2mag = 178.612\times10^6$ Hz — $FastYloop2(fres2mag) = 6.803\times10^{-3} + 2.660j\times10^{-3}$ S

$\frac{2\cdot\pi\cdot fres2mag}{c0}\cdot a1val = 1.048$ — $|FastYloop2(fres2mag)| = 7.305\times10^{-3}$ S

$|FastZloop2(fres2mag)| = 136.896$ Ω

second

$fres4ph := root(Im(FastYloop2(fx)), fx, 300\cdot10^6, 400\cdot10^6)$ — $fres4ph = 372.329\times10^6$ Hz — $FastYloop2(fres4ph) = 4.849\times10^{-3}$ S

$f := fres4ph$ — $|FastYloop2(fres4ph)| = 4.849\times10^{-3}$ S

Given $f > 300\cdot10^6$ $f < 400\cdot10^6$ — $|FastZloop2(fres4ph)| = 206.243$ Ω

$fres4mag := Maximize(ModFastYloop2, f)$ — $fres4mag = 351.585\times10^6$ Hz — $FastYloop2(fres4mag) = 5.411\times10^{-3} + 2.363j\times10^{-3}$ S

$\frac{2\cdot\pi\cdot fres4mag}{c0}\cdot a1val = 2.063$ — $|FastYloop2(fres4mag)| = 5.904\times10^{-3}$ S

$|FastZloop2(fres4mag)| = 169.366$ Ω