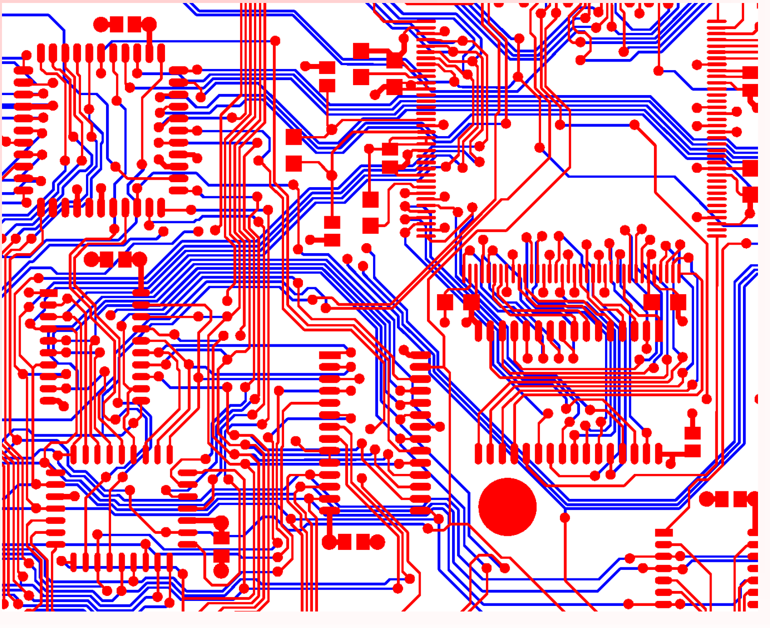
Seminar 32

Tutorial on Echo and Crosstalk in Printed Circuit Boards and Multi-Chip Modules — Lecture Slides

Third Edition

Frédéric Broydé & Evelyne Clavelier



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Tutorial on Echo and Crosstalk in Printed Circuit Boards and Multi-Chip Modules — Lecture Slides

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Foreword

This tutorial explains and compares classical and innovative techniques for controlling crosstalk and echo in printed circuit assemblies and multi-chip modules.

The first part of the tutorial presents propagation models based on the theory of multiconductor transmission lines (MTLs). This theoretical part uses matrix algebra, but is not difficult to follow. It is focused on the following points which are essential for the applications considered in the second part: concisely presenting the MTL model; identifying a few common misconceptions on modal voltages and currents; comparing biorthonormal eigenvectors with associated eigenvectors; and explaining the total decoupling of the telegrapher's equations.

In a second part, this theoretical framework is used to describe and analyze most known techniques for reducing crosstalk and echo in a uniform multiconductor interconnection. Here, the purpose is the reduction of the number of transmission conductors and their spacing. The effect of discontinuities such as vias, connectors, etc, is not covered. The following schemes are considered: multiple single-ended links, multiple differential links, links implementing modal transmission schemes and multichannel pseudo-differential links.

Howard L. Heck
Principal engineer at Intel Corporation
Hillsboro, Oregon
September 2011

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Part A — Propagation models

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1. Introduction and definitions

❐ An electrical link used for signal transmission comprises a physical device called interconnection. We emphasize the case where the interconnection is built in a printed circuit board (PCB) or multi-chip module (MCM).

❐ We shall use multiconductor transmission line (MTL) theory to explain and compare classical and innovative techniques for controlling crosstalk and echo.

❐ This tutorial is derived from a much more comprehensive training seminar of Excem, the Seminar 33. Many questions could not be included in the present tutorial.

❐ In a first part, we present propagation models based on MTL theory. This allows us to:

- define and study modal voltages and currents;
- compare biorthonormal eigenvectors with associated eigenvectors;
- explain the total decoupling of the telegrapher's equations.

❒ In a second part, this background is used to describe and analyze most known techniques for reducing crosstalk and echo in a uniform multiconductor interconnection. We will focus on innovative techniques.

❒ In circuit theory, interconnections are represented using a 0-D model: the node.

❒ The transmission line (TL) model of a two-conductor interconnection may be regarded as:

- ◆ a simplified 1-D model as regards propagation along the interconnection;
- ◆ a static 2-D model for the computation of the p.u.l. parameters.

❒ The MTL model extends the TL model to problems involving multiple conductors.

❒ Why are we devoting time to the MTL model when full-wave 3-D software is available?

❒ First reason: the more comprehensive the model, the smaller the problems it can handle.

❒ Second reason: optimization is useful only when a small number of dominant parameters has been identified.

❒ Third reason: invention occurs only when the inventor has a simple mental image of the problem and the effect of each main parameter.

❒ The best model and problem combination is the one that gives the best result for your objectives, your budget and your deadlines:

- for exploring innovative solution, analytical formula provide more insight;
- at the initial design stage, no detailed 3-D configuration is specified;
- for the analysis of a finalized design, the 3-D data is available (and large).

❒ If we use basic circuit theory, where each interconnection is a node, our model ignores the actual behavior of interconnections: it is inaccurate at high frequencies.

❒ If we use an enhanced circuit theory where some lumped elements (e.g. a stray capacitance, or a pi network) are used to model the longer interconnections:

- ◆ our model is more accurate in the lumped-element region [35, ch. 3];
- ◆ an LC pi network nearing resonance indicates that we have left this region.

❒ If we use a TL model for the longer interconnections:

- ◆ our model describes propagation and reflections;
- ◆ it allows us to reduce echo and to compensate losses;
- ◆ it ignores the effect of nearby conductors, which cannot be controlled.

❒ If we use a MTL model for the longer interconnections:

- ◆ our model accurately describes the interactions between conductors;
- ◆ it allows us to control echo, internal crosstalk and losses.

❒ In this tutorial:

◆ interactions between conductors within a multiconductor interconnection are treated using a MTL model;

◆ circuit models are used for other parts of the link, with caution;

◆ interactions between conductors belonging to different parallel interconnections are treated using a single MTL model;

◆ other interactions involving conductors are not considered;

◆ emission and immunity, as defined in electromagnetic compatibility (EMC), are not considered.

❒ *Uniform* means invariant along the interconnection or TL or MTL.

❒ *Homogeneous* means invariant in a cross-section of the interconnection or TL or MTL.

2. The 2-conductor transmission line in the frequency domain

❒ We consider an interconnection with 1 transmission conductor (TC) placed close to a reference conductor (GC) used as a reference for voltage measurements.

❒ Important: the GC is not necessarily a ground conductor or a combination of ground conductors.

❒ We define:

- ◆ the curvilinear abscissa z, the interconnection extending from $z = 0$ to $z = \mathcal{L}$;
- ◆ the natural current i as the current flowing on the TC, toward $z = \mathcal{L}$;
- ◆ the natural voltage v as the voltage between the TC and the GC.

❒ i and v are z-dependent.

❐ Except when otherwise stated, we shall consider frequency domain quantities and $\sqrt{\;}$ is always used to denote the principal root.

❐ We assume that the interconnection can be modeled as a TL. The TL model uses:

- a p.u.l. impedance Z' and a p.u.l. admittance Y';
- the telegrapher's equations

$$\begin{cases} \dfrac{dv}{dz} = -Z'i \\ \dfrac{di}{dz} = -Y'v \end{cases} \tag{1}$$

❐ Z' and Y' are frequency dependent. Z' and Y' must each represent passive linear systems. Thus, their real part is nonnegative.

❐ The TL is lossless if and only if $Z' = j\omega L'$ and $Y' = j\omega C'$ where L' and C' are real. In this case, L' and C' must be frequency independent.

❒ The TL is said to be uniform if Z' and Y' are independent of z.

❒ Assuming a uniform TL, we can derive two second order differential equations

$$\begin{cases} \dfrac{d^2 v}{dz^2} - Y'Z' v = 0 \\ \dfrac{d^2 i}{dz^2} - Y'Z' i = 0 \end{cases} \tag{2}$$

❒ The general solution of (2) can be written

$$\begin{cases} v = v_{0+}\, e^{-z\gamma} + v_{0-}\, e^{z\gamma} \\ i = i_{0+}\, e^{-z\gamma} + i_{0-}\, e^{z\gamma} \end{cases} \tag{3}$$

where v_{0+}, v_{0-}, i_{0+} and i_{0-} are z-independent scalars determined by the boundary conditions at $z = 0$ and $z = \mathcal{L}$ and where the propagation constant γ is given by:

$$\gamma = \sqrt{Y'Z'} \tag{4}$$

❒ $v_{0+}\, e^{-\gamma z}$ and $i_{0+}\, e^{-\gamma z}$ propagate with the propagation constant γ toward the far end; $v_{0-}\, e^{\gamma z}$ and $i_{0-}\, e^{\gamma z}$ propagate with the propagation constant $-\gamma$ toward the near end.

❒ The characteristic impedance Z_C is defined by

$$Z_C = \sqrt{\frac{Z'}{Y'}} \tag{5}$$

and it is such that

$$\begin{cases} v_{0+} = Z_C\, i_{0+} \\ v_{0-} = -Z_C\, i_{0-} \end{cases} \tag{6}$$

❒ We define

- the voltage traveling toward the far end, given by $v_+ = v_{0+}\, e^{-\gamma z}$;
- the voltage traveling toward the near end, given by $v_- = v_{0-}\, e^{\gamma z}$;
- the current traveling toward the far end, given by $i_+ = i_{0+}\, e^{-\gamma z}$;
- the current traveling toward the near end, given by $i_- = i_{0-}\, e^{\gamma z}$.

❒ We have

$$\begin{cases} v = v_+ + v_- \\ i = i_+ + i_- \end{cases} \qquad \begin{cases} v_+ = Z_C\, i_+ \\ v_- = -Z_C\, i_- \end{cases} \tag{7}$$

and, consequently

$$v_+ = \frac{v + Z_C\, i}{2} \qquad v_- = \frac{v - Z_C\, i}{2} \tag{8}$$

❒ For a lossless TL, $Z' = j\omega L'$ and $Y' = j\omega C'$ where L' and C' are real. Thus,

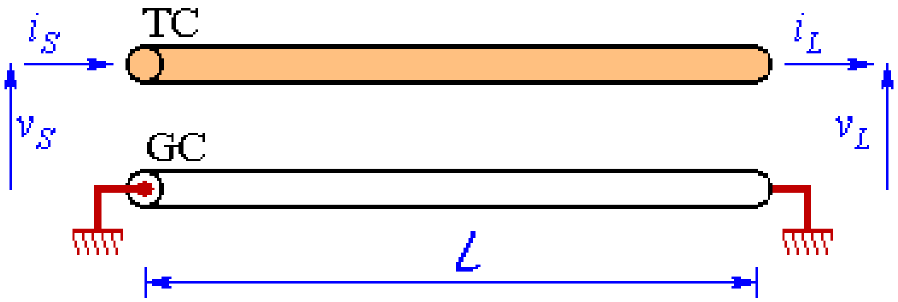
$$\gamma = j\omega\,\sqrt{L'C'} \qquad Z_C = \sqrt{\frac{L'}{C'}} \tag{9}$$

so that $c = 1/\sqrt{L'C'}$ is the propagation velocity in the TL. The travel time is $\tau = \dfrac{\mathcal{L}}{c}$.

❒ For a lossless TL in an homogeneous dielectric, we also have

$$L'C' = \frac{\varepsilon_r}{c_0^2} = \frac{1}{c^2} \tag{10}$$

where ε_r is the relative permittivity of the dielectric.



❒ A scattering matrix of the interconnection, denoted by $\mathcal{S}(\mathcal{L})$ and defined by

$$\begin{pmatrix} v_S - Z_C i_S \\ v_L + Z_C i_L \end{pmatrix} = \mathcal{S}(\mathcal{L}) \begin{pmatrix} v_S + Z_C i_S \\ v_L - Z_C i_L \end{pmatrix} \tag{11}$$

is given by

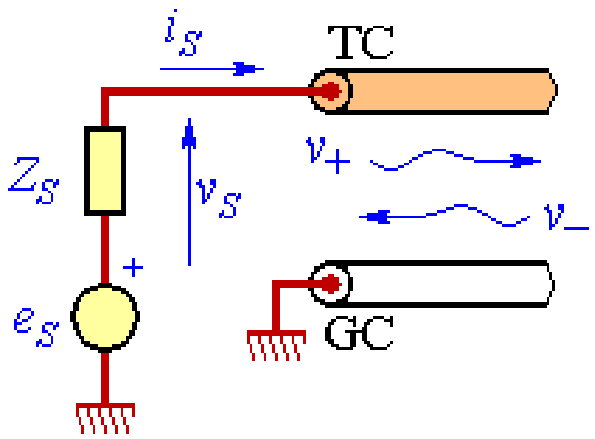
$$\mathcal{S}(\mathcal{L}) = e^{-\gamma \mathcal{L}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{12}$$

Proof. This is a direct consequence of (3) and (8). □

❒ $\mathcal{S}(\mathcal{L})$ is the usual scattering matrix of a 2-port only when Z_C is real.

3. Problems involving a TL and linear terminations

❒ At the near end, in the configuration shown, we have



$$\begin{cases} v_S = e_S - Z_S i_S \\ v_S = v_+ + v_- \\ i_S = \dfrac{v_+ - v_-}{Z_C} \end{cases} \tag{13}$$

we obtain

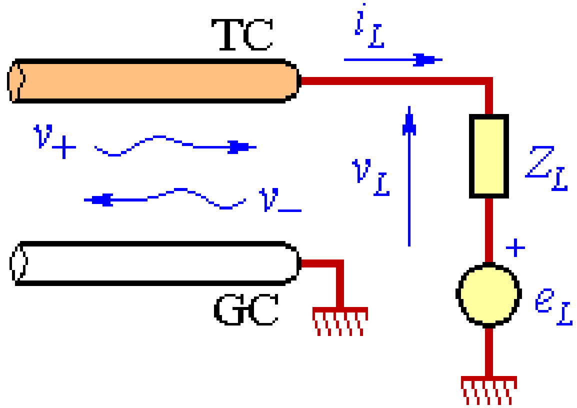
$$v_+ = \frac{Z_C}{Z_S + Z_C} e_S + \rho_S \, v_- = \frac{1 - \rho_S}{2} e_S + \rho_S \, v_- \tag{14}$$

where, in this configuration, the voltage reflection coefficient is defined by

$$\rho_S = \frac{Z_S - Z_C}{Z_S + Z_C} \tag{15}$$

❒ At the far end, in the configuration shown, we have

$$\begin{cases} v_L = e_L + Z_L i_L \\ v_L = v_+ + v_- \\ i_L = \dfrac{v_+ - v_-}{Z_C} \end{cases} \tag{16}$$



we obtain

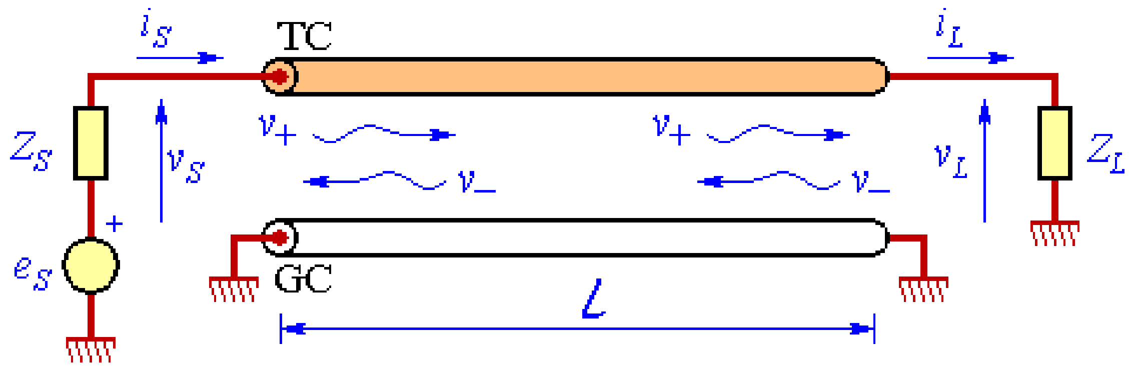
$$v_- = \frac{Z_C}{Z_L + Z_C} e_L + \rho_L v_+ = \frac{1-\rho_L}{2} e_L + \rho_L v_+ \tag{17}$$

where, in this configuration, the voltage reflection coefficient is defined by

$$\rho_L = \frac{Z_L - Z_C}{Z_L + Z_C} \tag{18}$$

❒ It is also possible to define current reflection coefficients.

❒ A termination is matched when it produces no reflection, i.e. for $Z_S = Z_C$ or $Z_L = Z_C$ (we are referring to reflectionless matching, as opposed to conjugate matching).



❐ Three possible approaches to find v_S and v_L in the configuration shown above:

- ◆ using the boundary conditions to find v_{0+}, v_{0-}, i_{0+} and i_{0-} in (3);
- ◆ using the scattering matrix defined by (12) and the reflection coefficients;
- ◆ using the chain matrix (not studied in this tutorial).

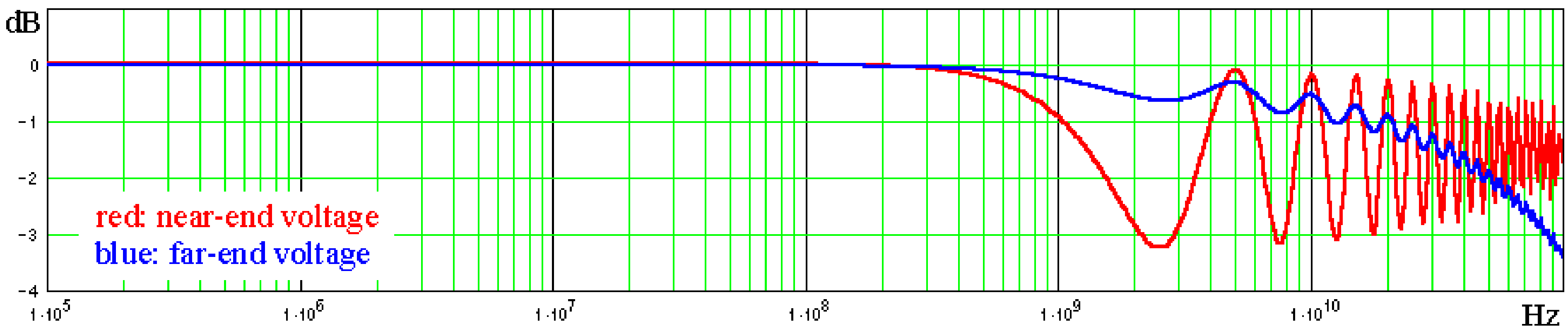
❐ Following the second approach, we consider multiple reflections occurring at the near end and at the far end, and multiple propagation through the TL. We get:

$$\begin{cases} v_+(0) = e_S \dfrac{Z_C}{Z_S + Z_C} \displaystyle\sum_{p=0}^{\infty} \left(\rho_L \rho_S\, e^{-2\gamma \mathcal{L}}\right)^p \\ v_+(\mathcal{L}) = e_S \dfrac{Z_C\, e^{-\gamma \mathcal{L}}}{Z_S + Z_C} \displaystyle\sum_{p=0}^{\infty} \left(\rho_L \rho_S\, e^{-2\gamma \mathcal{L}}\right)^p \end{cases} \quad \text{and} \quad \begin{cases} v_S = v_+(0)\left[1 + \rho_L e^{-2\gamma \mathcal{L}}\right] \\ v_L = v_+(\mathcal{L})\left[1 + \rho_L\right] \end{cases} \qquad (19)$$

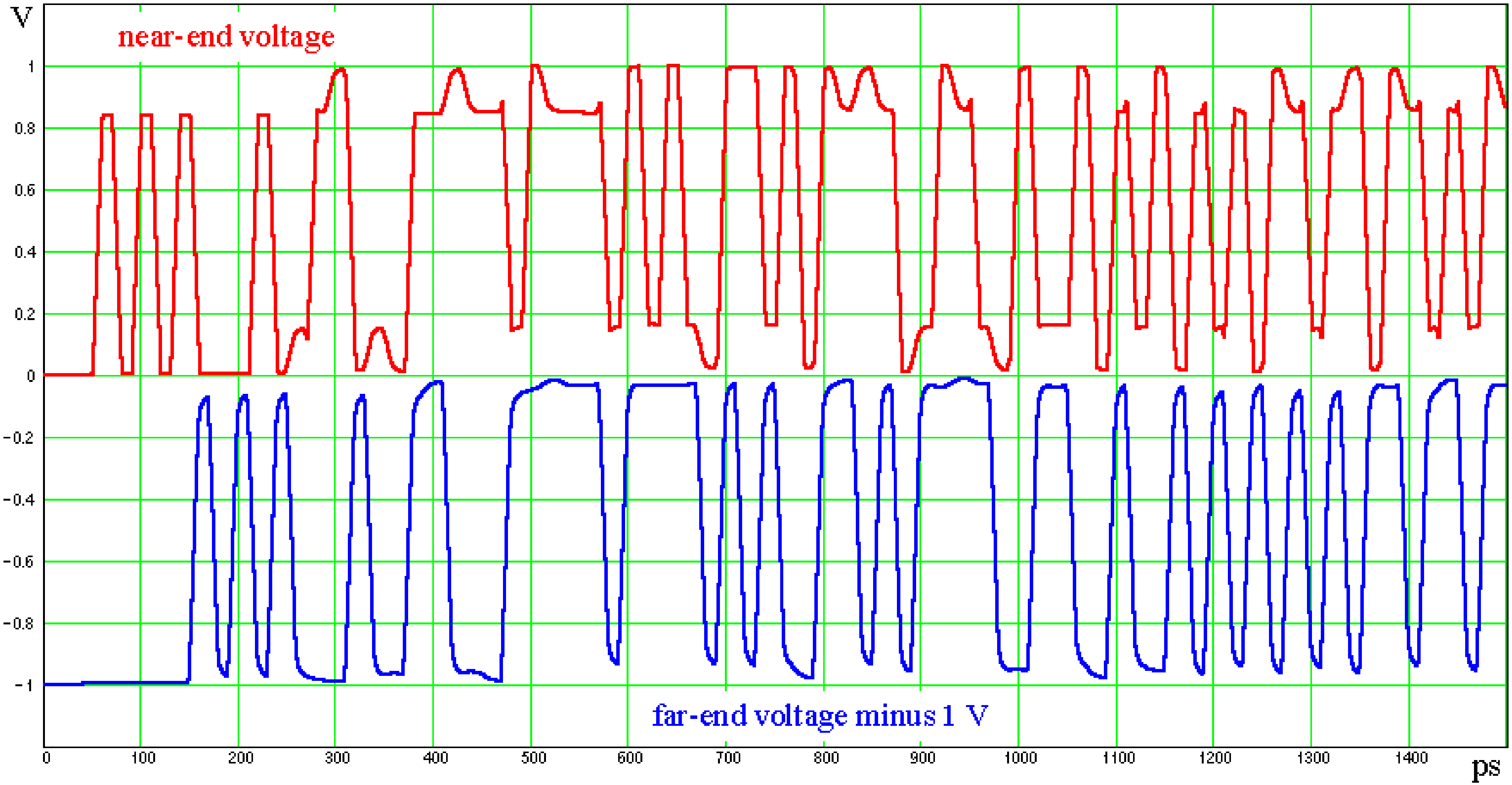
Thus, we obtain

$$\begin{cases} v_S = \dfrac{1+\rho_L\, e^{-2\gamma L}}{1-\rho_L \rho_S\, e^{-2\gamma L}}\, \dfrac{1-\rho_S}{2}\, e_S \\ v_L = \dfrac{1+\rho_L}{1-\rho_L \rho_S\, e^{-2\gamma L}}\, e^{-\gamma L}\, \dfrac{1-\rho_S}{2}\, e_S \end{cases} \qquad (20)$$

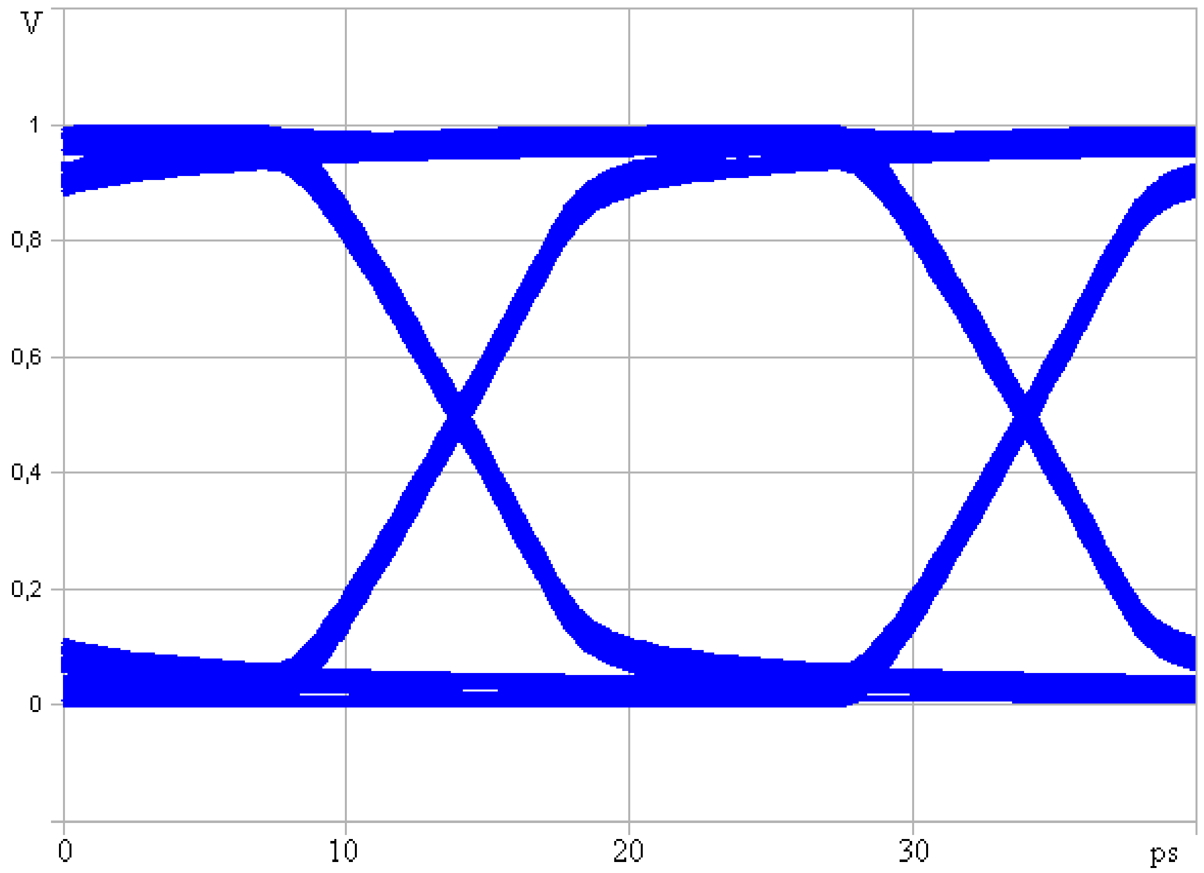
❒ An example involving a 20-mm long microstrip built on an enhanced high-speed multifunctional epoxy material. Taking resistive losses and dielectric losses into account in (20) yields:



❒ Time domain results can also be obtained, here for the unipolar non-return-to-zero level (NRZ-L) sequence 0010101000100110001111101111010100110101111001100100101010101011...:

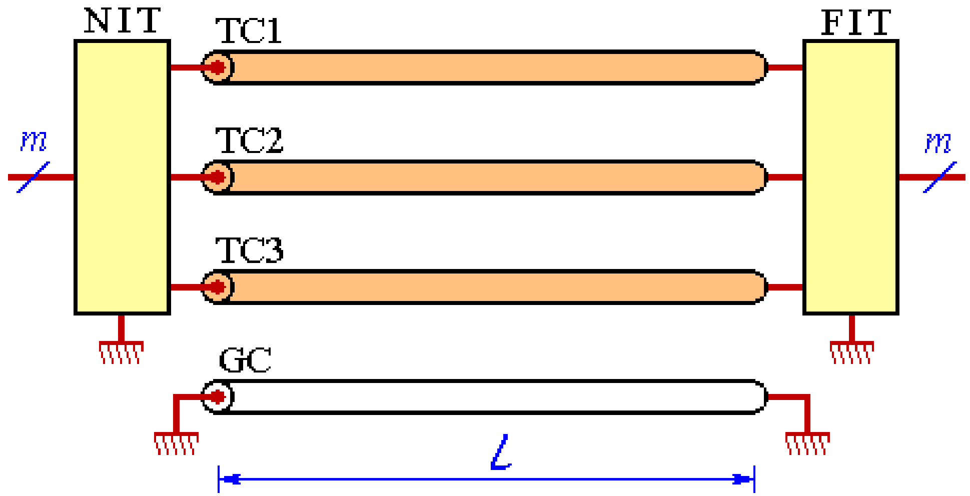


❒ And also eye diagrams, here the eye diagram at the far end:



❒ All results shown were obtained using a standard numerical computation program.

4. Telegrapher's equations of a uniform MTL and modal decomposition



❒ We consider a link providing *m* channels comprising:

- ◆ an interconnection having *n* TCs and a GC, where $n \geq m$;
- ◆ a near-end interface and termination device (NIT);
- ◆ a far-end interface and termination device (FIT).

❑ The GC is used as a reference for voltage measurements.

❑ Important: the GC is not necessarily a ground conductor or a combination of ground conductors.

❑ We number the TCs from 1 to n, and we define:

- ◆ the curvilinear abscissa z, the interconnection extending from $z = 0$ to $z = \mathcal{L}$;
- ◆ the natural current i_j as the current flowing on the TC j, toward $z = \mathcal{L}$;
- ◆ the natural voltage v_j as the voltage between the TC j and the GC;
- ◆ the column vector $\mathbf{i}$ of the natural currents $i_1, \ldots, i_n$, which depends on z;
- ◆ the column vector $\mathbf{v}$ of the natural voltages $v_1, \ldots, v_n$, which depends on z.

❑ Except when otherwise stated, we consider frequency domain quantities. We use $\mathbf{1}_n$ to denote the identity matrix of size $n \times n$, and $\mathbf{0}_{n\,n}$ to denote the zero matrix of size $n \times n$.

❒ We assume that the interconnection can be modeled as a MTL. The $(n+1)$-conductor MTL model uses [1]:

- a p.u.l. impedance matrix $\mathbf{Z}'$ and a p.u.l. admittance matrix $\mathbf{Y}'$;
- the telegrapher's equations

$$\begin{cases} \dfrac{d\,\mathbf{v}}{dz} = -\mathbf{Z}'\,\mathbf{i} \\ \dfrac{d\,\mathbf{i}}{dz} = -\mathbf{Y}'\,\mathbf{v} \end{cases} \tag{21}$$

❒ $\mathbf{Z}'$ and $\mathbf{Y}'$ are frequency-dependent symmetric matrices of size $n \times n$. $\mathbf{Z}'$ and $\mathbf{Y}'$ must each represent a passive linear system. Thus, their real parts are positive semidefinite [11, § 7.1]. We shall assume that $\mathbf{Z}'$ and $\mathbf{Y}'$ are invertible.

❒ The MTL is lossless if and only if $\mathbf{Z}' = j\omega\mathbf{L}'$ and $\mathbf{Y}' = j\omega\,\mathbf{C}'$ where $\mathbf{L}'$ and $\mathbf{C}'$ are real matrices of size $n \times n$. In this case, $\mathbf{L}'$ and $\mathbf{C}'$ must be frequency independent.

❐ The MTL is said to be uniform if $\mathbf{Z}'$ and $\mathbf{Y}'$ are independent of z.

❐ Assuming a uniform MTL, we can derive two second order differential equations

$$\begin{cases} \dfrac{d^2\mathbf{v}}{dz^2} - \mathbf{Z}'\mathbf{Y}'\,\mathbf{v} = \mathbf{0} \\ \dfrac{d^2\,\mathbf{i}}{dz^2} - \mathbf{Y}'\mathbf{Z}'\,\mathbf{i} = \mathbf{0} \end{cases} \tag{22}$$

❐ $\mathbf{Z}'\mathbf{Y}'$ and $\mathbf{Y}'\mathbf{Z}'$ are similar [11, § 1.3.20]. We shall assume that $\mathbf{Z}'\mathbf{Y}'$ is diagonalizable. In this case, there exist two invertible matrices **T** and **S** such that:

$$\begin{cases} \mathbf{T}^{-1}\mathbf{Y}'\mathbf{Z}'\,\mathbf{T} = \Gamma^2 \\ \mathbf{S}^{-1}\mathbf{Z}'\,\mathbf{Y}'\mathbf{S} = \Gamma^2 \end{cases} \qquad \text{where} \qquad \Gamma = \mathrm{diag}_n(\gamma_1, \ldots, \gamma_n) \tag{23}$$

is the diagonal matrix of order n of the propagation constants γ_i, chosen with an argument lying in $]-\pi/2, \pi/2]$, so that the γ_i are principal square roots.

❒ **T** and **S** define a *modal transform* for the natural currents and the natural voltages. We define $\mathbf{v}_M$ and $\mathbf{i}_M$ by

$$\begin{cases} \mathbf{v} = \mathbf{S}\,\mathbf{v}_M \\ \mathbf{i} = \mathbf{T}\,\mathbf{i}_M \end{cases} \tag{24}$$

where

- ◆ we use $\mathbf{i}_M$ to denote the vector of the n modal currents $i_{M\,1},\ldots,\ i_{M\,n}$;
- ◆ we use $\mathbf{v}_M$ to denote the vector of the n modal voltages $v_{M\,1},\ldots,\ v_{M\,n}$;
- ◆ we call **S** the *transition matrix from modal voltages to natural voltages*;
- ◆ we call **T** the *transition matrix from modal currents to natural currents*.

❒ Using (23) and (24), (22) takes on a form which contains n times (2):

$$\begin{cases} \dfrac{d^2\mathbf{v}_M}{dz^2} - \Gamma^2\,\mathbf{v}_M = \mathbf{0} \\ \dfrac{d^2\,\mathbf{i}_M}{dz^2} - \Gamma^2\,\mathbf{i}_M = \mathbf{0} \end{cases} \tag{25}$$

❒ The general solution of (25) is

$$\begin{cases} \mathbf{v}_M = e^{-z\Gamma}\,\mathbf{v}_{M0+} + e^{z\Gamma}\,\mathbf{v}_{M0-} \\ \mathbf{i}_M = e^{-z\Gamma}\,\mathbf{i}_{M0+} + e^{z\Gamma}\,\mathbf{i}_{M0-} \end{cases} \tag{26}$$

where $\mathbf{v}_{M0+}$, $\mathbf{v}_{M0-}$, $\mathbf{i}_{M0+}$ and $\mathbf{i}_{M0-}$ are z-independent vectors depending on the boundary conditions at $z = 0$ and $z = \mathcal{L}$.

❒ For a function $f(u)$ of the variable $u \in \mathbb{C}$ and a diagonal matrix $\mathrm{diag}_n(a_1,..., a_n)$, we define $f(\mathrm{diag}_n(a_1,..., a_n)) = \mathrm{diag}_n(f(a_1),..., f(a_n))$. This was used in (26).

❒ For any $\alpha \in \{1,..., n\}$, a modal current $i_{M\alpha}$ and a modal voltage $v_{M\alpha}$ may propagate with the propagation constant γ_α toward the far end, or with the opposite propagation constant $-\gamma_\alpha$ toward the near end.

❒ The column vectors of $\mathbf{S}$ (respectively, of $\mathbf{T}$) are defined as linearly independent eigenvectors of $\mathbf{Z}'\mathbf{Y}'$ (respectively, of $\mathbf{Y}'\mathbf{Z}'$). Consequently, $\mathbf{S}$ and $\mathbf{T}$ are not uniquely defined by (23).

5. The characteristic impedance matrix

❒ For a wave traveling toward the far end, the column vector of the modal voltages is $\mathbf{v}_{M+} = e^{-z\Gamma}\,\mathbf{v}_{M0+}$ and the column vector of the modal currents is $\mathbf{i}_{M+} = e^{-z\Gamma}\,\mathbf{i}_{M0+}$.

❒ The modal characteristic impedance matrix $\mathbf{Z}_{MC}$ is defined by

$$\mathbf{v}_{M+} = \mathbf{Z}_{MC}\,\mathbf{i}_{M+} \tag{27}$$

and given by [108]

$$\mathbf{Z}_{MC} = \Gamma^{-1}\,\mathbf{S}^{-1}\mathbf{Z}'\,\mathbf{T} = \Gamma\,\mathbf{S}^{-1}\,\mathbf{Y}'^{-1}\,\mathbf{T} = \mathbf{S}^{-1}\,\mathbf{Y}'^{-1}\,\mathbf{T}\,\Gamma = \mathbf{S}^{-1}\mathbf{Z}'\,\mathbf{T}\,\Gamma^{-1} \tag{28}$$

❒ For a wave traveling toward the near end, the column vector of the modal voltages is $\mathbf{v}_{M-} = e^{z\Gamma}\,\mathbf{v}_{M0-}$ and the column vector of the modal currents is $\mathbf{i}_{M-} = e^{z\Gamma}\,\mathbf{i}_{M0-}$, so that

$$\mathbf{v}_{M-} = -\mathbf{Z}_{MC}\,\mathbf{i}_{M-} \tag{29}$$

❒ The modal characteristic impedance matrix depends on the choice of **S** and **T**.

❒ We can now define the characteristic impedance matrix $\mathbf{Z}_C$ of the multiconductor transmission line, as:

$$\mathbf{Z}_C = \mathbf{S}\,\mathbf{Z}_{MC}\mathbf{T}^{-1} \tag{30}$$

❒ Using (28), we get [108]

$$\mathbf{Z}_C = \mathbf{S}\,\boldsymbol{\Gamma}^{-1}\,\mathbf{S}^{-1}\mathbf{Z}' = \mathbf{S}\,\boldsymbol{\Gamma}\,\mathbf{S}^{-1}\mathbf{Y}'^{-1} = \mathbf{Y}'^{-1}\mathbf{T}\boldsymbol{\Gamma}\mathbf{T}^{-1} = \mathbf{Z}'\,\mathbf{T}\boldsymbol{\Gamma}^{-1}\mathbf{T}^{-1} \tag{31}$$

❒ For a wave traveling toward the far end, the column vector of the natural voltages is $\mathbf{v}_+ = \mathbf{S}\mathbf{v}_{M+} = \mathbf{S}\; e^{-z\Gamma}\;\mathbf{v}_{M0+}$ and the column vector of the modal currents is $\mathbf{i}_+ = \mathbf{T}\,\mathbf{i}_{M+} = \mathbf{T}\; e^{-z\Gamma}\;\mathbf{i}_{M0+}$. We find:

$$\mathbf{v}_+ = \mathbf{Z}_C\,\mathbf{i}_+ \tag{32}$$

❒ For a wave traveling toward the near end, the column vector of the natural voltages is $\mathbf{v}_- = \mathbf{S}\,\mathbf{v}_{M-} = \mathbf{S}\,e^{z\Gamma}\,\mathbf{v}_{M0-}$ and the column vector of the modal currents is $\mathbf{i}_- = \mathbf{T}\,\mathbf{i}_{M-} = \mathbf{T}\,e^{z\Gamma}\,\mathbf{i}_{M0-}$. We find:

$$\mathbf{v}_- = -\mathbf{Z}_C\,\mathbf{i}_- \tag{33}$$

❒ Since (32) holds for any value $\mathbf{i}_+$, it can be used as a definition of $\mathbf{Z}_C$. Thus, $\mathbf{Z}_C$ is unique and does not depend on the choice of the matrices **S** and **T**.

❒ For a lossless MTL, real and frequency-independent matrices **T** and **S** can be computed [6], and we have

$$\gamma_\alpha = \frac{j\omega}{c_\alpha} \tag{34}$$

where the positive real c_α is the propagation velocity of the mode α. Thus, $\mathbf{Z}_C$ is real.

❒ The minimum travel time is $\tau_{\min} = \dfrac{\mathcal{L}}{\max(c_\alpha)}$.

❒ Special case of a lossless interconnection in which the electric field sees an homogeneous dielectric:

◆ we may consider that the interconnection has an homogeneous dielectric of relative permittivity ε_r, for which it can be shown that [1] [101],

$$\mathbf{C}' = \varepsilon_r \varepsilon_0 \mu_0 \mathbf{L}'^{-1} \tag{35}$$

where ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum, so that

$$\mathbf{Z}'\mathbf{Y}' = \mathbf{Y}'\mathbf{Z}' = -\omega^2 \varepsilon_r \varepsilon_0 \mu_0 \mathbf{1}_n \tag{36}$$

◆ we can use $\mathbf{S} = \mathbf{T} = \mathbf{1}_n$ so that, using $\varepsilon_0 \mu_0 c_0^2 = 1$, we obtain

$$\mathbf{\Gamma} = \frac{j\omega}{c_0 / \sqrt{\varepsilon_r}} \mathbf{1}_n \qquad \mathbf{Z}_C = \frac{\mathbf{C}'^{-1}}{c_0 / \sqrt{\varepsilon_r}} = \frac{c_0}{\sqrt{\varepsilon_r}} \mathbf{L}' \tag{37}$$

There is only one propagation constant: we have a completely degenerate MTL.

6. Biorthonormal eigenvectors and associated eigenvectors

❒ $\mathbf{Z}'$ and $\mathbf{Y}'$ being symmetric, if a diagonalization of the matrix $\mathbf{Y}'\mathbf{Z}'$ produces a matrix $\mathbf{T}$ satisfying the first line of (23), i.e. $\mathbf{T}^{-1}\mathbf{Y}'\mathbf{Z}'\mathbf{T} = \Gamma^2$, we find that a solution of the second line of (23), i.e. $\mathbf{S}^{-1}\mathbf{Z}'\mathbf{Y}'\mathbf{S} = \Gamma^2$ is

$$\mathbf{S} = {}^t\mathbf{T}^{-1} \tag{38}$$

where ${}^t\mathbf{A}$ is used to denote the transpose of a matrix $\mathbf{A}$.

❒ In other words, we can always compute the eigenvectors $\mathbf{T}_i$ of $\mathbf{Y}'\mathbf{Z}'$ and the eigenvectors $\mathbf{S}_j$ of $\mathbf{Z}'\mathbf{Y}'$ in such a way that they satisfy the relation

$${}^t\mathbf{S}_i\,\mathbf{T}_j = \delta_{ij} \tag{39}$$

where δ_{ij} is Kronecker's symbol. This is called the *biorthonormal property* [19].

❒ The possibility of using (38) in (31) shows that $\mathbf{Z}_C$ is symmetric.

❒ Note that:

- ◆ biorthonormal eigenvectors can be used with lossy and lossless MTLs;
- ◆ (38) is not a property of (23), it is only a possible choice.

❒ If a diagonalization of the matrix $\mathbf{Y}'\mathbf{Z}'$ produces a matrix $\mathbf{T}$ satisfying the first line of (23), i.e. $\mathbf{T}^{-1}\mathbf{Y}'\mathbf{Z}'\mathbf{T} = \Gamma^2$, we find that a solution of the second line of (23), i.e. $\mathbf{S}^{-1}\mathbf{Z}'\mathbf{Y}'\mathbf{S} = \Gamma^2$ is

$$\mathbf{S} = j\omega\, c_K\, \mathbf{Y}'^{-1}\mathbf{T} \tag{40}$$

where c_K is an arbitrary scalar different from zero, which may depend on frequency, and which has the dimensions of p.u.l. capacitance [4, eq. (19c)] [8, Appendix].

❒ When **S** and **T** are defined by (23) and (40), we say that they are *associated*, and that the eigenvectors contained in **S** and **T** (i.e. their column vectors) are associated [14] [16] [20] [26] [31] [42] [43].

❒ Using (21) and (24), we can write

$$\begin{cases} \dfrac{d\,\mathbf{v}_M}{dz} = -\mathbf{Z}'_M\,\mathbf{i}_M \\ \dfrac{d\,\mathbf{i}_M}{dz} = -\mathbf{Y}'_M\,\mathbf{v}_M \end{cases} \quad \text{where} \quad \begin{cases} \mathbf{Z}'_M = \mathbf{S}^{-1}\mathbf{Z}'\,\mathbf{T} \\ \mathbf{Y}'_M = \mathbf{T}^{-1}\mathbf{Y}'\mathbf{S} \end{cases} \tag{41}$$

This defines the modal p.u.l. impedance matrix $\mathbf{Z}'_M$ and the modal p.u.l. admittance matrix $\mathbf{Y}'_M$. For associated eigenvectors, we find

$$\mathbf{Z}'_M = \left(j\omega\, c_K\ \mathbf{Y}'^{-1}\mathbf{T}\right)^{-1}\mathbf{Z}'\,\mathbf{T} = \frac{1}{j\omega\, c_K}\mathbf{T}^{-1}\mathbf{Y}'\mathbf{Z}'\,\mathbf{T} = \frac{\Gamma^2}{j\omega\, c_K} \tag{42}$$

and

$$\mathbf{Y}'_M = \mathbf{T}^{-1}\mathbf{Y}'\left(j\omega\, c_K\ \mathbf{Y}'^{-1}\,\mathbf{T}\right) = j\omega\, c_K\,\mathbf{1}_n \tag{43}$$

Thus, we have shown that, for associated eigenvectors:

- ◆ the modal p.u.l. impedance matrix $\mathbf{Z}'_M = \mathbf{S}^{-1}\mathbf{Z}'\mathbf{T}$ is diagonal; and
- ◆ the modal p.u.l. admittance matrix $\mathbf{Y}'_M = \mathbf{T}^{-1}\mathbf{Y}'\mathbf{S}$ is diagonal.

❐ For associated eigenvectors, the modal characteristic impedance matrix is diagonal and given by

$$\mathbf{Z}_{MC} = \frac{1}{j\omega\ c_K}\ \Gamma \tag{44}$$

❐ Generalized associated eigenvectors are defined by the requirement

$$\mathbf{S} = j\omega\ \mathbf{Y}'^{-1}\ \mathbf{T}\,\mathbf{c}_K \tag{45}$$

where $\mathbf{c}_K$ is an arbitrary invertible diagonal matrix, possibly frequency dependent, and having the dimensions of p.u.l. capacitance.

❐ We find that, for generalized associated eigenvectors:

- the modal p.u.l. impedance matrix $\mathbf{Z}'_M = \mathbf{S}^{-1}\mathbf{Z}'\mathbf{T}$ is diagonal;
- the modal p.u.l. admittance matrix $\mathbf{Y}'_M = \mathbf{T}^{-1}\mathbf{Y}'\mathbf{S}$ is diagonal;
- the modal characteristic impedance matrix is diagonal and its value is

$$\mathbf{Z}_{MC} = \frac{1}{j\omega}\ \mathbf{c}_K^{-1}\ \Gamma \tag{46}$$

7. The choice of eigenvectors and total decoupling

❒ For associated eigenvectors, for a wave propagating in a given direction and for any $\alpha \in \{1,..., n\}$, by (27), (29) and (44) we have:

$$v_{M\alpha} = \frac{\varepsilon_D}{j\omega\, c_K}\, \gamma_\alpha\, i_{M\alpha} \tag{47}$$

where ε_D is equal to 1 if the wave propagates toward the far end, or to -1 if the wave propagates toward the near end.

❒ For generalized associated eigenvectors, using (27), (29) and (46), we find:

$$v_{M\alpha} = \frac{\varepsilon_D}{j\omega\, c_{K\alpha\alpha}}\, \gamma_\alpha\, i_{M\alpha} \tag{48}$$

where $c_{K\alpha\beta}$ denotes an entry of $\mathbf{c}_K$.

❒ According to (26) and to (47) or (48), the propagation of $v_{M\,\alpha}$ and $i_{M\,\alpha}$ can be viewed as the propagation on a ficticious 2-conductor transmission line having the propagation constant γ_α and the characteristic impedance $\gamma_\alpha / j\omega c_K$ or $\gamma_\alpha / j\omega c_{K\,\alpha\,\alpha}$.

❒ We say that total decoupling occurs when a particular choice of **T** and **S** leads to (47) or (48), so that an equivalent circuit comprising n independent 2-conductor TLs may be defined for the $(n + 1)$ conductor MTL.

❒ We see that the diagonalization of $\mathbf{Y}'\mathbf{Z}'$ and $\mathbf{Z}'\mathbf{Y}'$ in (23) provides a decoupling in (26), but it need not provide total decoupling.

❒ Theorem: Total decoupling means that $\mathbf{Z}_{MC}$ is diagonal. It only occurs for generalized associated eigenvectors, i.e. eigenvectors complying with (45).

Proof. The $c_{K\,\alpha\,\alpha}$ being arbitrary scalars, (48) means that $\mathbf{Z}_{MC}$ is diagonal. By (28), we have $\mathbf{S} = \mathbf{Y}'^{-1}\mathbf{T}\,\Gamma\,\mathbf{Z}_{MC}^{-1}$ which complies with (45) for $j\omega\,\mathbf{c}_K = \Gamma\,\mathbf{Z}_{MC}^{-1}$. □

❒ In the literature, we find that

◆ $\mathbf{T}$ and $\mathbf{S}$ satisfy $\mathbf{S} = {}^t\mathbf{T}^{-1}$ [10] [18, § 4.3.2] [21, § 6.2.6] [88, § 4.4.1];

◆ the modes are orthogonal, i.e. the eigen-voltages (the columns of $\mathbf{S}$) or the eigen-currents (the columns of $\mathbf{T}$) are orthogonal [13] [17, col. 1] [23, col. 4] [37] [38] [88, § 4.4] [96] [98];

◆ the modal impedance matrix $\mathbf{Z}'_M = \mathbf{S}^{-1}\mathbf{Z}'\mathbf{T}$ and/or the modal admittance matrix $\mathbf{Y}'_M = \mathbf{T}^{-1}\mathbf{Y}'\mathbf{S}$ are diagonal [9] [10] [88, § 4.4.1];

◆ but these assertions need not be correct [19] [42, § X] [43, § V and § VI].

❒ Biorthonormal eigenvectors, defined by the relation $\mathbf{S} = {}^t\mathbf{T}^{-1}$, are such that total decoupling need not be present, so that it need not lead us to an equivalent circuit based on n uncoupled 2-conductor transmission lines.

❒ However, when all γ_α are different from one another, i.e. when there is no degenerate propagation constant, any choice of $\mathbf{T}$ and $\mathbf{S}$ complies with (45), so that biorthonormal eigenvectors provide a total decoupling in this case.

❐ However, in the case of a lossless MTL, it is possible to compute a matrix **T** such that [6] [7] [18, § 4.4.3]

$$\mathbf{T}^{-1} = \varepsilon_0 \, {}^t\mathbf{T} \, \mathbf{C}'^{-1} \tag{49}$$

for which biorthonormal eigenvectors comply with (45), so that they provide total decoupling (at the cost of a complex algorithm).

❐ Using (generalized) associated eigenvectors, because of total decoupling:

◆ any $(n + 1)$-conductor MTL has an equivalent circuit comprising voltage-controlled voltage sources, current-controlled current sources and n uncoupled 2-conductor transmission lines;

◆ in general, all electrical parameters of the equivalent circuit are complex and frequency dependent;

◆ if the MTL is lossless, all electrical parameters of the equivalent circuit are real and frequency independent. It can be used in SPICE [14] [16] [20].

❒ A interesting choice of generalized associated eigenvectors is given by

$$\mathbf{S} = \frac{1}{z_K}\,\mathbf{Z}_C\mathbf{T} \qquad (50)$$

where z_K is an arbitrary nonzero scalar, which may depend on frequency, and which has the dimensions of impedance [108].

❒ Example A: This example relates to a multiconductor microstrip interconnection having 8 TCs, built on a polyimide substrate. We neglect losses.

The worksheet of Annex A shows that, in this example:

- ◆ there are no degenerate eigenvalues (see § 2);
- ◆ $\mathbf{Z}_C$ is real, not diagonal, and may be realized with a network of $n\,(n+1)/2$ resistors, some of which being obviously superfluous (see § 3 and § 4);
- ◆ the eigen-voltages and the eigen-currents are not orthogonal (see § 5);
- ◆ for associated eigenvectors, $\mathbf{Z}'_M$ and $\mathbf{Y}'_M$ are diagonal (see § 6);

◆ for our choice of biorthonormal eigenvectors, $\mathbf{Z}_{MC}$, $\mathbf{Z'}_M$ and $\mathbf{Y'}_M$ are diagonal (see § 7 and § 8).

❒ Example B: This example relates to the multiconductor stripline interconnection having 8 TCs, built in a polyimide substrate. We neglect losses.

The worksheet of Annex B shows that, in this example:

◆ we assume an homogeneous dielectric, so that (35) is used to compute $\mathbf{C'}$ and completely degenerate eigenvalues are obtained (see § 1 and § 2);

◆ $\mathbf{Z}_C$ is real, not diagonal, and may be realized with a network of $n\ (n + 1)/2$ resistors, many of which being obviously superfluous (see § 3 and § 4);

◆ for our choice of associated eigenvectors, the eigen-voltages are not orthogonal while the eigen-currents are orthogonal (see § 5);

◆ for associated eigenvectors, $\mathbf{Z'}_M$ and $\mathbf{Y'}_M$ are diagonal (see § 6);

◆ for our choice of biorthonormal eigenvectors, $\mathbf{Z}_{MC}$, $\mathbf{Z'}_M$ and $\mathbf{Y'}_M$ are not diagonal (see § 7 and § 8).

8. Propagation in the frequency domain

❒ According to (26), $\mathbf{v}_M$ and $\mathbf{i}_M$ are given by

$$\begin{cases} \mathbf{v}_M = \mathbf{v}_{M+} + \mathbf{v}_{M-} \\ \mathbf{i}_M = \mathbf{i}_{M+} + \mathbf{i}_{M-} \end{cases} \tag{51}$$

where

$$\begin{cases} \mathbf{v}_{M+} = e^{-z\Gamma}\,\mathbf{v}_{M0+} \\ \mathbf{i}_{M+} = e^{-z\Gamma}\,\mathbf{i}_{M0+} \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{v}_{M-} = e^{z\Gamma}\,\mathbf{v}_{M0-} \\ \mathbf{i}_{M-} = e^{z\Gamma}\,\mathbf{i}_{M0-} \end{cases} \tag{52}$$

where

- $\mathbf{v}_{M+}$ is the column vector of the modal voltages traveling toward the far end,
- $\mathbf{v}_{M-}$ is the column vector of the modal voltages traveling toward the near end,
- $\mathbf{i}_{M+}$ is the column vector of the modal currents traveling toward the far end,
- $\mathbf{i}_{M-}$ is the column vector of the modal currents traveling toward the near end,
- $\mathbf{v}_{M0+}$, $\mathbf{v}_{M0-}$, $\mathbf{i}_{M0+}$ and $\mathbf{i}_{M0-}$ are z-independent column vectors depending on the boundary conditions at $z = 0$ and $z = \mathcal{L}$.

❒ Thus, a *modal scattering matrix*, denoted by $S_M(z)$ and defined by

$$\begin{pmatrix} \mathbf{v}_{M-}(0) \\ \mathbf{v}_{M+}(z) \end{pmatrix} = S_M(z) \begin{pmatrix} \mathbf{v}_{M+}(0) \\ \mathbf{v}_{M-}(z) \end{pmatrix} \tag{53}$$

is given by

$$S_M(z) = \begin{pmatrix} 0 & e^{-z\Gamma} \\ e^{-z\Gamma} & 0 \end{pmatrix} \tag{54}$$

❒ This result is similar to (11), which applies to a TL.

❒ By (27) and (29), we have

$$\mathbf{v}_{M+} = \frac{\mathbf{v}_M + \mathbf{Z}_{MC}\,\mathbf{i}_M}{2} \qquad \mathbf{v}_{M-} = \frac{\mathbf{v}_M - \mathbf{Z}_{MC}\,\mathbf{i}_M}{2} \tag{55}$$

❒ Using (51), (52), $\mathbf{v} = \mathbf{S}\,\mathbf{v}_M$ and $\mathbf{i} = \mathbf{T}\,\mathbf{i}_M$, we find that $\mathbf{v}$ and $\mathbf{i}$ are given by

$$\begin{cases} \mathbf{v} = \mathbf{v}_+ + \mathbf{v}_- \\ \mathbf{i} = \mathbf{i}_+ + \mathbf{i}_- \end{cases} \tag{56}$$

where

$$\begin{cases} \mathbf{v}_+ = \mathbf{S}\, e^{-z\Gamma}\, \mathbf{S}^{-1}\, \mathbf{v}_{0+} \\ \mathbf{i}_+ = \mathbf{T}\, e^{-z\Gamma}\, \mathbf{T}^{-1}\, \mathbf{i}_{0+} \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{v}_- = \mathbf{S}\, e^{z\Gamma}\, \mathbf{S}^{-1}\, \mathbf{v}_{0-} \\ \mathbf{i}_- = \mathbf{T}\, e^{z\Gamma}\, \mathbf{T}^{-1}\, \mathbf{i}_{0-} \end{cases} \tag{57}$$

where

- $\mathbf{v}_+$ is the column vector of the natural voltages traveling toward the far end,
- $\mathbf{v}_-$ is the column vector of the natural voltages traveling toward the near end,
- $\mathbf{i}_+$ is the column vector of the natural currents traveling toward the far end,
- $\mathbf{i}_-$ is the column vector of the natural currents traveling toward the near end,
- $\mathbf{v}_{0+}$, $\mathbf{v}_{0-}$, $\mathbf{i}_{0+}$ and $\mathbf{i}_{0-}$ are z-independent column vectors depending on the boundary conditions at $z = 0$ and $z = \mathcal{L}$.

❒ $\mathbf{v}_+$, $\mathbf{v}_-$, $\mathbf{i}_+$ and $\mathbf{i}_-$ are independent of the choice of eigenvectors.

❒ Thus, a *scattering matrix*, denoted by $\mathcal{S}(z)$ and defined by

$$\begin{pmatrix} \mathbf{v}_-(0) \\ \mathbf{v}_+(z) \end{pmatrix} = \mathcal{S}(z) \begin{pmatrix} \mathbf{v}_+(0) \\ \mathbf{v}_-(z) \end{pmatrix} \tag{58}$$

is given by

$$\mathcal{S}(z) = \begin{pmatrix} 0 & \mathbf{S}e^{-z\Gamma}\mathbf{S}^{-1} \\ \mathbf{S}e^{-z\Gamma}\mathbf{S}^{-1} & 0 \end{pmatrix} = \begin{pmatrix} 0 & e^{-z\sqrt{\mathbf{Z}'\mathbf{Y}'}} \\ e^{-z\sqrt{\mathbf{Z}'\mathbf{Y}'}} & 0 \end{pmatrix} \tag{59}$$

❒ By (32) and (33), we have

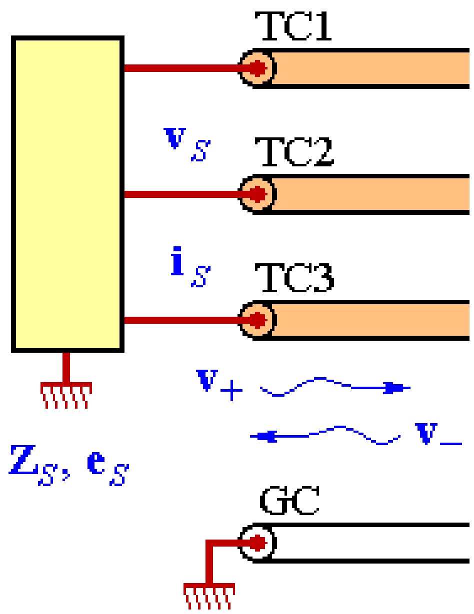
$$\mathbf{v}_+ = \frac{\mathbf{v} + \mathbf{Z}_C\,\mathbf{i}}{2} \qquad\qquad \mathbf{v}_- = \frac{\mathbf{v} - \mathbf{Z}_C\,\mathbf{i}}{2} \tag{60}$$

❒ $\mathcal{S}(\mathcal{L})$ is the usual scattering matrix of a $2n$-port only when $\mathbf{Z}_C$ is real and diagonal.

9. Matched termination circuit and pseudo-matched impedances

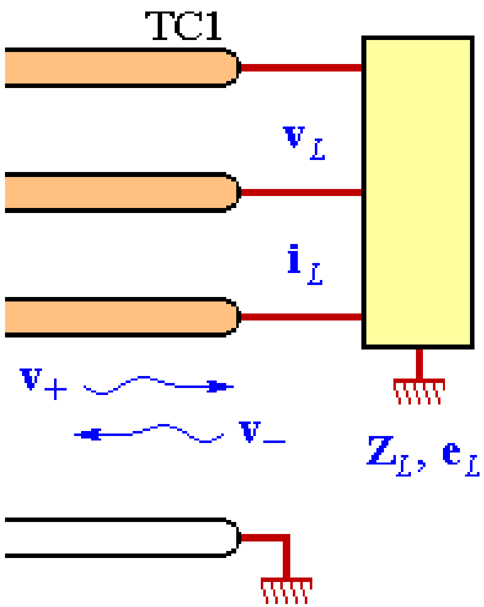
❒ We consider, at the near end, a passive linear (n + 1)-terminal circuit of impedance matrix $\mathbf{Z}_S$ and open-circuit voltage $\mathbf{e}_S$. We find:

$$\mathbf{v}_+ = \mathbf{Z}_C(\mathbf{Z}_S + \mathbf{Z}_C)^{-1}\mathbf{e}_S + \mathbf{P}_S\,\mathbf{v}_- = \frac{\mathbf{1}_n - \mathbf{P}_S}{2}\mathbf{e}_S + \mathbf{P}_S\,\mathbf{v}_- \tag{61}$$



where $\mathbf{P}_S$ is the matrix of the voltage reflection coefficients for this configuration, given by

$$\mathbf{P}_S = \mathbf{Z}_C(\mathbf{Z}_S + \mathbf{Z}_C)^{-1}(\mathbf{Z}_S - \mathbf{Z}_C)\mathbf{Z}_C^{-1} = (\mathbf{Z}_S - \mathbf{Z}_C)(\mathbf{Z}_S + \mathbf{Z}_C)^{-1} \tag{62}$$



❒ For the corresponding case at the far end, we find:

$$\begin{aligned}\mathbf{v}_- &= \mathbf{Z}_C\left(\mathbf{Z}_L+\mathbf{Z}_C\right)^{-1}\mathbf{e}_L+\mathbf{P}_L\,\mathbf{v}_+\\ &= \frac{\mathbf{1}_n-\mathbf{P}_L}{2}\,\mathbf{e}_L+\mathbf{P}_L\,\mathbf{v}_+\end{aligned} \tag{63}$$

where $\mathbf{P}_L$ is the matrix of the voltage reflection coefficients for this configuration, given by

$$\mathbf{P}_L=\mathbf{Z}_C\left(\mathbf{Z}_L+\mathbf{Z}_C\right)^{-1}\left(\mathbf{Z}_L-\mathbf{Z}_C\right)\mathbf{Z}_C^{-1}=\left(\mathbf{Z}_L-\mathbf{Z}_C\right)\left(\mathbf{Z}_L+\mathbf{Z}_C\right)^{-1} \tag{64}$$

❒ A matched termination circuit produces no reflection. It satisfies

- at the far end, $\mathbf{Z}_L=\mathbf{Z}_C$ or equivalently $\mathbf{P}_L=\mathbf{0}_{n\,n}$;
- at the near end, $\mathbf{Z}_S=\mathbf{Z}_C$ or equivalently $\mathbf{P}_S=\mathbf{0}_{n\,n}$.

❒ Note that we are again referring to reflectionless matching, as opposed to hermitian matching which provides maximum power transfer [5].

❒ For an interconnection in which TC-to-TC coupling is not negligible, a matched termination circuit has a non-diagonal impedance matrix (with respect to the GC).

❒ A termination circuit made of *n* impedors each connected between a TC and the GC has a diagonal impedance matrix (with respect to the GC). The impedance of an impedor intended to minimize the detrimental effects of reflections is referred to as pseudo-matched impedance [40] [42] [43].

❒ Pseudo-matched impedances may be chosen as the diagonal elements of $\mathbf{Z}_C$ [14] [16]. This is an arbitrary choice.

❒ A second choice of pseudo-matched impedances requires that the diagonal elements of $\mathbf{P}_L$ or $\mathbf{P}_S$ are equal to zero. This is referred to as *diagonal matching* [24].

❒ In diagonal matching, if the incident wave exists on a single TC, there is no reflected wave on this TC. Consequently, the termination circuit produces crosstalk, but no echo (however, it may indirectly contribute to echo).

❒ A third choice of pseudo-matched impedances requires that the maximum (absolute) column sum norm $|||\mathbf{P}|||_1$ of $\mathbf{P} = \mathbf{P}_S$ or $\mathbf{P} = \mathbf{P}_L$ be minimized. If $\mathbf{P} = [\rho_{\alpha\,\beta}]$, this matrix norm [11, § 5.6.4] [25, p. 1148] is defined by

$$|||\mathbf{P}|||_1 = \max_j \sum_{i=1}^{n} \left|\rho_{i\,j}\right| \tag{65}$$

❒ $|||\bullet|||_1$ is the matrix norm induced by the L_1-norm for vectors, defined by

$$\left\|\mathbf{v}\right\|_1 = \sum_{i=1}^{n} \left|v_i\right| \tag{66}$$

Thus, for a non-zero incident wave **v**

$$\frac{\left\|\mathbf{P\,v}\right\|_1}{\left\|\mathbf{v}\right\|_1} \leq \max_{\mathbf{x}\neq\mathbf{0}} \frac{\left\|\mathbf{P\,x}\right\|_1}{\left\|\mathbf{x}\right\|_1} = |||\mathbf{P}|||_1 \tag{67}$$

❒ A fourth choice of pseudo-matched impedances requires that the maximum (absolute) row sum norm $|||\mathbf{P}|||_\infty$ of $\mathbf{P} = \mathbf{P}_S$ or $\mathbf{P} = \mathbf{P}_L$ be minimized. If $\mathbf{P} = [\rho_{\alpha\,\beta}]$, this matrix norm is defined by

$$|||\mathbf{P}|||_\infty = \max_i \sum_{j=1}^{n} \left|\rho_{i\,j}\right| \tag{68}$$

❒ $|||\bullet|||_\infty$ is the matrix norm induced by the L_∞-norm for vectors, defined by

$$\left\|\mathbf{v}\right\|_\infty = \max_i \left|v_i\right| \tag{69}$$

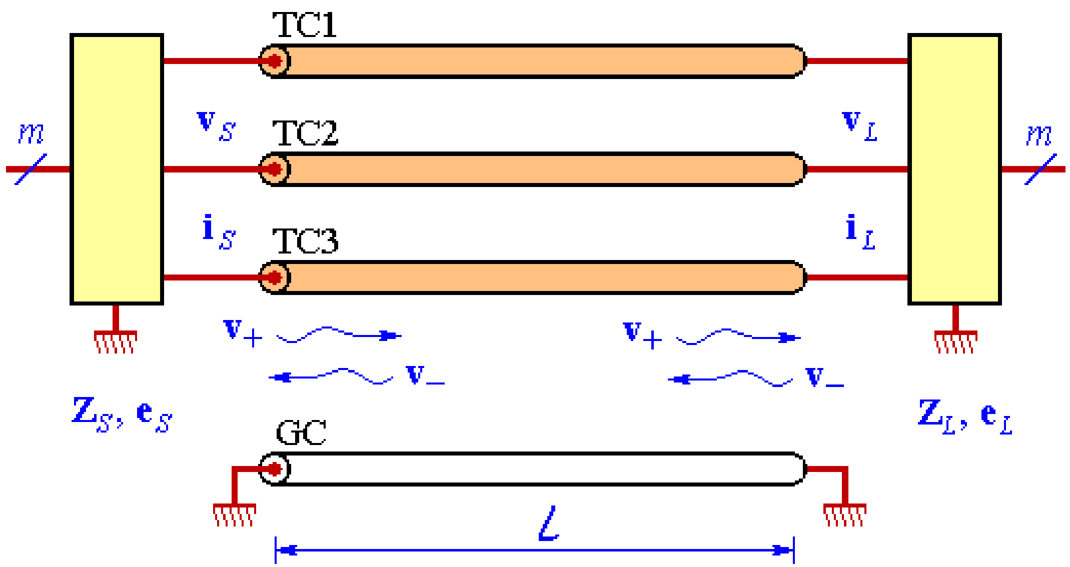
Thus, for a non-zero incident wave **v**

$$\frac{\left\|\mathbf{P\,v}\right\|_\infty}{\left\|\mathbf{v}\right\|_\infty} \le \max_{\mathbf{x}\neq\mathbf{0}} \frac{\left\|\mathbf{P\,x}\right\|_\infty}{\left\|\mathbf{x}\right\|_\infty} = |||\mathbf{P}|||_\infty \tag{70}$$

❒ In the special case where losses are neglected, $\mathbf{Z}_C$ is real and frequency independent, so that the proposed pseudo-matched impedances are resistances.

10. Problems involving a MTL and linear terminations

In this configuration, $\mathbf{e}_S$ and $\mathbf{e}_L$ are the vectors of the open-circuit voltages at the near end and at the far end, respectively.



❒ Three possible approaches to find $\mathbf{v}_S$ and $\mathbf{v}_L$ in the configuration shown above:

- ◆ using the boundary conditions to obtain $\mathbf{v}_{M0+}$, $\mathbf{v}_{M0-}$, $\mathbf{i}_{M0+}$ and $\mathbf{i}_{M0-}$ in (24) and (26);
- ◆ using the scattering matrix defined by (59) and the matrices $\mathbf{P}_S$ and $\mathbf{P}_L$;
- ◆ using the chain matrix (not studied in this tutorial).

❒ Following the second approach, we consider multiple reflections occurring at the ends and multiple propagations. For the case $\mathbf{e}_S \neq \mathbf{0}$ and $\mathbf{e}_L = \mathbf{0}$, we get:

$$\begin{cases} \mathbf{v}_+(0) = \left\{ \sum_{p=0}^{\infty} \left(\mathbf{P}_S \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \, \mathbf{P}_L \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \right)^p \right\} \mathbf{Z}_C \left(\mathbf{Z}_S + \mathbf{Z}_C \right)^{-1} \mathbf{e}_S \\ \mathbf{v}_+(\mathcal{L}) = e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \left\{ \sum_{p=0}^{\infty} \left(\mathbf{P}_S \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \, \mathbf{P}_L \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \right)^p \right\} \mathbf{Z}_C \left(\mathbf{Z}_S + \mathbf{Z}_C \right)^{-1} \mathbf{e}_S \end{cases} \tag{71}$$

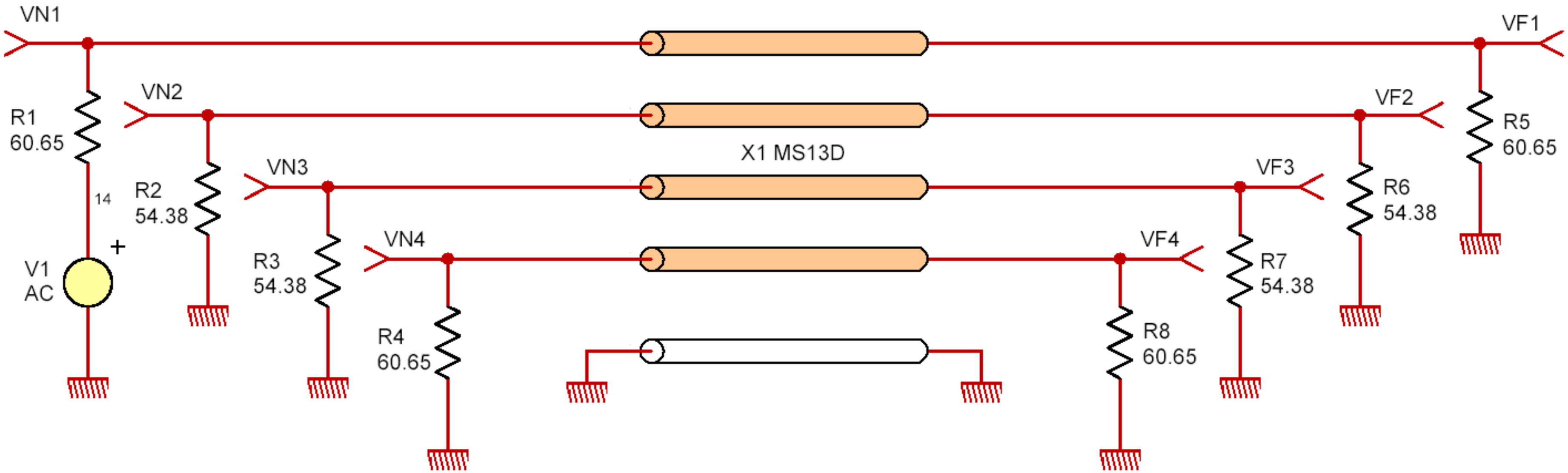
and

$$\begin{cases} \mathbf{v}_S = \left[\mathbf{1}_n + e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \, \mathbf{P}_L \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \right] \mathbf{v}_+(0) \\ \mathbf{v}_L = \left[\mathbf{1}_n + \mathbf{P}_L \right] \mathbf{v}_+(\mathcal{L}) \end{cases} \tag{72}$$

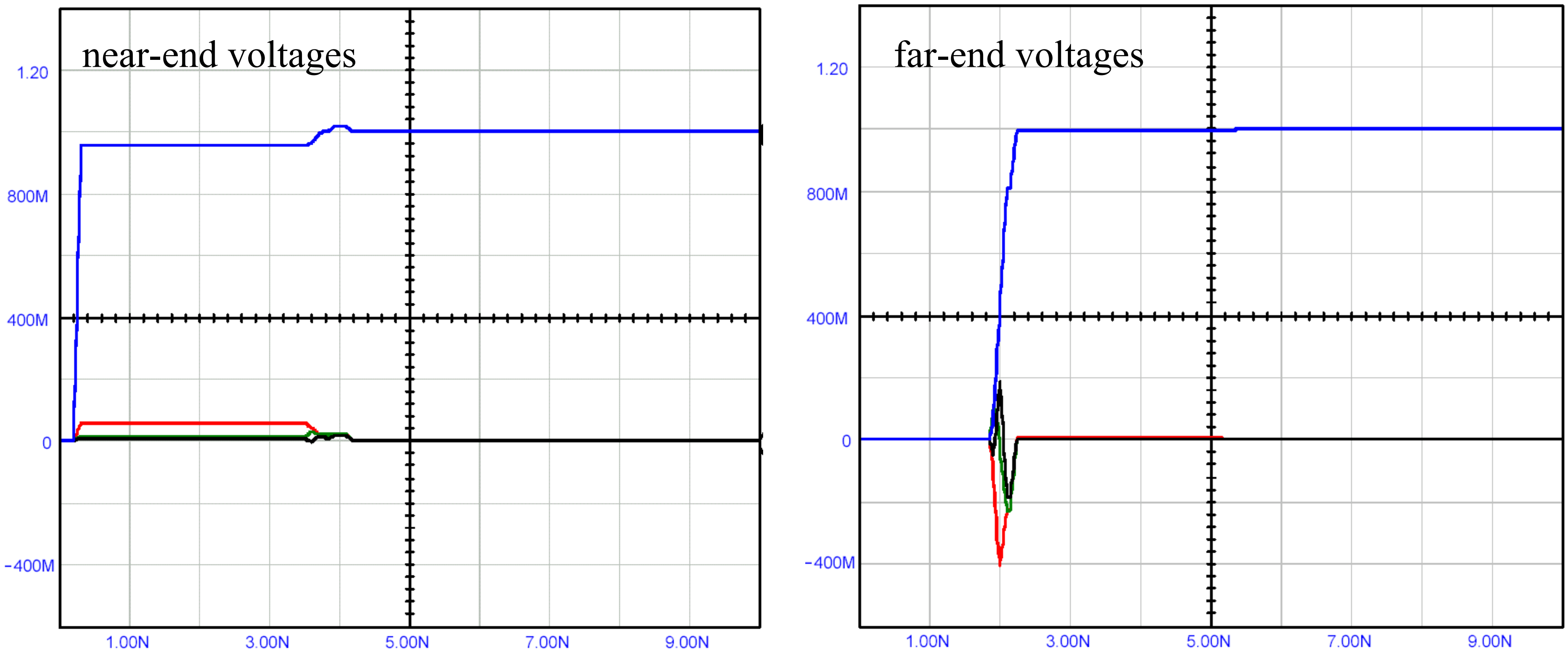
Using [11, § 5.6.16], we obtain

$$\begin{cases} \mathbf{v}_S = \left(\mathbf{1}_n + e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \, \mathbf{P}_L \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \right) \left(\mathbf{1}_n - \mathbf{P}_S \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \, \mathbf{P}_L \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \right)^{-1} \dfrac{\mathbf{1}_n - \mathbf{P}_S}{2} \mathbf{e}_S \\ \mathbf{v}_L = \left(\mathbf{1}_n + \mathbf{P}_L \right) e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \left(\mathbf{1}_n - \mathbf{P}_S \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \, \mathbf{P}_L \, e^{-\mathcal{L}\sqrt{\mathbf{Z}'\mathbf{Y}'}} \right)^{-1} \dfrac{\mathbf{1}_n - \mathbf{P}_S}{2} \mathbf{e}_S \end{cases} \tag{73}$$

❒ Example: a 300-mm long multiconductor microstrip built on a general-purpose FR-4 material, having 4 TCs, used for single-ended transmission with pseudo-matched terminations such that $|||\mathbf{P}_S|||_\infty = |||\mathbf{P}_L|||_\infty \approx 0.130$ and $|||\mathbf{P}_S|||_1 = |||\mathbf{P}_L|||_1 \approx 0.138$.

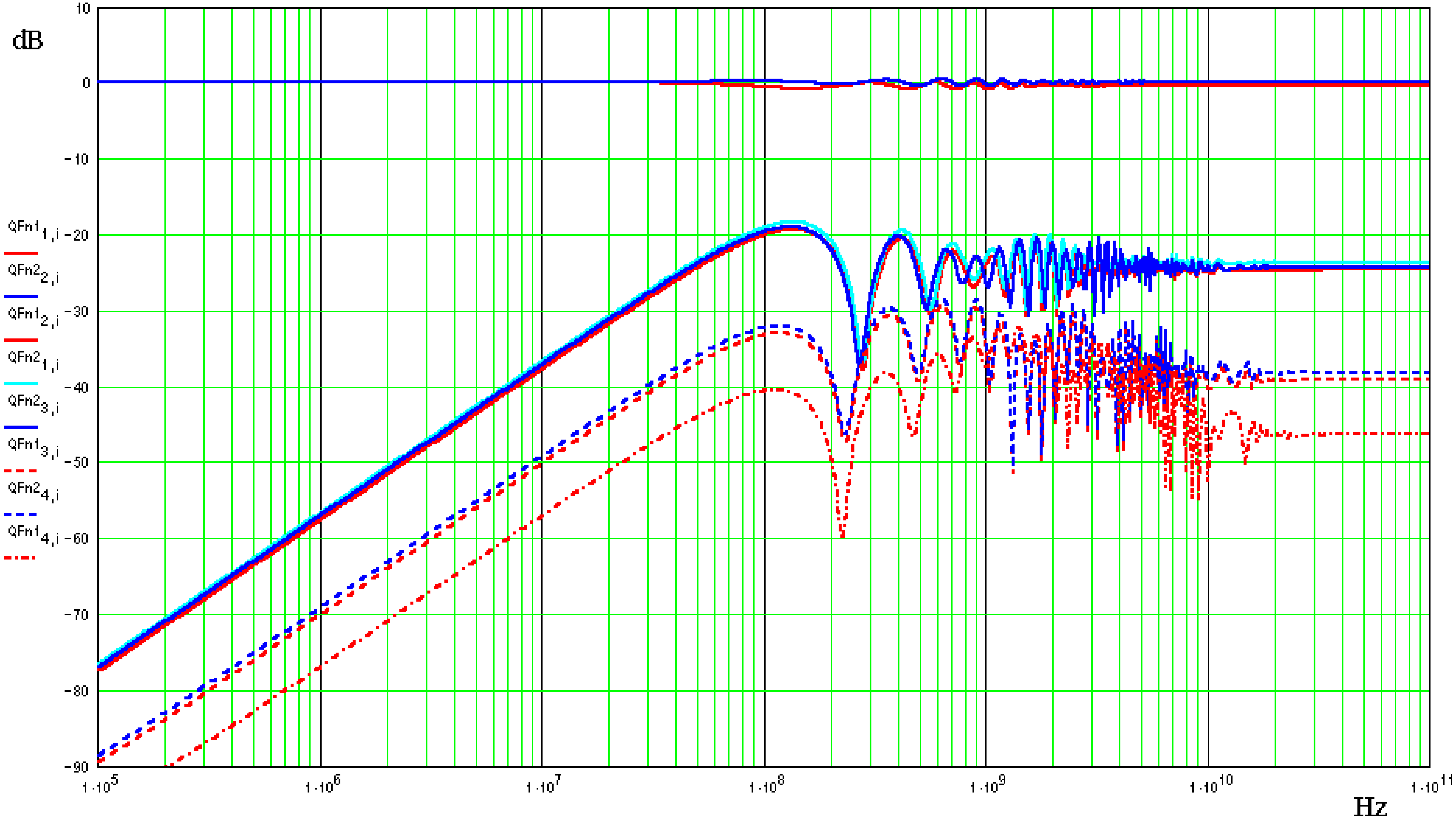


Simulation using a circuit simulation program (based on Berkeley SPICE 3F.2) and a lossless MTL model generated by SpiceLine [14] [16] [20] [26].

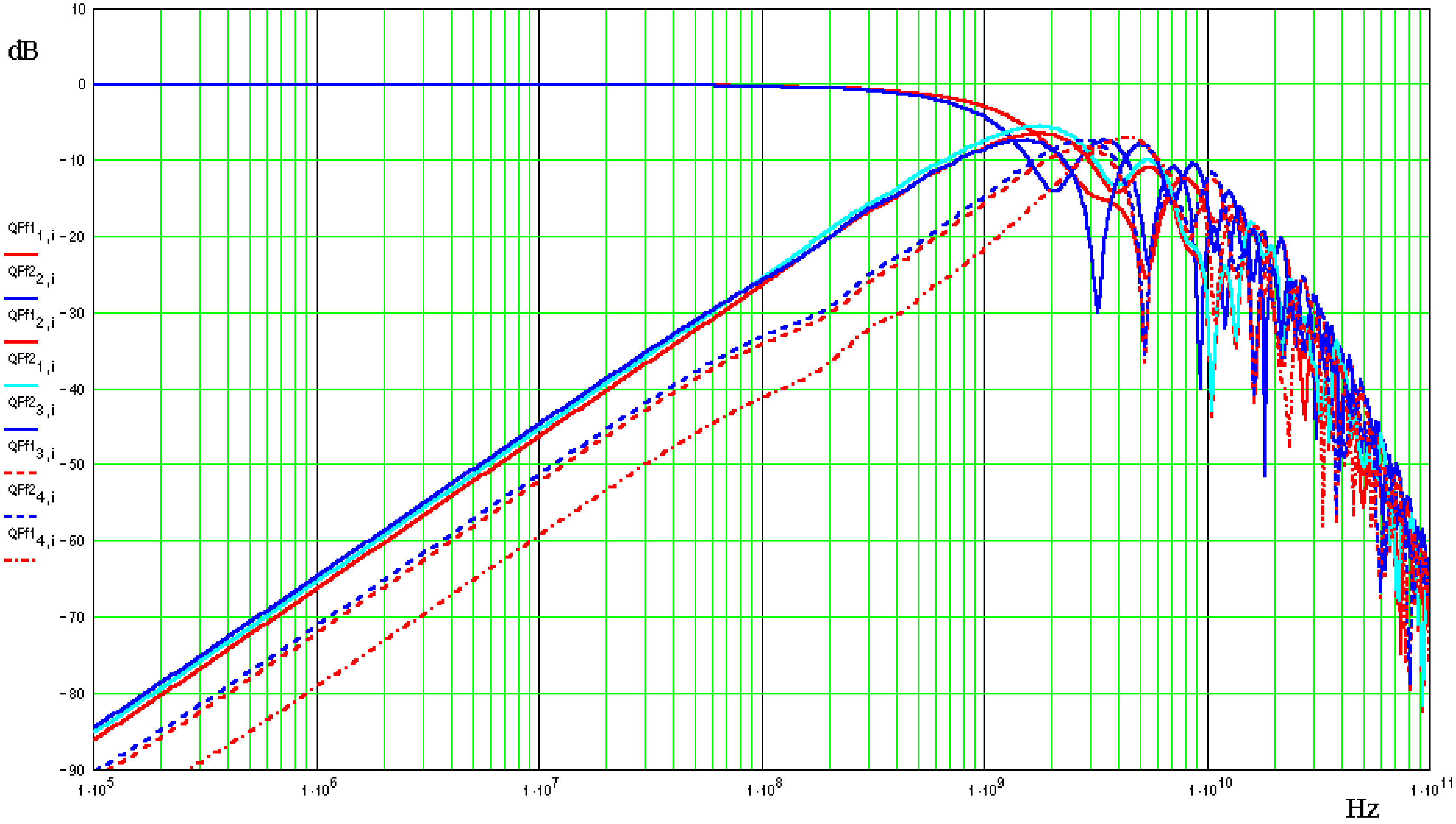


Voltages in mV versus time in ns. TC1: blue curves. TC2: red curves. TC3: green curves. TC4: black curves.

In the frequency domain, some near-end voltages obtained with a standard numerical computation program, resistive and dielectric losses being taken into account:

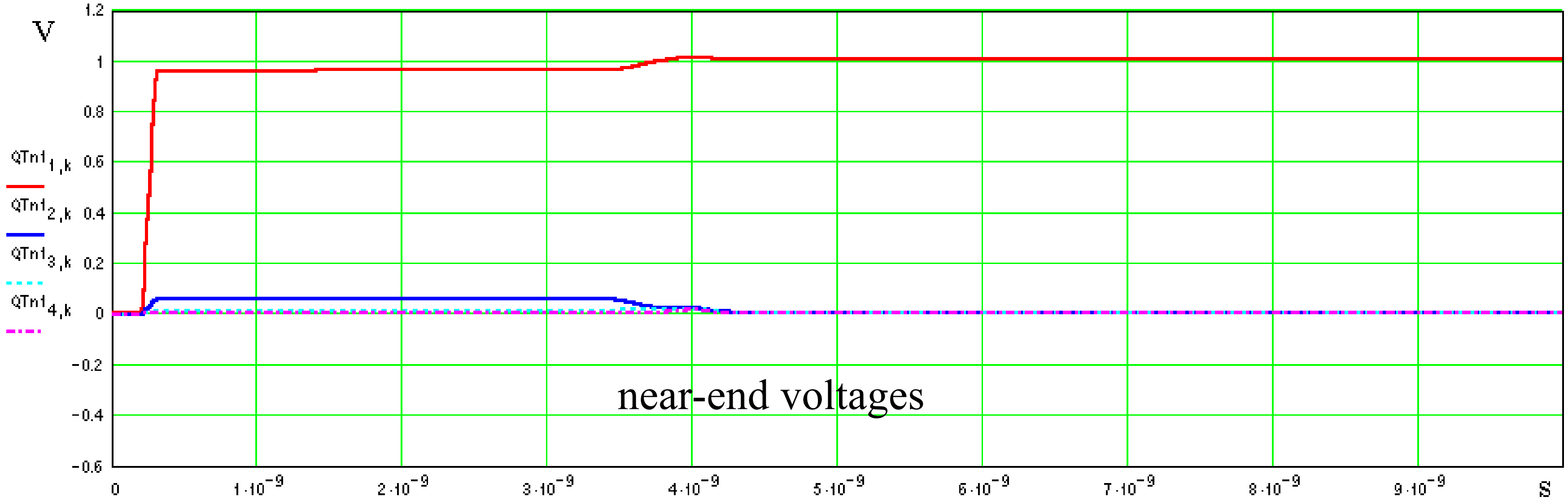


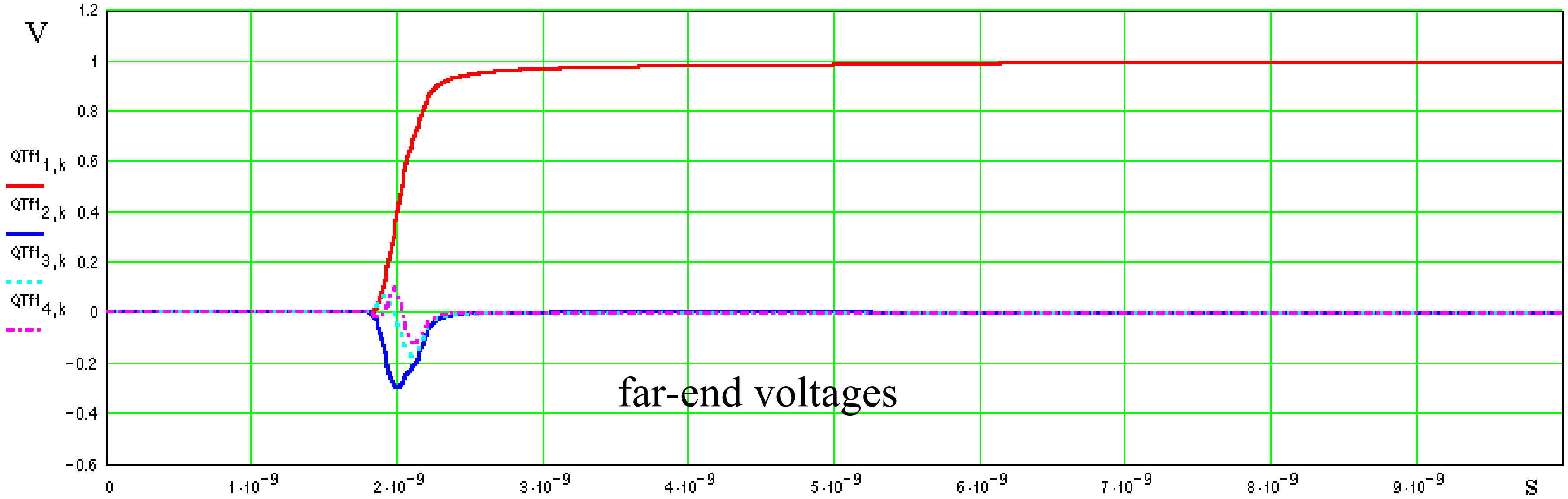
Some voltages at the far end, computed with the same tool, resistive and dielectric losses being taken into account:



Time domain results using the same tool,
resistive and dielectric losses being taken into account.

EXCEM





11. The degradation of transmitted signals

❒ The degradation of signals transmitted through a linear multichannel link is the result of five phenomena related to the properties of the interconnection:

◆ *attenuation* of the wanted signal;

◆ *echo*, the detrimental phenomenon by which a signal sent or received at an end of the link, in one of the channels, is followed by the reception of a delayed noise on the same channel, at the same end of the link;

◆ *internal crosstalk*, the detrimental phenomenon by which a signal sent in one of the channels produces noise in another channel;

◆ *other linear distortions* of the wanted signal, which may be due to the variation of the propagation velocity with frequency (dispersion) or other causes;

◆ *thermal noise*.

❒ We have not included the *propagation delay* in this list, but this phenomenon might also be a problem.

❒ In the case of a signal applied at the near end,

- ◆ *near-end crosstalk* (NEXT) is the internal crosstalk occurring at the near end;
- ◆ *far-end crosstalk* (FEXT) is the internal crosstalk occurring at the far end.

❒ *Reflection* is the phenomenon by which a wave propagating in a given direction, in one or more TCs, produces a wave propagating in the opposite direction.

❒ Reflections may be responsible for echo and/or internal crosstalk.

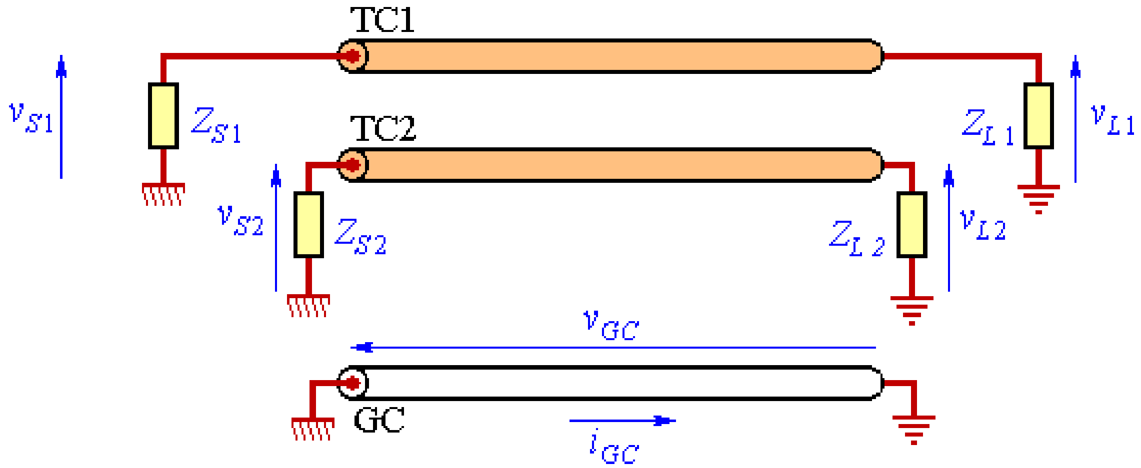
❒ If the interconnection is uniform, reflection can only be caused by the items connected at its ends.

❒ *TC-to-TC coupling* collectively designates p.u.l. mutual capacitance between the TCs and p.u.l. mutual impedance between loops comprising the TCs and the GC.

❒ TC-to-TC coupling may be responsible for internal crosstalk.

❒ Detrimental interactions between the link and other circuits of the chip, MCM, SiP or PCB in which the link is built are referred to as *external crosstalk*.

❒ External crosstalk is often associated with a current i_{GC} flowing in the GC because of such other circuit, causing a voltage drop v_{GC} along the GC, often referred to as *ground shift* or *noisy ground*.



❒ This explanation of external crosstalk is valid at dc and very low frequencies, but it is not compatible with the MTL model.

- This is not a deficiency of the MTL model:
  - at high frequencies, the voltages between the conductors are defined unambiguously only in a cross-section of the interconnection;
  - a TC must be added to allow i_{GC} to flow, and included in the MTL model.

- Possible causes of external crosstalk degrading transmission in a given link:
  - a conductor parallel to the interconnection, excited at the near end;
  - a conductor parallel to the interconnection, excited at the far end;
  - a conductor crossing the interconnection (below or above the TCs);
  - common-mode coupling at the near end (sending end);
  - common-mode coupling at the far end (receiving end).

- The mitigation techniques for internal crosstalk apply to the first two cases.

❒ Crossing at a right angle produces a local coupling, which can often be modeled with a circuit model only comprising stray capacitances.

❒ Common-mode coupling at the near end or at the far end typically happens in the line drivers or line receivers of an IC, because of currents produced by other circuits of the IC, flowing in a common impedance across which unwanted voltages referred to as *bounces* appear (e.g. the so-called *ground bounce* and *power bounce*).

❒ The common impedance is often caused by mutual inductance in the reference or power supply conductors within the IC package, or the corresponding leads.

❒ The currents having the worst effects are often caused by multiple switching in the IC, producing the so-called *simultaneous switching output* (SSO) noise.

❒ Once the common impedance has been reduced to the smaller practical value, a crosstalk cancellation scheme must be used to reduce external noise in the channel.

12. Single-ended parallel links

❒ *Single-ended transmission*: any transmission scheme in which a single TC is allocated to each channel, and in which a global net such as GND or VCC, referred to as the GC, is used as a voltage reference for receiving signals.

❒ A *single-ended link* uses single-ended transmission. It must be such that

- ◆ TC-to-TC coupling is small;
- ◆ the near-end interface and termination device (NIT) and the far-end interface and termination device (FIT) do not introduce any significant couplings between the voltages or currents in different TCs.

❒ Thus, for small signals, in any state, the impedance matrices of the NIT and FIT with respect to ground may be regarded as diagonal (or almost diagonal) matrices.

❒ A single-ended link may use:

- ◆ voltage-mode signaling (low impedance NIT, high impedance FIT);
- ◆ current-mode signaling (high impedance NIT, low impedance FIT);
- ◆ pseudo-matched impedances at the NIT and/or at the FIT.

❒ Voltage-mode and current mode circuits can be used in the line drivers and line receivers of different types of single-ended links. For instance:

- ◆ a voltage-mode line driver fitted with a series resistor (series termination) can provide a suitable pseudo-matched impedance;
- ◆ a current-mode line driver or a voltage-mode line receiver fitted with a parallel termination can provide a suitable pseudo-matched impedance.

❒ A single-ended link can be point-to-point or multidrop.

❒ Each TC of a single-ended point-to-point link can be used for unidirectional (simplex) transmission, alternate bidirectional (half duplex) transmission or simultaneous bidirectional (full duplex) transmission.

❒ Each TC of a single-ended multidrop link can be used for unidirectional transmission or as a bus.

❒ The interconnection model underlying the concept of single-ended links excludes TC-to-TC coupling. Thus, possible models for preliminary link design are:

- ◆ one ideal node for each TC, in the case of a short interconnection;
- ◆ a lumped elements model for each TC, for longer but electrically short interconnections;
- ◆ a TL model for each TC;
- ◆ a uniform TL model for each TC if the interconnection is uniform.

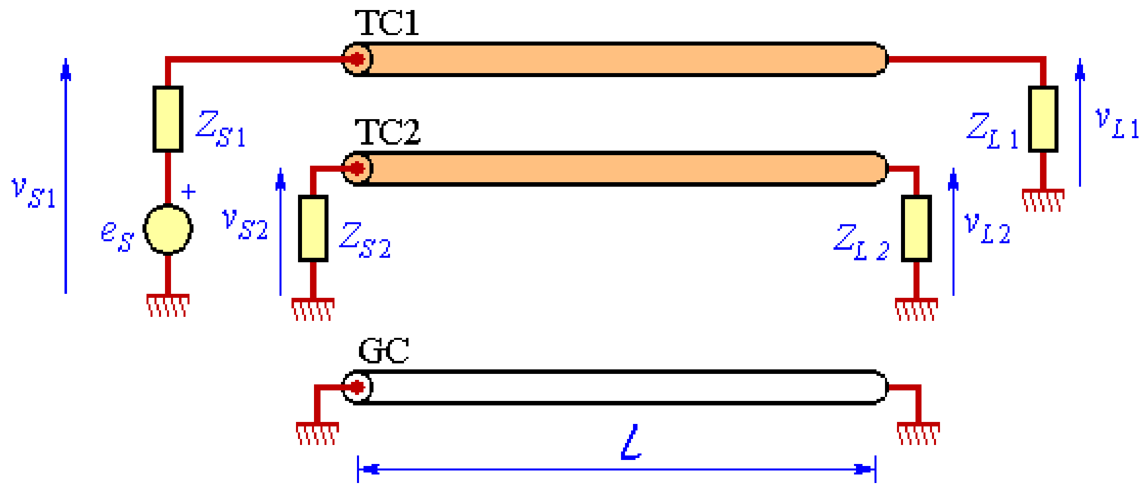
❒ Since the underlying interconnection model does not include TC-to-TC coupling, actual non-zero TC-to-TC coupling is likely to produce crosstalk.

❒ In the case $n \geq 2$, it is advisable to take TC-to-TC coupling into account at the analysis stage. It is possible to use:

- ◆ a lumped element model for an electrically short interconnection;
- ◆ the weak coupling approximation applied to a lossless MTL model;
- ◆ an exact solution of a lossless MTL model;
- ◆ an exact solution of a MTL model taking losses into account.

❒ The weak coupling approximation provides closed-form solutions which reveal the underlying physics and can be used for comparing design options.

❒ We can use a first order perturbation theory where the undisturbed solution corresponds to uncoupled TLs.



❒ For the single-ended link shown, let us assume a uniform and lossless interconnection such that

$$\mathbf{Z}' = j\omega \begin{pmatrix} L'_{11} & L'_{12} \\ L'_{12} & L'_{22} \end{pmatrix} \quad \text{and} \quad \mathbf{Y}' = j\omega \begin{pmatrix} C'_{11} & C'_{12} \\ C'_{12} & C'_{22} \end{pmatrix} \tag{74}$$

❒ Let us define

$$Z_{01} = \sqrt{\frac{L'_{11}}{C'_{11}}} \;,\; Z_{02} = \sqrt{\frac{L'_{22}}{C'_{22}}} \;,\; c_1 = \frac{1}{\sqrt{L'_{11}C'_{11}}} \text{ and } c_2 = \frac{1}{\sqrt{L'_{22}C'_{22}}} \tag{75}$$

which would be characteristic impedances and propagation velocities if coupling was not present.

❒ Let us define the capacitive coupling coefficient ξ_2 and the relative magnetic coupling coefficient Ξ_2 as:

$$\xi_2 = -\frac{C'_{12}}{C'_{22}} \quad \text{and} \quad \Xi_2 = -\frac{L'_{12}C'_{11}}{L'_{22}C'_{12}} \tag{76}$$

❒ For $Z_{S1} = Z_{L1} = Z_{01}$, $Z_{S2} = Z_{L2} = Z_{02}$ and $c_1 = c_2 = c$, we get the result of Jarvis [2]:

$$v_{S2} = \frac{e_S \xi_2}{8}\left(\Xi_2 + 1\right)\left(1 - e^{-2j\frac{\omega}{c}\mathcal{L}}\right) \tag{77}$$

$$v_{L2} = -j\frac{\omega}{c}\,\frac{e_S \xi_2}{4}\left(\Xi_2 - 1\right)\mathcal{L}\, e^{-j\frac{\omega}{c}\mathcal{L}} \tag{78}$$

❒ We can check that the maximum value of $|v_{S2}/v_{L1}|$ is $\frac{\xi_2}{2}\left(\Xi_2 + 1\right)$, first achieved at $\omega = \frac{\pi c}{2\mathcal{L}}$. This maximum is independent of $\mathcal{L}$.

❒ In the time domain, in the same case, the Jarvis formulas are:

$$v_{S2}(t) = \frac{\xi_2}{8}(\Xi_2 + 1)\left(e_S(t) - e_S\left(t - \frac{2\mathcal{L}}{c} \right) \right) \tag{79}$$

and

$$v_{L2}(t) = -\frac{\xi_2}{4c}(\Xi_2 - 1)\mathcal{L}\frac{de_S}{dt}\left(t - \frac{\mathcal{L}}{c} \right) \tag{80}$$

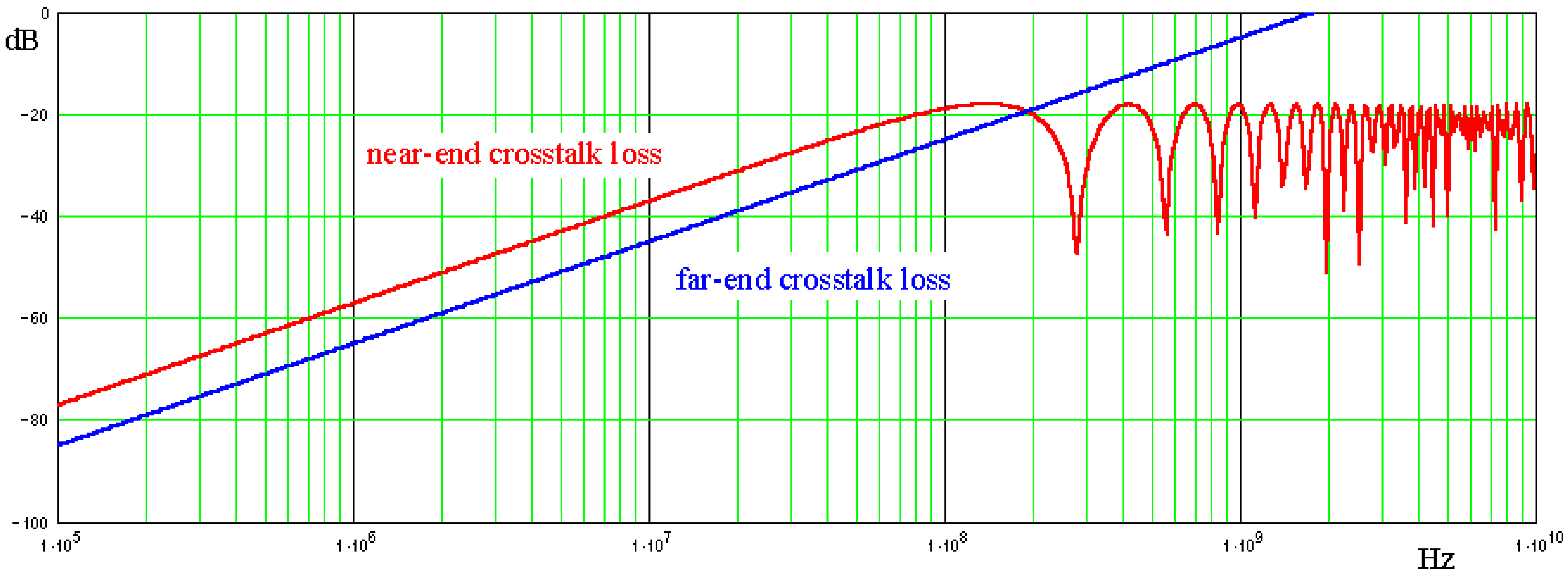
❒ This approximation predicts that:

- $v_{S2}(t)$ is proportional to $e_S(t) - e_S(t - 2\mathcal{L}/c)$ and otherwise independent of $\mathcal{L}$;
- $v_{S2}(t)$ is related to a reflection taking place at the far end;
- $v_{L2}(t)$ is proportional to $\mathcal{L}$ and to the time derivative of $e_S(t - \mathcal{L}/c)$;
- $v_{L2}(t)$ vanishes for $\Xi_2 = 1$;
- thus, for $\Xi_2 = 1$, we have a *directional coupling*.

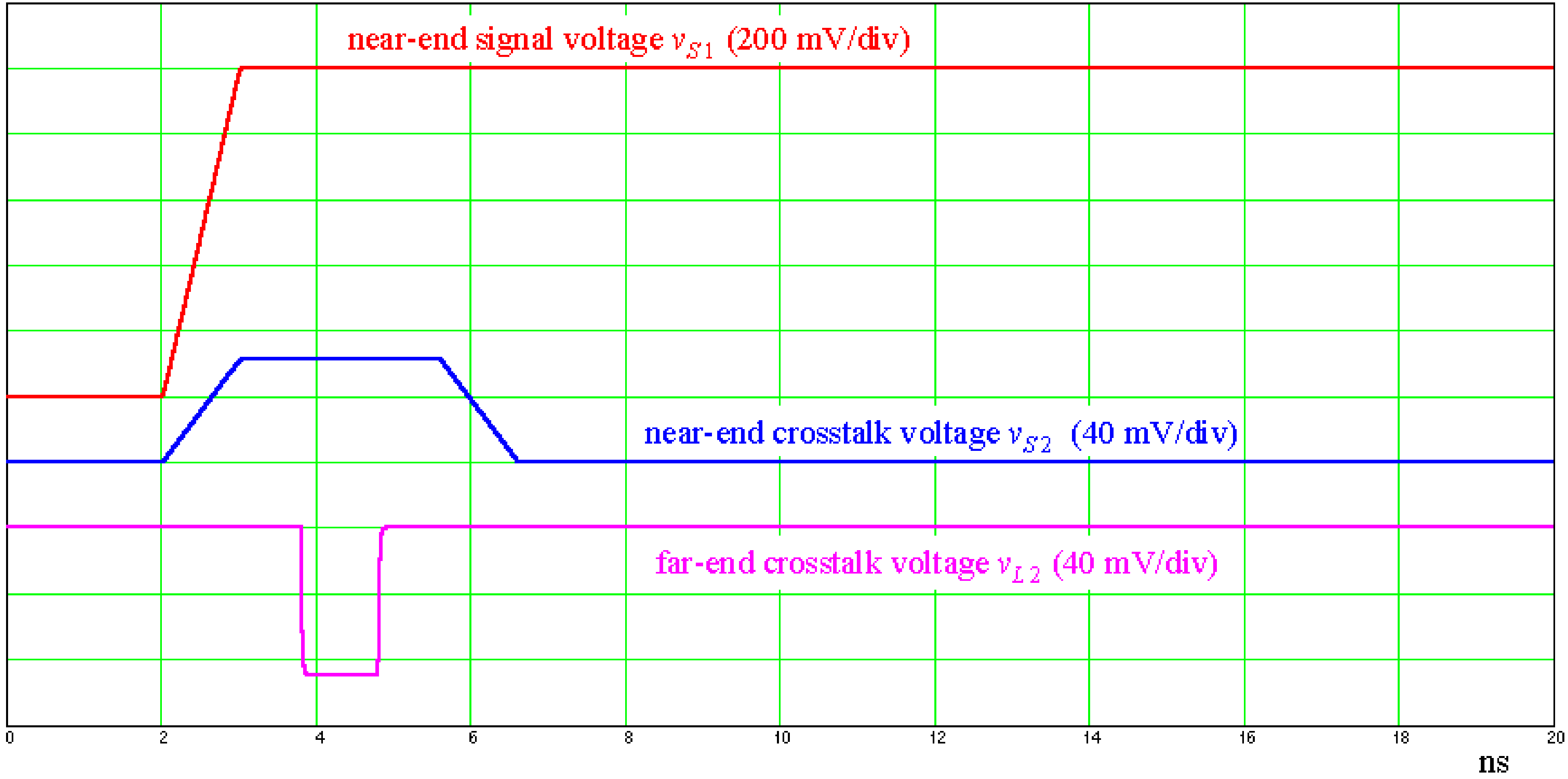
❒ In a multiconductor stripline structure, we have $\Xi_2 = 1$. In a multiconductor microstrip structure, we have $\Xi_2 > 1$.

❒ Z_{01} and Z_{02} are not characteristic impedances but pseudo-matched impedances.

❒ An example of NEXT loss $|v_{L1}/v_{S2}|$ and FEXT loss $|v_{L1}/v_{L2}|$ in a 300-mm long multiconductor microstrip built on FR-4, having 2 TCs, according to the weak coupling approximation.



In the time domain, for the step having 0%-100% rise time of 1 ns, we obtain the classical waveforms of this approximation:



❒ If we do not assume $Z_{S1} = Z_{L1} = Z_{01}$, it is useful to define the parameters

$$\rho_{S1} = \frac{Z_{S1} - Z_{01}}{Z_{S1} + Z_{01}} \quad \text{and} \quad \rho_{L1} = \frac{Z_{L1} - Z_{01}}{Z_{L1} + Z_{01}} \tag{81}$$

and

$$K_{MR1} = \sum_{p=0}^{\infty} \left(\rho_{S1} \rho_{L1} e^{-2j\frac{\omega}{c_1}\mathcal{L}} \right)^p = \frac{1}{1 - \rho_{S1} \rho_{L1} e^{-2j\frac{\omega}{c_1}\mathcal{L}}} \tag{82}$$

❒ If we do not assume $Z_{S2} = Z_{L2} = Z_{02}$, it is useful to define the parameters

$$\rho_{S2} = \frac{Z_{S2} - Z_{02}}{Z_{S2} + Z_{02}} \quad \text{and} \quad \rho_{L2} = \frac{Z_{L2} - Z_{02}}{Z_{L2} + Z_{02}} \tag{83}$$

and

$$K_{MR2} = \sum_{p=0}^{\infty} \left(\rho_{S2} \rho_{L2} e^{-2j\frac{\omega}{c_2}\mathcal{L}} \right)^p = \frac{1}{1 - \rho_{S2} \rho_{L2} e^{-2j\frac{\omega}{c_2}\mathcal{L}}} \tag{84}$$

❒ If we only assume negligible losses, weak coupling and $c_1 = c_2 = c$, we get:

$$v_{S2} = \frac{e_S(1-\rho_{S1})(1+\rho_{S2})K_{MR1}K_{MR2}\xi_2}{4}\left\{\begin{array}{l}\left(1+\rho_{L1}\rho_{L2}e^{-2j\frac{\omega}{c}\mathcal{L}}\right)\dfrac{\Xi_2+1}{2}\left(1-e^{-2j\frac{\omega}{c}\mathcal{L}}\right)\\ -(\rho_{L1}+\rho_{L2})\dfrac{j\omega}{c}(\Xi_2-1)\mathcal{L}\,e^{-2j\frac{\omega}{c}\mathcal{L}}\end{array}\right\} \tag{85}$$

and

$$v_{L2} = \frac{e_S(1-\rho_{S1})(1+\rho_{L2})K_{MR1}K_{MR2}\xi_2}{4}\left\{\begin{array}{l}(\rho_{L1}+\rho_{S2})\dfrac{\Xi_2+1}{2}\left(1-e^{-2j\frac{\omega}{c}\mathcal{L}}\right)\\ -\left(1+\rho_{L1}\rho_{S2}e^{-2j\frac{\omega}{c}\mathcal{L}}\right)\dfrac{j\omega}{c}(\Xi_2-1)\mathcal{L}\end{array}\right\}e^{-j\frac{\omega}{c}\mathcal{L}} \tag{86}$$

❒ In the case where $\mathcal{L}$ is not electrically short, a practical link is such that, for reducing echo, the NIT and/or the FIT must provide a low reflection coefficient at at least one end of each TC. Thus, we can assume that $K_{MR1} \approx 1$ and $K_{MR2} \approx 1$.

❒ The internal crosstalk mitigation approaches taught by (85) and (86) are:

- ◆ increasing the distance between the TCs relative to the distance between a TC and the GC, to obtain a decrease in ξ_{12} ;
- ◆ making Ξ_2 close to 1;
- ◆ making Z_{L1} close to Z_{01} reduces the absolute value of 2 terms in (85) and 2 terms in (86);
- ◆ making Z_{L2} close to Z_{02} reduces the absolute value of 2 terms in (85);
- ◆ making Z_{S2} close to Z_{02} reduces the absolute value of 2 terms in (86);
- ◆ decreasing as much as possible the bandwidth of the line receivers;
- ◆ decreasing as much as possible the bandwidth of the line drivers.

❐ Eq. (73) shows that, for a lossless MTL, at a time $t < 2\,\tau_{\min}$ from an excitation, $\mathbf{v}_S$ is only determined by $\mathbf{Z}_C\,(\mathbf{Z}_S + \mathbf{Z}_C)^{-1}\,\mathbf{e}_S$. This exact result is compatible with (79) and (85): $\mathcal{L}$ plays no role and ω plays no role except, possibly, in $\mathbf{Z}_S$.

❐ Using (73), it can be shown that, for a completely degenerate MTL seeing pseudo-matched impedances at the ends of each TC, there is no FEXT to the first order in the weak coupling approximation. This result is compatible with (80).

❐ Thus, in a long single-ended link terminated with pseudo-matched impedances,

- ◆ the cause of NEXT is the lack of matching at the near end; and
- ◆ the cause of FEXT is the propagation of modes at unequal velocities.

❐ It is often difficult to use low-swing transmission in the single-ended links of a digital IC, due to common-mode coupling at the near end or at the far end.

❐ A compensation or equalization scheme, used to flatten the channel gain in the bandwidth used for transmission, unfortunately increases the FEXT.

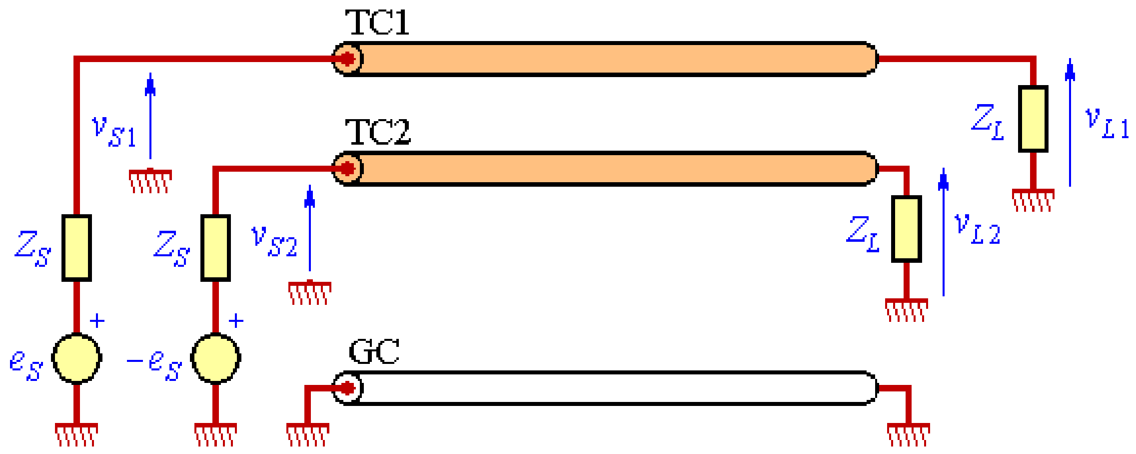
13. Multichannel differential links

❐ A balanced pair comprises 2 TCs having the same *averaged* p.u.l. impedance and p.u.l. admittance with respect to the GC.

❐ In a link using a 2-TC interconnection, a NIT or FIT is balanced if the signal terminals present the same admittance with respect to the GC.

❐ Using a balanced pair and balanced NIT and FIT, *it seems that* a differential-mode source and a common-mode source produce opposite and equal voltages with respect to the GC, respectively, at each z.

For instance, for $L'_{11} = L'_{22}$ and $C'_{11} = C'_{22}$, in the link shown, by symmetry we have $-v_{S2} = v_{S1}$ and $-v_{L2} = v_{L1}$.



❒ A single-channel differential link uses this configuration with:

◆ a transmitting circuit using a differential-mode source for signaling (differential line driver); and

◆ a receiving circuit sensitive to differential mode signals and insensitive to the common-mode voltages (differential line receiver).

❒ This crosstalk cancellation scheme may effectively reduce the effect of 3 causes of external crosstalk:

◆ a conductor crossing the interconnection;

◆ common-mode coupling at the near end;

◆ common-mode coupling at the far end.

❒ However, this is only true if other parallel conductors do not disturb the symmetry of the TCs. For instance, the following geometry may cause problems:

❒ A perfectly balanced pair comprises 2 TCs such that [55]:

◆ the TCs have the same *averaged* p.u.l. impedance and p.u.l. admittance with respect to the GC;

◆ the excitation of the pair in differential mode induces no voltage and injects no current in any other conductor.

❒ Reciprocity entails that a voltage appearing on or a current flowing in such other conductor produces no differential mode excitation in the perfectly balanced pair.

❒ A single-channel differential link using a perfectly balanced pair implements a crosstalk cancellation scheme which reduces all causes of external crosstalk.

❒ The second condition for a perfectly balanced pair may be satisfied using a sufficient distance from other parallel conductors, or using a frequent twisting [34]:

❒ The twisted pair behaves as a uniform interconnection only if the length of a twist is much smaller than the wavelength, hence the use of *averaged* in the definitions.

❒ Let us study the propagation in an isolated pair which need not be balanced.

- We write:

$$\mathbf{Z}' = \begin{pmatrix} Z'_{11} & Z'_{12} \\ Z'_{12} & Z'_{22} \end{pmatrix} \quad \text{and} \quad \mathbf{Y}' = \begin{pmatrix} Y'_{11} & Y'_{12} \\ Y'_{12} & Y'_{22} \end{pmatrix} \tag{87}$$

so that

$$\mathbf{Z}'\,\mathbf{Y}' = \begin{pmatrix} Z'_{11}Y'_{11} + Z'_{12}Y'_{12} & Z'_{11}Y'_{12} + Z'_{12}Y'_{22} \\ Z'_{12}Y'_{11} + Z'_{22}Y'_{12} & Z'_{12}Y'_{12} + Z'_{22}Y'_{22} \end{pmatrix} \tag{88}$$

- We find that the propagation constants are the solutions of the equation

$$\gamma^2 = \frac{1}{2}\begin{pmatrix} Z'_{11}Y'_{11} + 2Z'_{12}Y'_{12} + Z'_{22}Y'_{22} \\ \pm\sqrt{\left(Z'_{11}Y'_{11} - Z'_{22}Y'_{22}\right)^2 + 4\left(Z'_{11}Y'_{12} + Z'_{12}Y'_{22}\right)\left(Z'_{22}Y'_{12} + Z'_{12}Y'_{11}\right)} \end{pmatrix} \tag{89}$$

◆ For $\left(Z'_{11}Y'_{11} - Z'_{22}Y'_{22}\right)^2 + 4\left(Z'_{11}Y'_{12} + Z'_{12}Y'_{22}\right)\left(Z'_{22}Y'_{12} + Z'_{12}Y'_{11}\right) = 0$, we have degenerate eigenvalues. Assuming that $\mathbf{Z}'\,\mathbf{Y}'$ is diagonalizable, we reach the conclusion that $\mathbf{Z}'\,\mathbf{Y}'$ is diagonal.

Thus, $\mathbf{S}$ can be any invertible matrix. Any non-trivial linear combination of natural voltages is a modal voltage. Same thing for the currents. We find:

$$\mathbf{Z}_C = \Gamma^{-1}\mathbf{Z}' = \frac{2}{Z'_{11}Y'_{11} + 2Z'_{12}Y'_{12} + Z'_{22}Y'_{22}} \begin{pmatrix} Z'_{11} & Z'_{12} \\ Z'_{12} & Z'_{22} \end{pmatrix} \tag{90}$$

◆ For $\left(Z'_{11}Y'_{11} - Z'_{22}Y'_{22}\right)^2 + 4\left(Z'_{11}Y'_{12} + Z'_{12}Y'_{22}\right)\left(Z'_{22}Y'_{12} + Z'_{12}Y'_{11}\right) \neq 0$, we have two distinct eigenvalues, denoted by γ_1 and γ_2. We find that we can use

$$\mathbf{S} = \begin{pmatrix} -Z'_{11}Y'_{12} - Z'_{12}Y'_{22} & -Z'_{11}Y'_{12} - Z'_{12}Y'_{22} \\ Z'_{11}Y'_{11} + Z'_{12}Y'_{12} - \gamma_1^2 & Z'_{11}Y'_{11} + Z'_{12}Y'_{12} - \gamma_2^2 \end{pmatrix} \tag{91}$$

There is no simple result for $\mathbf{Z}_C = \mathbf{S}\,\Gamma^{-1}\mathbf{S}^{-1}\,\mathbf{Z}'$.

❒ For a perfectly balanced pair, we have $Z'_{11} = Z'_{22}$ and $Y'_{11} = Y'_{22}$ so that (89) leads us to:

$$\begin{cases} \gamma_1 = \sqrt{Z'_{11}Y'_{11} + Z'_{12}Y'_{12} - Z'_{11}Y'_{12} - Z'_{12}Y'_{11}} = \sqrt{\left(Z'_{11} - Z'_{12}\right)\left(Y'_{11} - Y'_{12}\right)} \\ \gamma_2 = \sqrt{Z'_{11}Y'_{11} + Z'_{12}Y'_{12} + Z'_{11}Y'_{12} + Z'_{12}Y'_{11}} = \sqrt{\left(Z'_{11} + Z'_{12}\right)\left(Y'_{11} + Y'_{12}\right)} \end{cases} \qquad (92)$$

❒ Here, we can use the traditional biorthonormal eigenvectors defined by

$$\mathbf{S} = \begin{pmatrix} 1/2 & 1 \\ -1/2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{T} = {}^t\mathbf{S}^{-1} = \begin{pmatrix} 1 & 1/2 \\ -1 & 1/2 \end{pmatrix} \qquad (93)$$

for which

- $v_{M\,1}$ is the differential-mode voltage $\quad v_{DM} = v_1 - v_2$;
- $i_{M\,1}$ is the differential-mode current $\quad i_{DM} = (i_1 - i_2)/2$;
- $v_{M\,2}$ is the common-mode voltage $\quad v_{CM} = (v_1 + v_2)/2$;
- $i_{M\,2}$ is the common-mode current $\quad i_{CM} = i_1 + i_2$.

❒ For the isolated balanced pair and this choice of **S** and **T**, we obtain

$$\mathbf{Z}_{MC} = \mathrm{diag}_2(Z_{DMC}, Z_{CMC}) \tag{94}$$

where

$$Z_{DMC} = 2\sqrt{\frac{Z'_{11} - Z'_{12}}{Y'_{11} - Y'_{12}}} \quad \text{and} \quad Z_{CMC} = \frac{1}{2}\sqrt{\frac{Z'_{11} + Z'_{12}}{Y'_{11} + Y'_{12}}} \tag{95}$$

Z_{DMC} and Z_{CMC} are traditionally referred to as the *differential-mode characteristic impedance* and the *common-mode characteristic impedance*, respectively.

❒ For an isolated balanced pair, the characteristic impedance matrix is given by:

$$\mathbf{Z}_C = \begin{pmatrix} Z_{CMC} + \frac{1}{4} Z_{DMC} & Z_{CMC} - \frac{1}{4} Z_{DMC} \\ Z_{CMC} - \frac{1}{4} Z_{DMC} & Z_{CMC} + \frac{1}{4} Z_{DMC} \end{pmatrix} \tag{96}$$

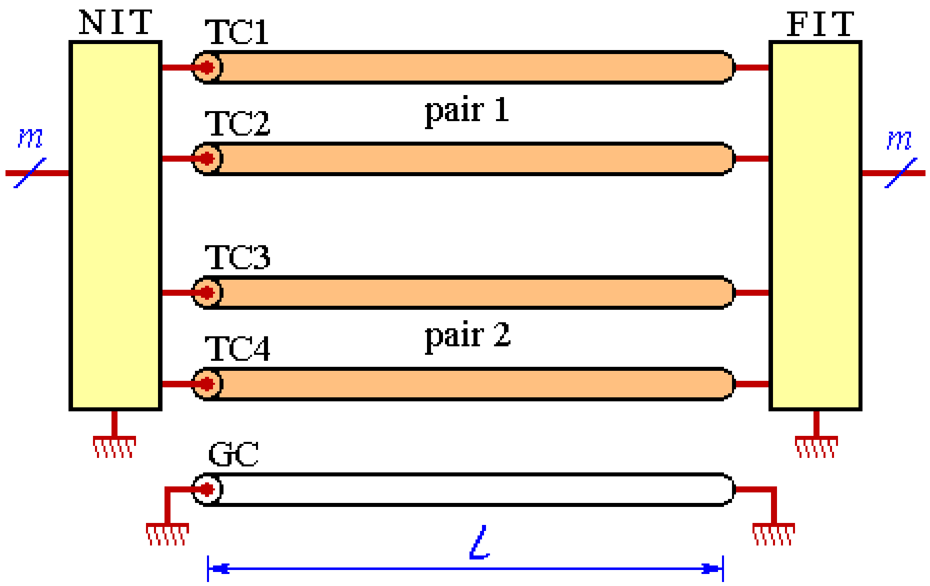
❒ Many link designs use terminations only intended to provide a suitable Z_{DM}.

❒ A perfectly balanced interconnection comprises p pairs such that [55]:

- ◆ the TCs of the same pair have the same *averaged* p.u.l. impedance and p.u.l. admittance with respect to the GC;
- ◆ the excitation of any pair in differential mode induces no voltage and injects no current in any other conductor.

❒ The second condition for a perfectly balanced interconnection may be satisfied using a sufficient distance from other parallel conductors and/or using a frequent transposition of the TCs of each pair.

❒ In transposition, the TCs of each pair exchange position at intervals along the interconnection so as to balance out, as exactly as possible, unwanted voltages and currents induced by adjacent circuits, while complying with the first condition.



❐ *Differential transmission*: any transmission scheme in which two TCs (*a pair*), a differential line driver and a differential line receiver are allocated to each channel.

❐ A *multichannel differential link* uses differential transmission. It must be such that:

- ◆ the interconnection nearly behaves as a perfectly balanced interconnection;
- ◆ each pair of signal terminals of the NIT or of the FIT, intended to be connected to an end of a single pair, is nearly perfectly balanced.

- Note that TC-to-TC coupling is not a problem:
    - between the TCs of the same pair;
    - if it occurs between the TCs of different pairs, because it is balanced out.

- A multichannel differential link using a perfectly balanced interconnection implements a crosstalk cancellation scheme which:
    - reduces internal crosstalk;
    - reduces the effect of all causes of external crosstalk.

- A differential link can be point-to-point or multidrop.

- Each pair of a differential point-to-point link can be used for unidirectional (simplex) transmission, alternate bidirectional (half duplex) transmission or simultaneous bidirectional (full duplex) transmission.

- Each pair of a differential multidrop link can be used for unidirectional transmission or as a bus.

- For p pairs, for $\alpha \in \{1,..., p\}$, we can define
  - p differential-mode voltages $v_{DM\,\alpha} = v_{2\alpha-1} - v_{2\alpha}$;
  - p differential-mode currents $i_{DM\,\alpha} = (i_{2\alpha-1} - i_{2\alpha})/2$;
  - p pair common-mode voltages $v_{CM\,\alpha} = (v_{2\alpha-1} + v_{2\alpha})/2$;
  - p pair common-mode currents $i_{CM\,\alpha} = i_{2\alpha-1} + i_{2\alpha}$.
- We do not imply that these variables are propagation modes of the interconnection.
- It can be shown that, for a perfectly balanced interconnection [55]:
  - the differential-mode voltages and the differential-mode currents are propagation modes of the interconnection;
  - thus, there is no coupling between the differential-mode variables of a pair and the differential-mode or pair common-mode variables of another pair;
  - thus, in the case of an ideal NIT and of an ideal FIT, there is no internal crosstalk.

❒ In a densely wired PCB or MCM, the interconnection is often far from being perfectly balanced.

❒ The interconnection model underlying the concept of differential links excludes any coupling between the differential-mode variables of a pair, on the one hand, and currents or voltages on other conductors, on the other. Thus, possible models are:

- ◆ one ideal node for each TC, in the case of a short interconnection;
- ◆ a lumped elements model for each pair, in the case of a longer but electrically short interconnections;
- ◆ a 3-conductor MTL model for each pair;
- ◆ a uniform 3-conductor MTL model for each pair, if the interconnection is uniform;
- ◆ a TL model for the differential-mode variable of each pair;
- ◆ a uniform TL model for the differential-mode variable of each pair, if the interconnection is uniform.

❐ Since the underlying interconnection model excludes the couplings between the differential-mode variables on a pair and the differential-mode or pair common-mode variables relating to another pair, actual non-zero values of these couplings are likely to produce internal crosstalk.

❐ In the case $n \geq 2$, it is advisable to take TC-to-TC coupling into account at the analysis stage. It is possible to use:

- ◆ a lumped element model for an electrically short interconnection;
- ◆ an exact solution of a lossless MTL model using natural variables;
- ◆ an exact solution of a lossless MTL model using the differential-mode and pair common-mode variables;
- ◆ an exact solution of a MTL model taking losses into account;
- ◆ an exact solution of a MTL model using the differential-mode and pair common-mode variables, taking losses into account.

❒ In the usual case where transposition is not used, a discussion of the design options for a multichannel differential link must take into account the length $\mathcal{L}$ and :

◆ the types of pairs (microstrip, edge-coupled stripline, broadside-coupled stripline, ...) and the tightness of the coupling between the TCs of each pair;

◆ the relative position of the pairs;

◆ the type of termination for each pair (unterminated, terminated at one or both ends, using pseudo-matched or floating impedors, or π or Y terminations);

◆ the type of differential line driver and differential line receivers (voltage-mode, current mode [58] [90] [99]).

❒ A differential link of a digital IC can use low-swing transmission if it is sufficiently immune to common-mode coupling at the near end and at the far end.

❒ A wide-band differential link can use a compensation or equalization schemes to flatten the channel gain, if the high-frequency crosstalk is sufficiently low [88].

14. Modal signaling

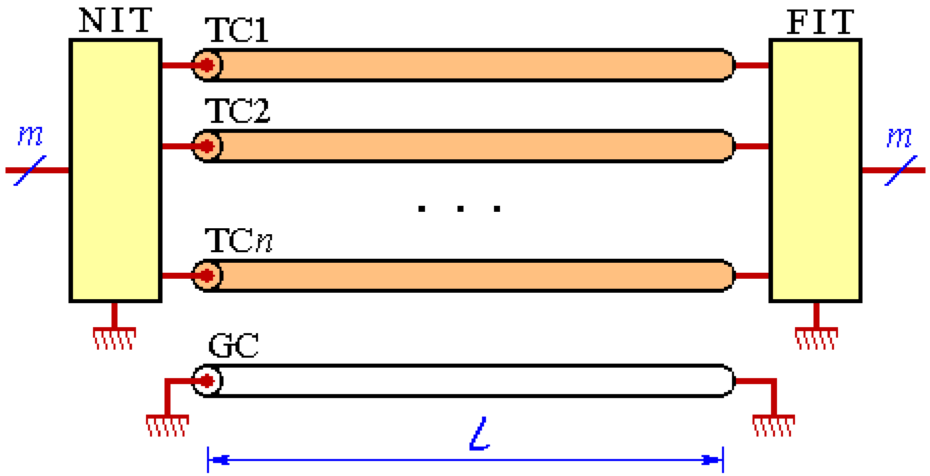
❒ Up to now, we have considered the modal decomposition as a step in the MTL theory used for computing voltages and currents in a uniform multiconductor interconnection having n TCs.

❒ In § 4, we used

$$\begin{cases} \mathbf{v}_M = e^{-z\Gamma}\, \mathbf{v}_{M\,0+} + e^{z\Gamma}\, \mathbf{v}_{M\,0-} \\ \mathbf{i}_M = e^{-z\Gamma}\, \mathbf{i}_{M\,0+} + e^{z\Gamma}\, \mathbf{i}_{M\,0-} \end{cases} \tag{26}$$

where $\mathbf{v}_{M\,0+}$, $\mathbf{v}_{M\,0-}$, $\mathbf{i}_{M\,0+}$ and $\mathbf{i}_{M\,0-}$ are z-independent vectors. We see that propagation entails an alteration of $\mathbf{v}_{M+} = e^{-z\Gamma}\, \mathbf{v}_{M\,0+}$, of $\mathbf{v}_{M-} = e^{z\Gamma}\, \mathbf{v}_{M\,0-}$, of $\mathbf{i}_{M+} = e^{-z\Gamma}\, \mathbf{i}_{M\,0+}$ and of $\mathbf{i}_{M-} = e^{z\Gamma}\, \mathbf{i}_{M\,0-}$, but no interference between the entries of each of these vectors.

❒ This suggests a modal signaling method for removing crosstalk in a m-channel link using a uniform multiconductor interconnection having $n \geq m$ TCs.



❒ In modal signaling:

◆ for each of the *m* transmission channels, we use a modal electrical variable (modal voltage or modal current), instead of a natural electrical variable (natural voltage or natural current) in single-ended signaling;

◆ the modal electrical variables used for transmission are all modal voltages or all modal currents;

◆ the NIT and the FIT must perform the necessary conversions, which are defined by a transition matrix from modal electrical variables to natural electrical variables, i.e., **S** or **T**.

❒ A differential link implements modal signaling, for $m = 1$ and $n = 2$. In this case, **S** and **T** are determined by the symmetry of the interconnection so that:

◆ they are frequency independent, according to (93);

◆ $\mathbf{v}_M$ and $\mathbf{i}_M$ are easily defined in the time and frequency domains.

❒ In most multiconductor interconnections such that $n \geq 3$, **S** and **T** are not fully determined by the symmetries and are frequency-dependent complex matrices.

❒ A simplified definition of the general ZXtalk method reads as follows:

◆ for each of the m transmission channels, we use a modal voltage or a modal current (modal signaling);

◆ the interconnection has n TCs, with $n \geq m$, and it is terminated with at least one matched or almost matched termination, i.e. a $(n + 1)$-terminal linear termination circuit having an impedance matrix approximating $\mathbf{Z}_C$.

❒ According to this simplified definition of the general ZXtalk method, no internal crosstalk and no echo occurs in the transmission channels since:

◆ total decoupling provides an independent propagation of each eigen-voltage with the associated eigen-current;

◆ the termination circuits absorb incident waves so that they do not create couplings between the modes.

❒ According to this simplified definition, at each frequency used for transmission,

◆ a transmitting circuit (TX circuit) must combine the input signals according to linear combinations defined by **S** or **T**;

◆ a receiving circuit (RX circuit) must combine the signals present on the TCs according to linear combinations defined by $\mathbf{S}^{-1}$ or $\mathbf{T}^{-1}$.

❒ Unfortunately, it is difficult to perform the modal variable to natural variable conversion when **S** and **T** are frequency-dependent complex matrices.

❐ In the RC region [35, ch. 3], we may consider that $\mathbf{Z}' \approx \mathbf{R}'$ and $\mathbf{Y}' \approx j\omega\,\mathbf{C}'$ where $\mathbf{R}'$ and $\mathbf{C}'$ are frequency-independent matrices. Thus,

◆ $\mathbf{Z}_C$ is in the form

$$\mathbf{Z}_C \approx \frac{1-j}{\sqrt{2\omega}}\mathbf{A}_C \tag{97}$$

where $\mathbf{A}_C$ is a real and frequency-independent matrix;

◆ this $\mathbf{Z}_C$ cannot be realized accurately over a wide relative bandwidth, using a lumped termination circuit;

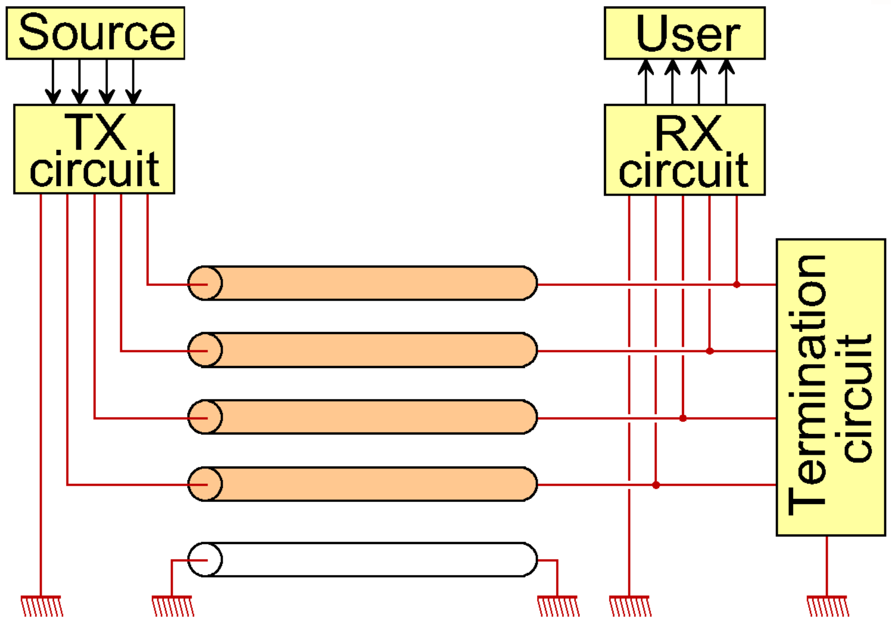
◆ thus, we are in trouble to implement the simplified definition of the general ZXtalk method at low frequencies.

❐ However,

◆ a reduction of crosstalk or echo is not needed at low frequencies;

◆ the simplified definition of the general ZXtalk method is easy to implement at high frequencies such that $\mathbf{Z}' \approx j\omega\mathbf{L}'$, where $\mathbf{Z}_C$ is real and frequency independent, and where $\mathbf{S}$ and $\mathbf{T}$ may be chosen real and frequency independent.

❒ According to a more general definition, the general ZXtalk method relates to an m-channel link such that [31] [32]:



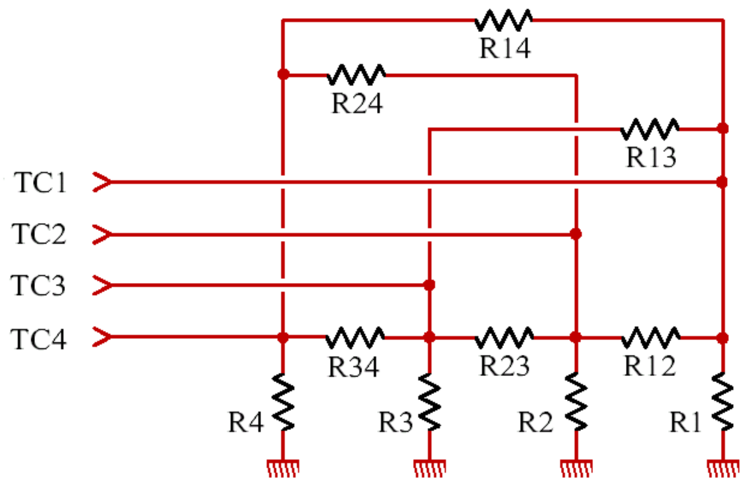
- ◆ the interconnection has n TCs, with $n \geq m$, and may be modeled as a uniform $(n + 1)$-conductor MTL, with a sufficient accuracy, in a known frequency band;
- ◆ the interconnection is connected at at least one end to a termination circuit having, in the known frequency band, an impedance matrix near $\mathbf{Z}_C$;
- ◆ a TX circuit delivers modal electrical variables defined by **S** or **T**, each modal electrical variable being mainly determined by one and only one input signal;
- ◆ a RX circuit delivers output signals, each of the output signals being mainly determined by one and only one of said modal electrical variables.

❒ The "known frequency band" may correspond to high frequencies such that $\mathbf{Z}' \approx j\omega\mathbf{L}'$. Thus, for the synthesis of the link, we can use a lossless MTL model [108].

❒ Consequently, we can use:

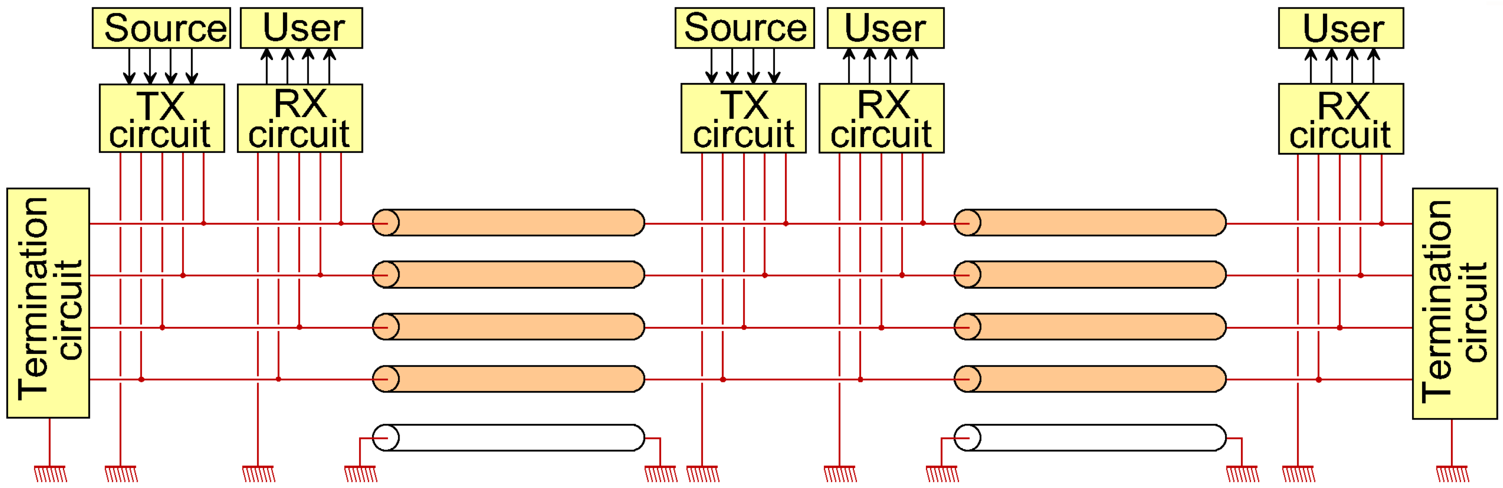
◆ a termination circuit made of $n\ (n + 1)\ /2$ resistors, as shown here for $n = 4$;

◆ TX circuits and RX circuits performing real and frequency-independent linear combinations.



❒ Outside the known frequency band, the MTL model used for the synthesis of the link is not required to be an accurate model of the actual interconnection.

❒ Of course, for the analysis of the link, we should use a MTL model providing accurate predictions in the whole frequency band used for transmission. This accurate model could include losses and departures from the assumption of uniformity.



❒ A link implementing the ZXtalk method can be point-to-point or multidrop. Above: a bus using parallel connection for the TX circuits and RX circuits.

❒ Each mode of a point-to-point link implementing the ZXtalk method can be used for simplex, half duplex or full duplex transmission.

❒ Each mode of a multidrop link implementing the ZXtalk method can be used for simplex transmission or as a bus.

❒ There are 8 possible designs, corresponding to the following equations:

Interface	Connection	Design using modal voltages (voltage-mode signaling)	Design using modal currents (current-mode signaling)
TX circuit	series (low impedance)	$\mathbf{v}_T = \pm a\, \mathbf{S}\, \mathrm{diag}_n(\alpha_1, \ldots, \alpha_n)\, \mathbf{x}_I$	$\mathbf{v}_T = a\, \mathbf{Z}_C \mathbf{T}\, \mathrm{diag}_n(\lambda_1, \ldots, \lambda_n)\, \mathbf{x}_I$
	parallel (high impedance)	$\mathbf{i}_T = a\, \mathbf{Z}_C^{-1} \mathbf{S}\, \mathrm{diag}_n(\alpha_1, \ldots, \alpha_n)\, \mathbf{x}_I$	$\mathbf{i}_T = \pm a\, \mathbf{T}\, \mathrm{diag}_n(\lambda_1, \ldots, \lambda_n)\, \mathbf{x}_I$
RX circuit	series (low impedance)	$\mathbf{x}_O = \pm\, \mathrm{diag}_n(\beta_1, \ldots, \beta_n) \mathbf{S}^{-1} \mathbf{Z}_C\, \mathbf{i}_R$	$\mathbf{x}_O = \mathrm{diag}_n(\mu_1, \ldots, \mu_n) \mathbf{T}^{-1}\, \mathbf{i}_R$
	parallel (high impedance)	$\mathbf{x}_O = \mathrm{diag}_n(\beta_1, \ldots, \beta_n) \mathbf{S}^{-1} \mathbf{v}_R$	$\mathbf{x}_O = \pm\, \mathrm{diag}_n(\mu_1, \ldots, \mu_n) \mathbf{T}^{-1} \mathbf{Z}_C^{-1} \mathbf{v}_R$

(98)

where $\mathbf{x}_I$, $\mathbf{v}_T$ and $\mathbf{i}_T$ are the column vectors of the input signals, output voltages and output currents of the TX circuit, respectively, a is the number of termination circuits; $\mathbf{v}_R$, $\mathbf{i}_R$ and $\mathbf{x}_O$ are the column vectors of the input voltages, input currents and output signals of the RX circuit, respectively; the α_i, λ_i, β_i, and the μ_i are arbitrary nonzero constants.

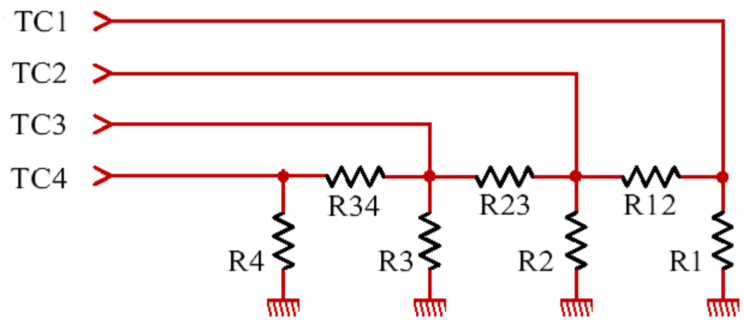
❒ Between a TX circuit and a RX circuit connected to the interconnection at $z = z_{TX}$ and $z = z_{RX}$, respectively, we have, in the known frequency band:

$$x_{Oi} = \alpha_i \, \beta_i \; e^{-\gamma_i \left| z_{RX} - z_{TX} \right|} \, x_{Ii} \qquad \text{or} \qquad x_{Oi} = \lambda_i \, \mu_i \; e^{-\gamma_i \left| z_{RX} - z_{TX} \right|} \, x_{Ii} \tag{99}$$

according to the case.

❒ It can be shown that voltage-mode signaling and current mode signaling are equivalent when generalized associated eigenvectors are used.

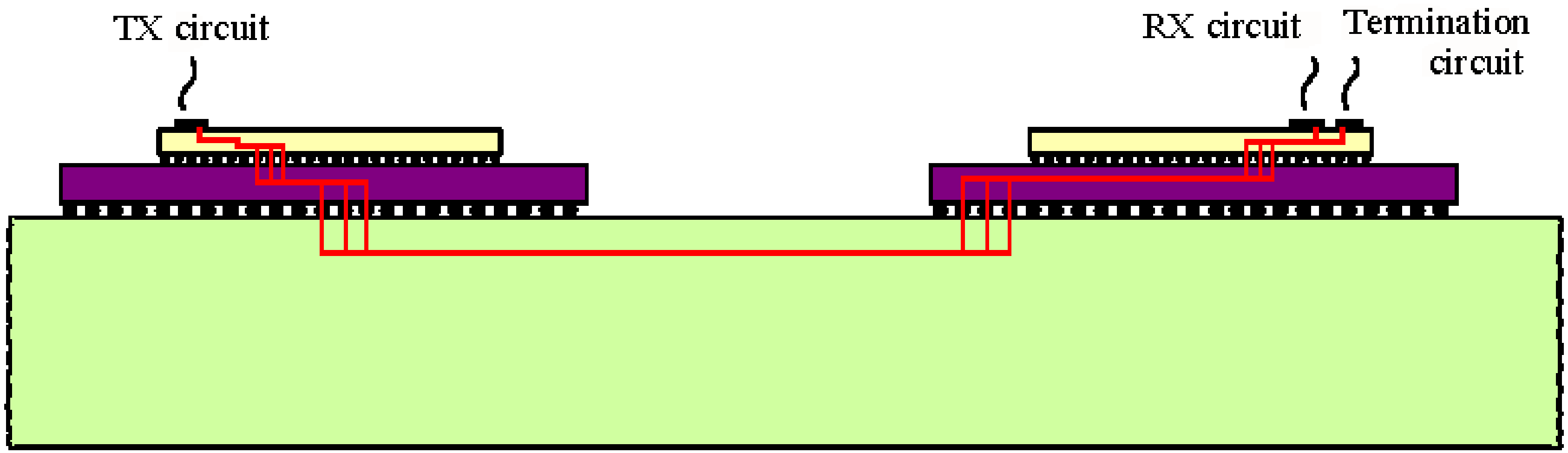
❒ For $n \geq 3$, a termination circuit can often use less than $n\,(n + 1)\,/2$ resistors, for instance only $2\,n - 1$ resistors.



❒ However, the general ZXtalk method is mostly appropriate for small values of n, because the complexity of the TX and RX circuits increases as n^2.

❒ Up to now, our definitions of modal signaling and of the ZXtalk method are based on a uniform MTL model.

❒ It is possible to extend the general ZXtalk method to some interconnections which cannot be modeled with a uniform MTL. This extension is useful when one wishes to consider an interconnection spanning several substrates.



❒ The basic ideas of this extension are the following [84] [109]:

◆ we assume that the interconnection can be modeled as a non-uniform MTL, so that, at each point z we can formally define $\mathbf{S}$, $\mathbf{T}$ and $\mathbf{Z}_C$;

◆ the interconnection is proportioned such that $\mathbf{S}$ and $\mathbf{Z}_C$ are uniform;

◆ in this case, total decoupling can be obtained with a uniform $\mathbf{T}$ given by

$$\mathbf{T} = z_K \, \mathbf{Z}_C^{-1} \, \mathbf{S} \tag{100}$$

where z_K is an arbitrary nonzero impedance, as in (50);

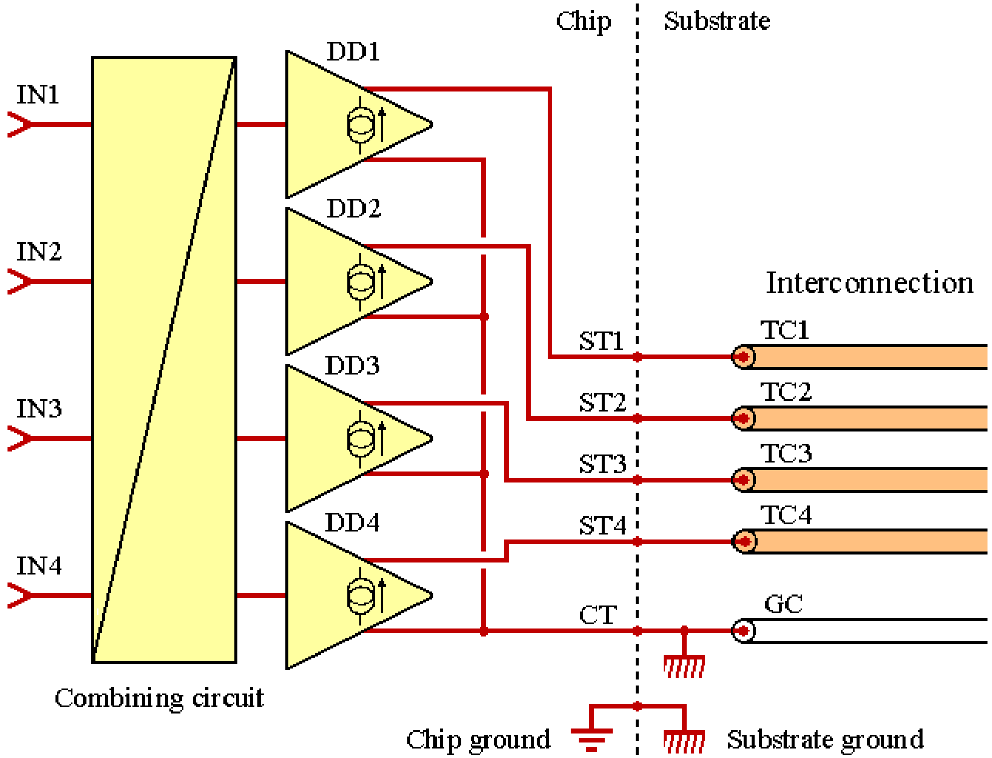
◆ we can then define $\mathbf{v}_M = \mathbf{S}^{-1} \, \mathbf{v}$ and $\mathbf{i}_M = \mathbf{T}^{-1} \, \mathbf{i}$, $\mathbf{v}_M$, and show that they satisfy

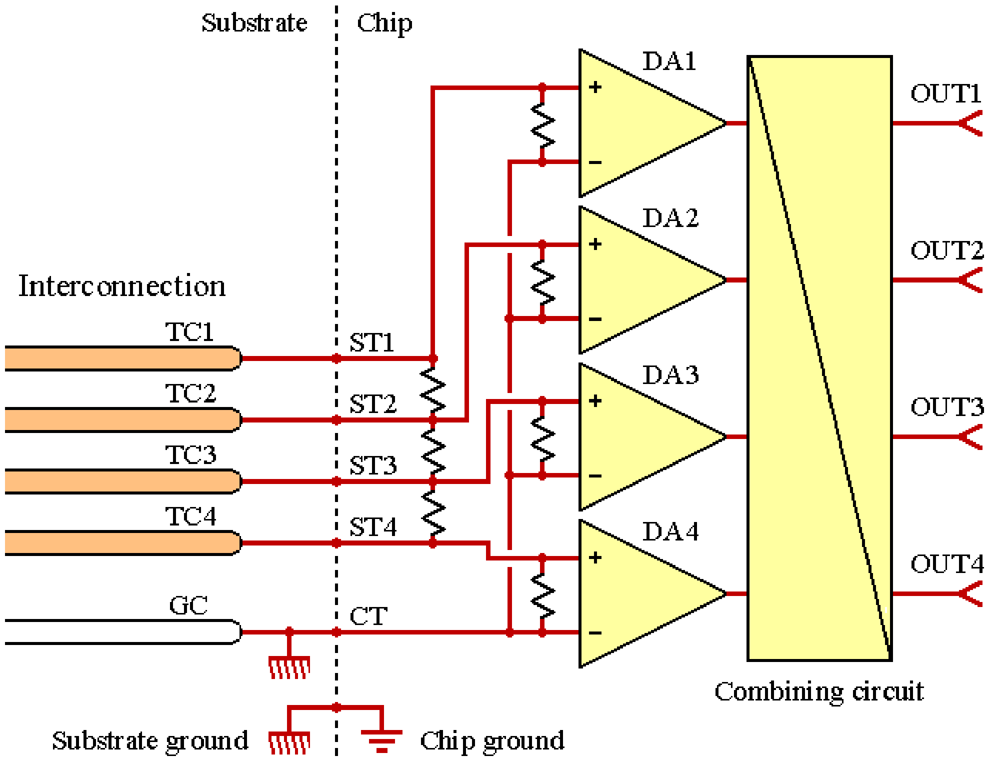
$$\begin{cases} \dfrac{d^2 \, \mathbf{v}_M}{dz^2} - \Gamma^2 \mathbf{v}_M = \dfrac{d\,\Gamma}{dz} \, \Gamma^{-1} \, \dfrac{d\,\mathbf{v}_M}{dz} \\ \dfrac{d^2 \, \mathbf{i}_M}{dz^2} - \Gamma^2 \, \mathbf{i}_M = \dfrac{d\,\Gamma}{dz} \, \Gamma^{-1} \, \dfrac{d\,\mathbf{i}_M}{dz} \end{cases} \tag{101}$$

◆ Γ and $d\,\Gamma/dz$ being diagonal matrices, these equations are decoupled.

❒ Modal signaling does not reduce external crosstalk. The dominant source of external crosstalk is usually common-mode coupling at the near end or at the far end.

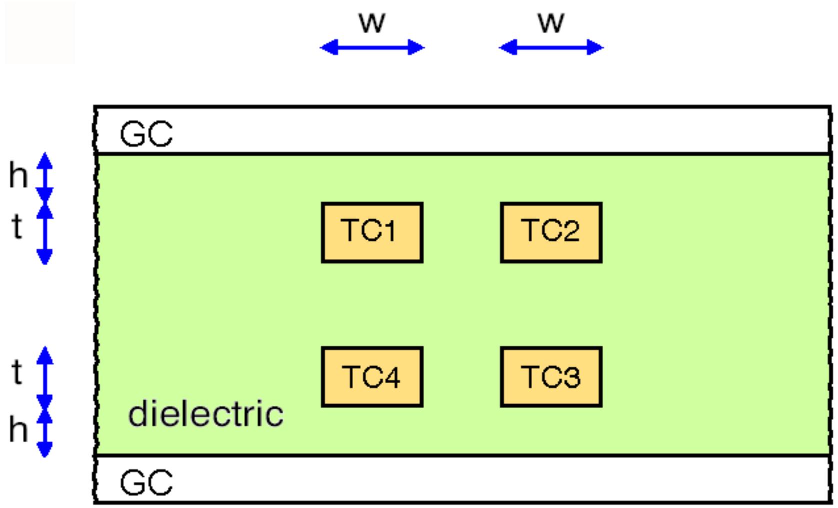
❒ This TX-circuit implementing the ZXtalk method is protected against common-mode coupling at the near end [67] [109].





❒ This RX-circuit and termination circuit implementing the ZXtalk method are protected against common-mode coupling at the far end [65] [109].

❒ Both designs use a CT terminal and substrate ground configuration which minimizes common impedances.

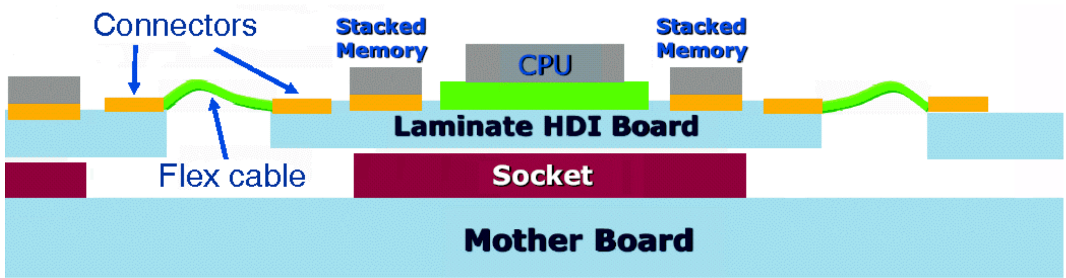


❐ A modal link and the ZXtalk method can easily use a differential pair [42] [44] [56] [59], the 4-TC configuration shown on the left [91] [95], or two differential pairs [56] [59] [97] [102].

❐ A modal link using an interconnection without symmetry (a multiconductor microstrip having 4 TCs) is described in [104].

❐ Two factors playing against the widespread use of modal links other than differential serial links:

- the lack of standardized interface;
- modal links using a large number of TCs are more relevant to configurations without bends causing unequal TC lengths.



❒ There are many types of links for which these factors are immaterial, for instance in the substrate of an MCM, or in flex top-side bridges between MCMs [63] [94] [100].

❒ Modal signaling is a special case of noise subtraction where the signal processing requirements are light because modal transforms are independent (or mildly dependent) of frequency, and independent of the interconnection length [109] [112].

❒ The ZXtalk method is a modal scheme in which these requirements are further reduced, because of the use of a matched (or nearly matched) termination.

❒ Some additional references about modal signaling [3] [12] [13] [17] [23] [40] [41] [43] [48] [62] [74] [87] [89] [93] [103] [105] [106] [107] [110] [111].

15. Modal signaling in a decoupled interconnection

❒ By definition, a completely degenerate interconnection (CDI) is such that $\gamma_1,..., \gamma_n$ may be regarded as equal, in a given frequency band. This for instance occurs when losses are negligible and the propagation medium is homogeneous, but we won't need these assumptions [15].

❒ In the given frequency band, (23) defining **T** and **S** becomes

$$\begin{cases} \mathbf{T}^{-1}\mathbf{Y}'\mathbf{Z}'\mathbf{T} = \gamma^2 \mathbf{1}_n \\ \mathbf{S}^{-1}\mathbf{Z}'\mathbf{Y}'\mathbf{S} = \gamma^2 \mathbf{1}_n \end{cases} \tag{102}$$

where γ is the common value of the propagation constants. In (102) the eigenvalues are completely degenerate. We have :

$$\mathbf{Y}'\mathbf{Z}' = \mathbf{Z}'\mathbf{Y}' = \gamma^2 \mathbf{1}_n \tag{103}$$

Thus, **S** and/or **T** may be chosen equal to $\mathbf{1}_n$, so that

$$\mathbf{Z}_C = \frac{1}{\gamma}\mathbf{Z}' = \gamma\ \mathbf{Y}'^{-1} \tag{104}$$

❒ This choice clearly simplifies the ZXtalk method, since linear combinations are no longer needed in the TX circuits and RX circuits.

❒ A special ZXtalk method for CDI was defined, in which it was required that, in the known frequency band, the propagation constants of the different propagation modes may be considered as substantially equal [33] [49] [80].

❒ However, this definition is too restrictive since the desired effect of being able to choose $\mathbf{S}$ and/or $\mathbf{T}$ equal to $\mathbf{1}_n$ is obtained if and only if $\mathbf{Z}'\mathbf{Y}'$ is diagonal (or, equivalently $\mathbf{Y}'\mathbf{Z}'$ is diagonal).

❒ This circumstance can be referred to as a *decoupled interconnection*.

❒ We therefore propose the following definition of a special ZXtalk method in which it is required that, in the known frequency band, the interconnection may be considered as decoupled.

❐ According to a general definition, the special ZXtalk method relates to an m-channel link such that:

◆ the interconnection has n TCs, with $n \geq m$, and may be modeled, with a sufficient accuracy, in a known frequency band, as a uniform $(n + 1)$-conductor MTL such that the transition matrix from modal electrical variables to natural electrical variables may be considered as substantially equal to $\mathbf{1}_n$;

◆ the interconnection is connected at at least one end to a termination circuit having, in the known frequency band, an impedance matrix near $\mathbf{Z}_C$;

◆ a TX circuit delivers natural electrical variables, each natural electrical variable being mainly determined by one and only one input signal;

◆ a RX circuit delivers output signals, each of the output signals being mainly determined by one and only one of said natural electrical variables.

❐ The special ZXtalk method corresponds to a particular implementation of the general ZXtalk method, in which $\mathbf{S}$ or $\mathbf{T}$ are chosen equal to $\mathbf{1}_n$.

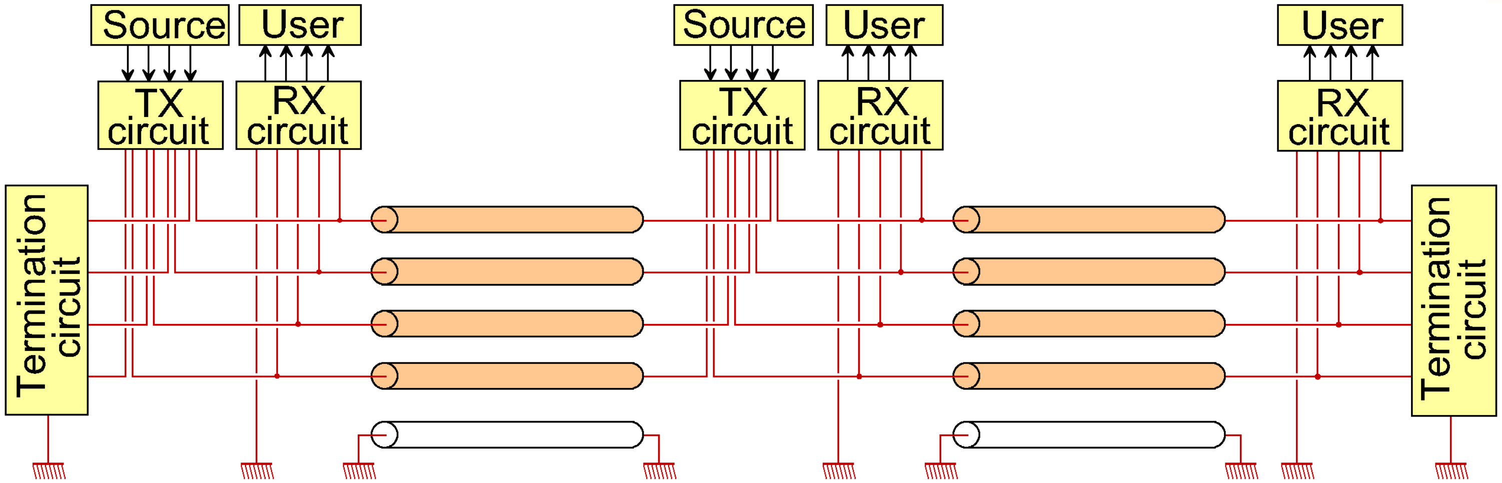
❒ There are 8 possible designs, corresponding to the following equations:

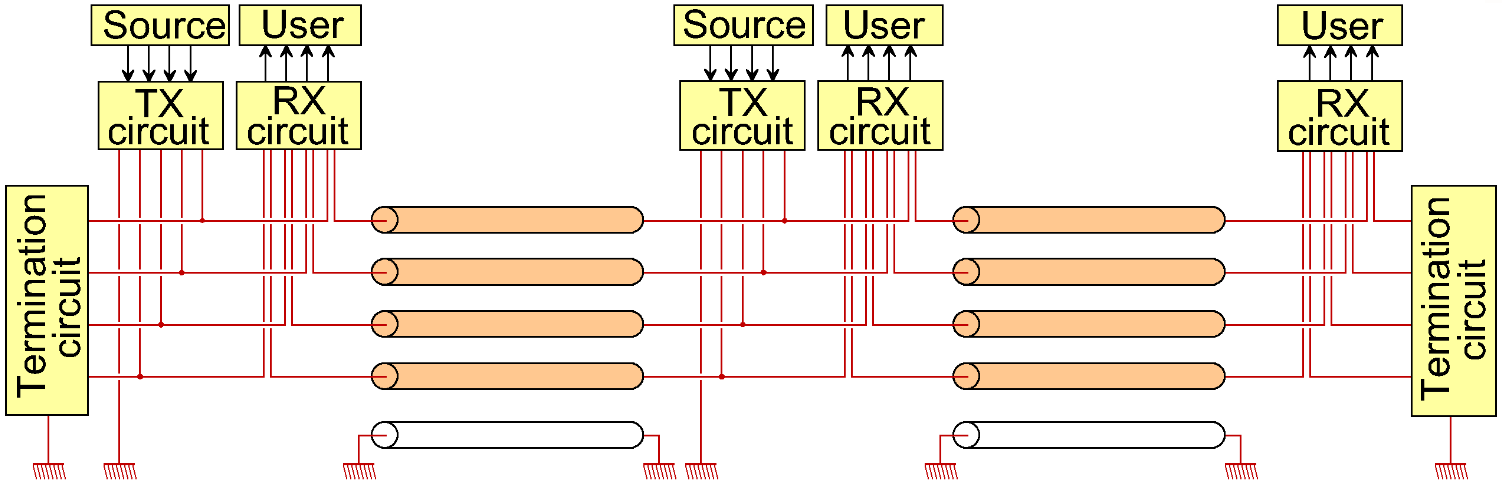
Interface	Connection	Design using modal voltages (voltage-mode signaling)	Design using modal currents (current-mode signaling)
TX circuit	series (low impedance)	$\mathbf{v}_T = \pm a \, \mathrm{diag}_n(\alpha_1, \ldots, \alpha_n) \, \mathbf{x}_I$	$\mathbf{v}_T = a \, \mathbf{Z}_C \, \mathrm{diag}_n(\lambda_1, \ldots, \lambda_n) \, \mathbf{x}_I$
	parallel (high impedance)	$\mathbf{i}_T = a \, \mathbf{Z}_C^{-1} \, \mathrm{diag}_n(\alpha_1, \ldots, \alpha_n) \, \mathbf{x}_I$	$\mathbf{i}_T = \pm a \, \mathrm{diag}_n(\lambda_1, \ldots, \lambda_n) \, \mathbf{x}_I$
RX circuit	series (low impedance)	$\mathbf{x}_O = \pm \, \mathrm{diag}_n(\beta_1, \ldots, \beta_n) \, \mathbf{Z}_C \, \mathbf{i}_R$	$\mathbf{x}_O = \mathrm{diag}_n(\mu_1, \ldots, \mu_n) \, \mathbf{i}_R$
	parallel (high impedance)	$\mathbf{x}_O = \mathrm{diag}_n(\beta_1, \ldots, \beta_n) \, \mathbf{v}_R$	$\mathbf{x}_O = \pm \, \mathrm{diag}_n(\mu_1, \ldots, \mu_n) \mathbf{Z}_C^{-1} \mathbf{v}_R$

(105)

where $\mathbf{x}_I$, $\mathbf{v}_T$ and $\mathbf{i}_T$ are the column vectors of the input signals, output voltages and output currents of the TX circuit, respectively, a is the number of termination circuits; $\mathbf{v}_R$, $\mathbf{i}_R$ and $\mathbf{x}_O$ are the column vectors of the input voltages, input currents and output signals of the RX circuit, respectively; the α_i, λ_i, β_i, and the μ_i are arbitrary nonzero constants.

❒ Two designs do not require linear combinations:

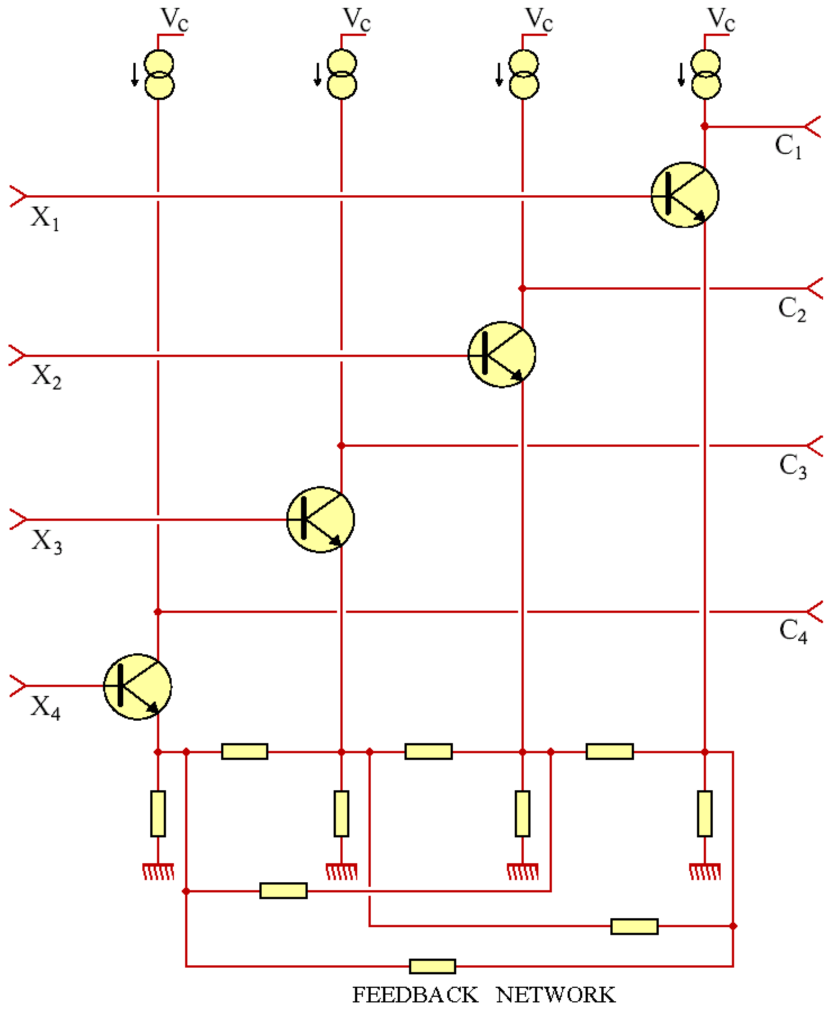


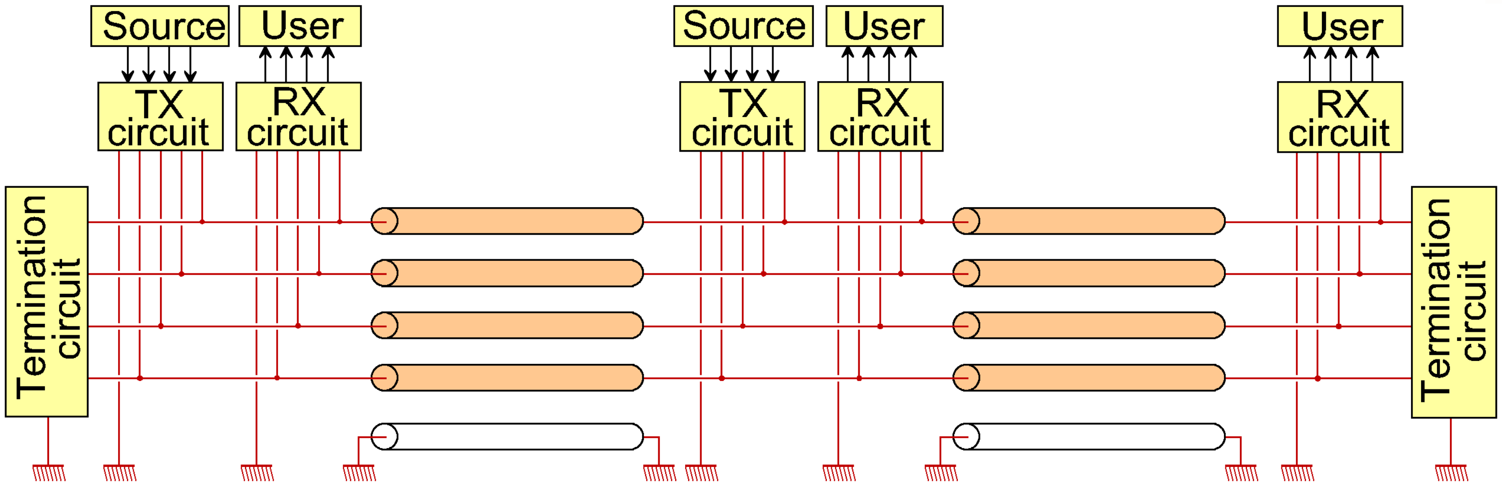


❒ In the designs shown above, the remaining difficulty is the floating TX circuits or RX circuits, connected in series with the interconnection.

❒ This difficulty is circumvented using a MIMO series-series feedback amplifier (MIMO-SSFA) that can be used to perform all linear combinations containing $\mathbf{Z}_C^{-1}$ in (105), in a wide bandwidth.

Structure of a simple 4-channel MIMO-SSFA using bipolar transistors ➜





❒ A design using a MIMO-SSFA in each TX circuit may have the TX circuits and RX circuits connected in parallel with the interconnection.

❒ It is often possible to design such a link so that the number of circuit elements in the termination circuits and TX circuits is proportional to *n* when *n* is large [49].

❒ Thus, this method is appropriate for reducing echo and internal crosstalk in the widest bandwidth, and it is applicable to massively parallel interconnections.

❐ We have seen that the FEXT is small, to the first order in the weak coupling approximation, in a single-ended link using a CDI and pseudo-matched impedances at the ends of each TC. Thus, the special ZXtalk method is mostly of interest in cases where:

- ◆ there is a strong coupling between the TCs; and/or
- ◆ the interconnection is used for bidirectional transmission.

❐ Like the general ZXtalk method, the special ZXtalk method can be extended to some interconnections which cannot be modeled with a uniform MTL.

❐ Suggested additional reading relating or relevant to the special ZXtalk method [37] [38] [39] [42] [43] [46] [48] [57] [60] [83] [84] [91] [96] [98].

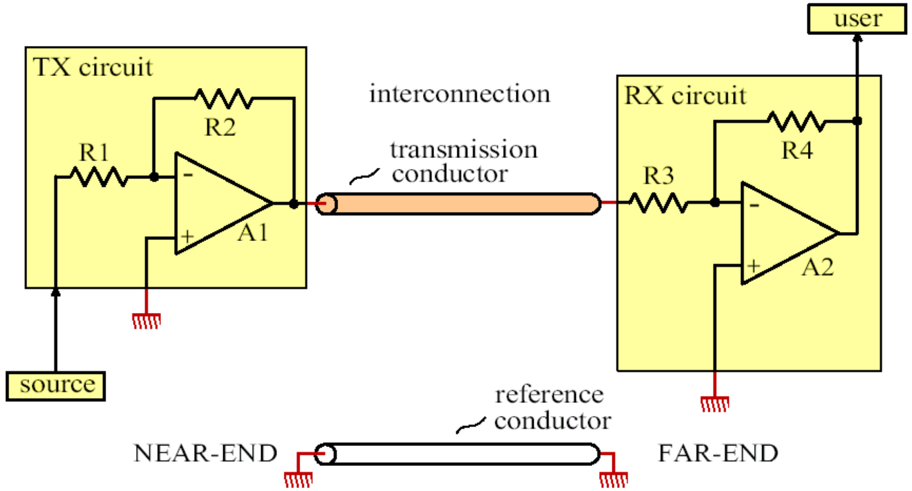
16. Pseudo-differential links

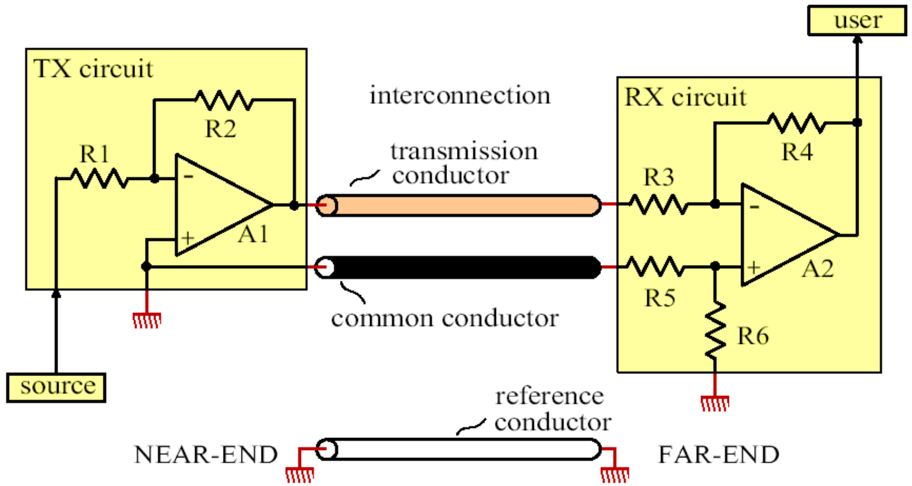
❒ A single-ended link is subject to external crosstalk with other circuits on the same chip, MCM or PCB.

❒ A simple pseudo-differential link (PDL) is protected against crosstalk using little additional hardware.

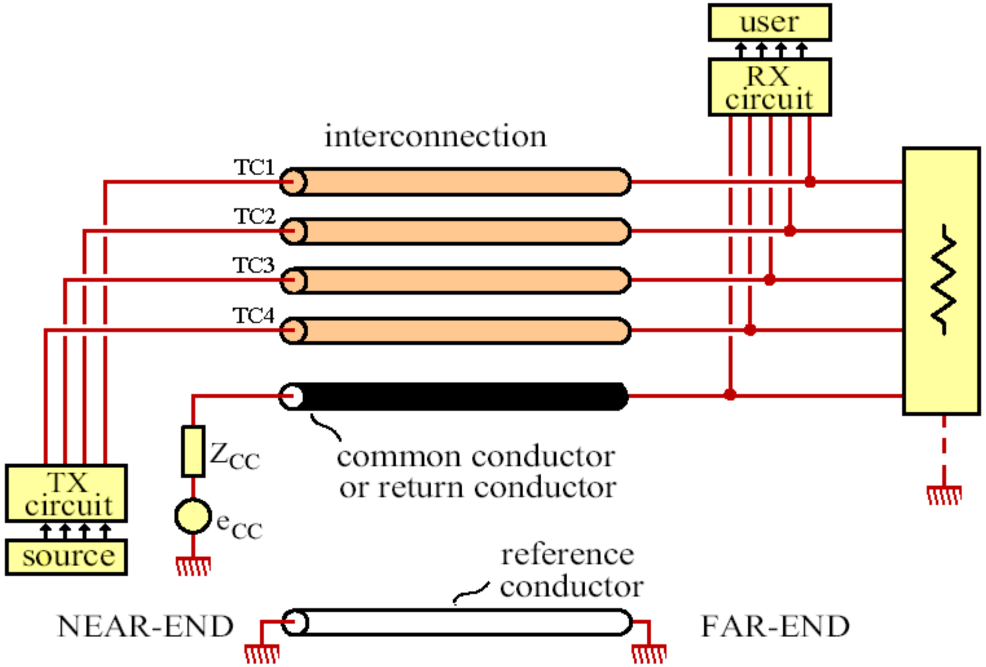
❒ A PDL providing m channels uses only m + 1 conductors to reduce external crosstalk in m channels.

❒ There are four possible PDL architectures.





❒ First architecture: a unidirectional PDL with voltage-driven common conductor (VDCC). The common conductor (CC) may be used as a return conductor (RC).

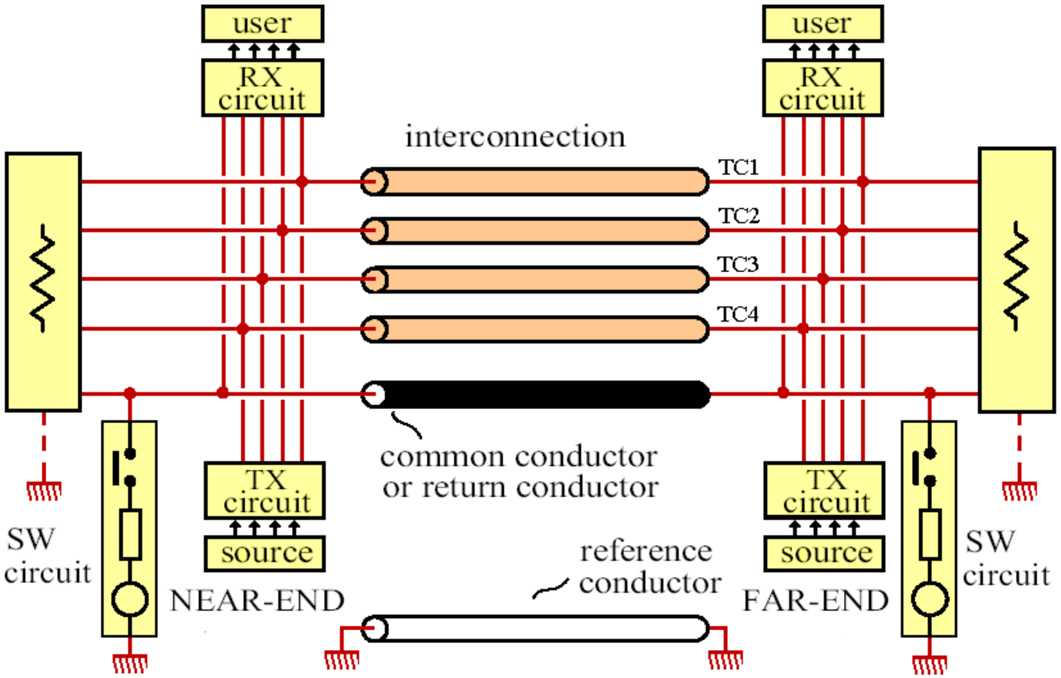


◆ The termination circuit may or may not be present.

◆ The TX circuit may be a conventional line driver (but this is not necessary).

◆ This VDCC architecture cannot be used in a bidirectional PDL.

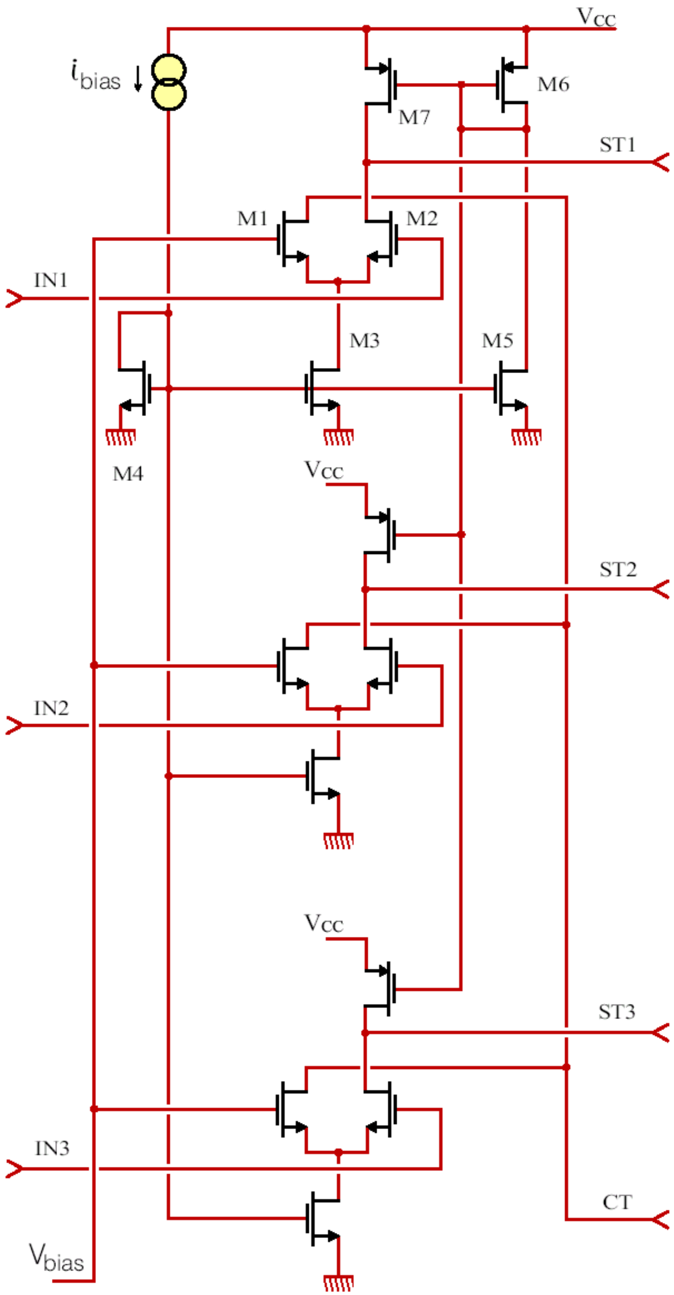
❐ Second architecture: a bidirectional PDL may be built using common terminal switching circuits (SW circuits).



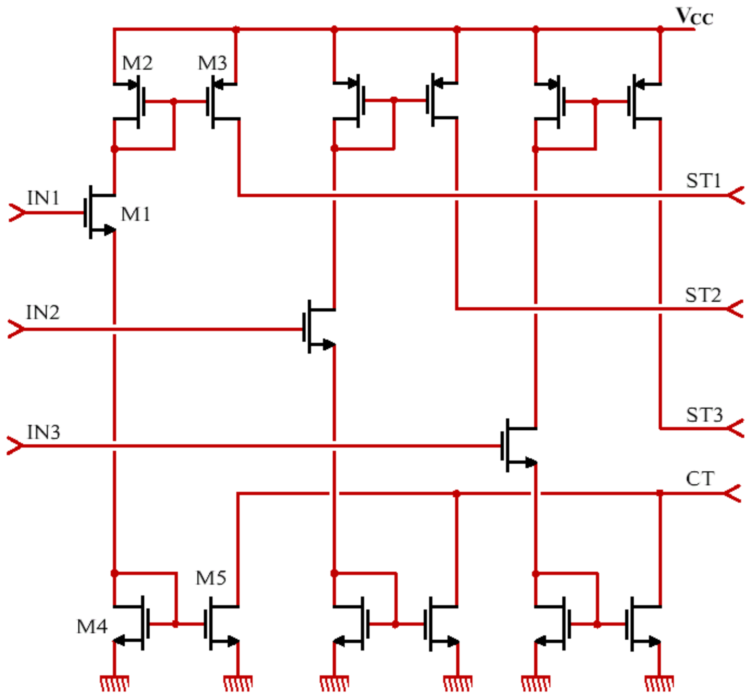
◆ One or more of the termination circuits may or may not be present.

◆ The TX circuits may be conventional 3-state line drivers.

◆ This SW circuit architecture cannot be used for full duplex operation.

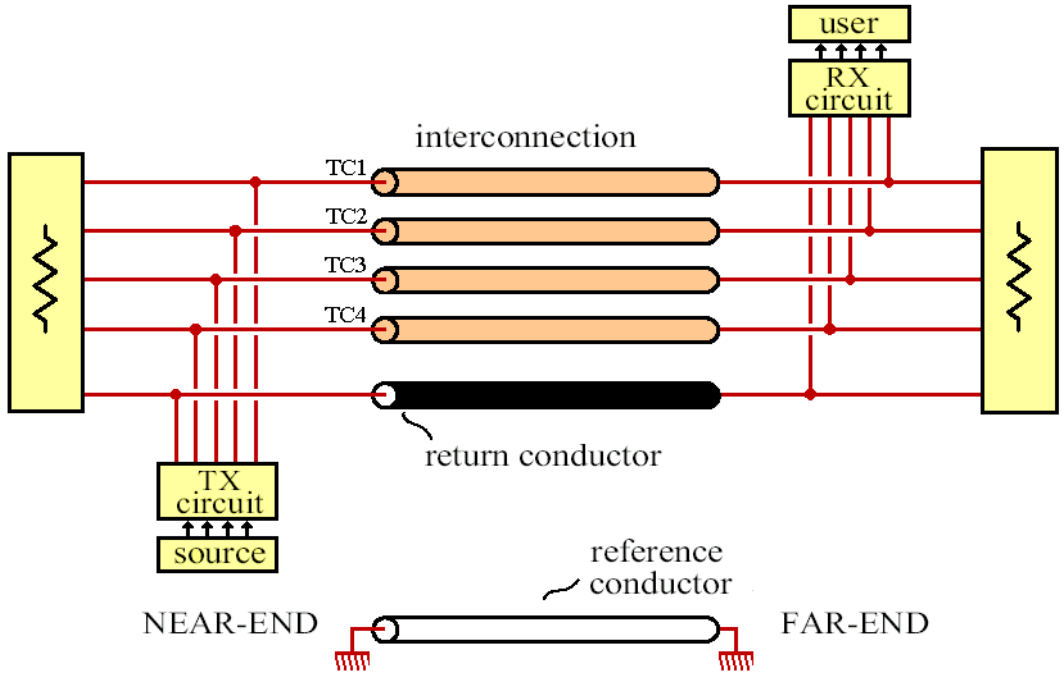


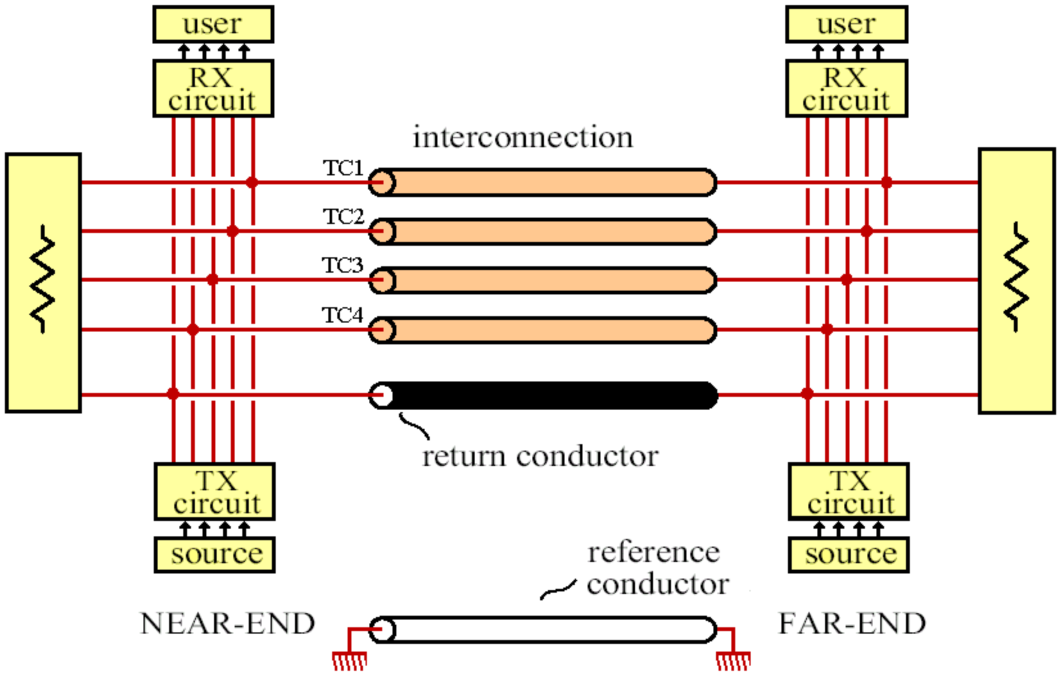
❒ PDLs may also use TX circuits producing a constant (or zero) common-mode current, such as the two 3-channels TX circuits shown here. The ST*n* output terminal is coupled to the TC*n*. The CT output terminal is coupled to the RC.



❒ Let us use i_j to denote the current flowing from ST j to the TC j, and i_C to denote the current flowing from CT to the RC. In the TX-circuits shown above, a balancing circuit controls i_C in such a way that the TX circuit does not cause any significant variation of the common-mode current $i_1 + ... + i_n + i_C$.

❒ Using such TX circuits, we can introduce constant common-mode current (CCMC) architectures





◆ A least one floating termination circuit is needed.

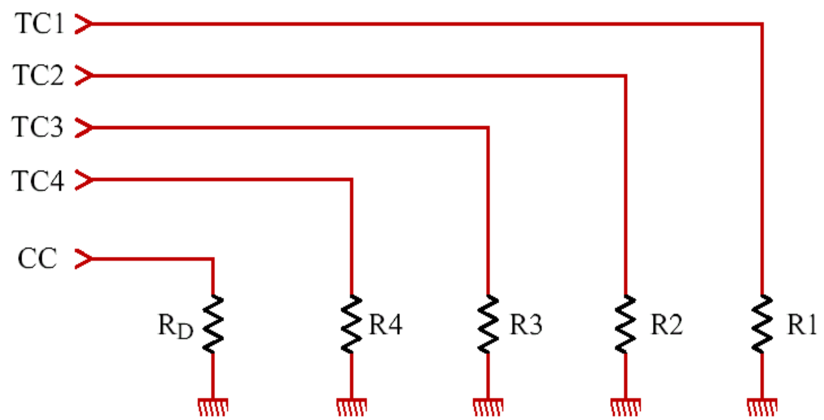
◆ The bidirectional CCMC architecture is compatible with full duplex signaling.

❒ At this stage, we have identified four PDL architectures: PDL with VDCC, PDL using SW circuits, unidirectional PDL operating at CCMC and bidirectional PDL operating at CCMC.

❒ We can define four types of termination circuit.

❒ Type 0: no termination.

❒ Type 1: termination circuit typically made of impedors connected to the GC or to a power supply node.

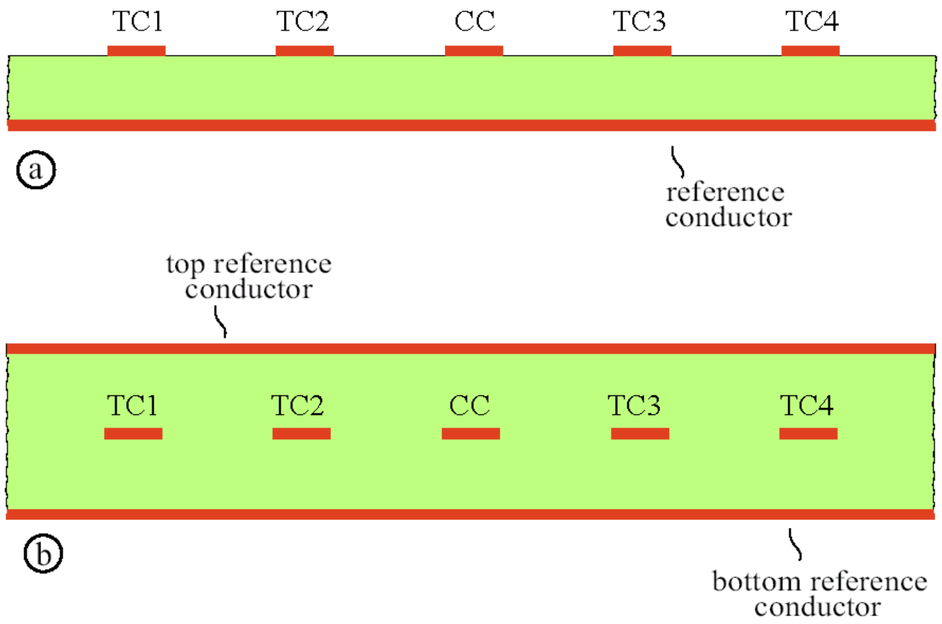


◆ Operates as intended only if the electric and magnetic fields of the signals are mainly located between the TCs and the GC.

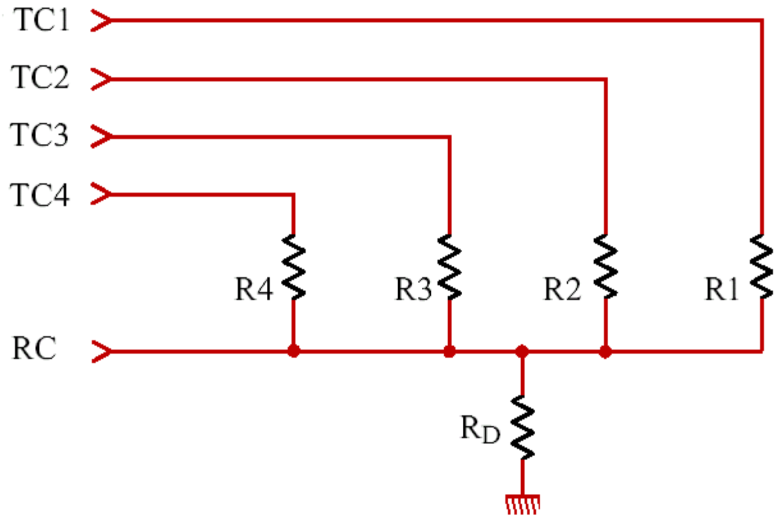
◆ The GC belongs to the signal path: this is a problem.

◆ Not compatible with the CCMC architecture.

❒ Example of 2 interconnection-ground structures compatible with type 1 termination circuits:



❐ Type 2: floating termination circuit made of impedors connected between a TC and the RC.



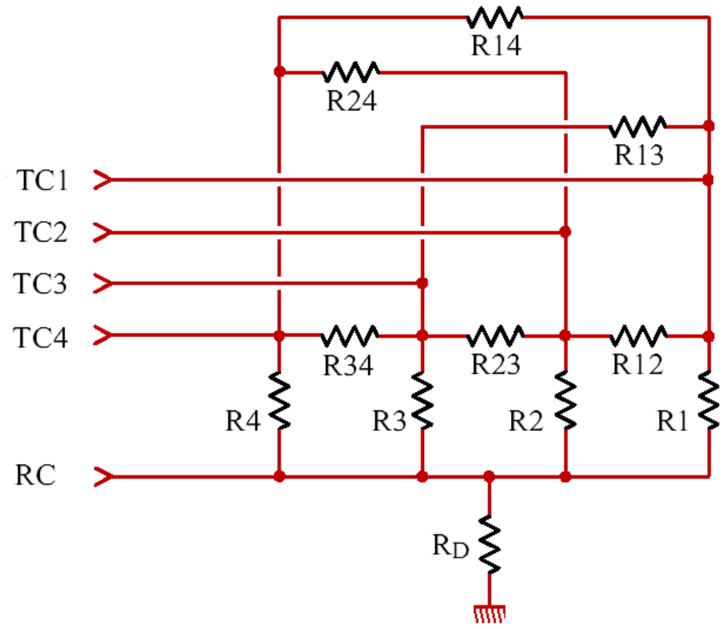
◆ The optional damping resistor R_D is not a part of the termination circuit.

◆ Operates as intended only if the electric and magnetic fields of the signals are mainly located between the TCs and the RC.

◆ Does not degrade the reduction of external crosstalk.

◆ Compatible with the CCMC architecture.

❒ Type 3: floating termination circuit comprising impedors connected between a TC and the RC and impedors connected between two TCs.



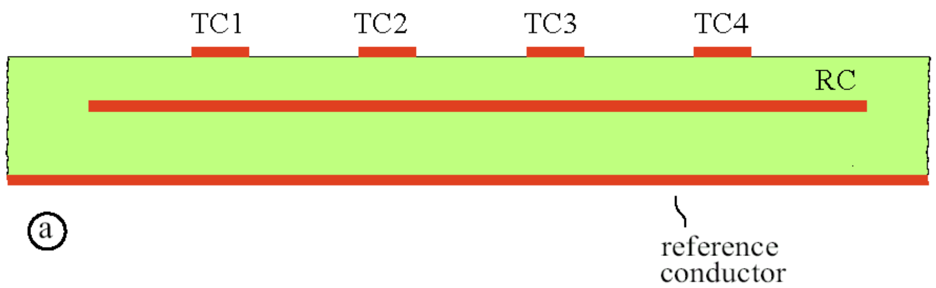
◆ The optional damping resistor R_D is not a part of the termination circuit.

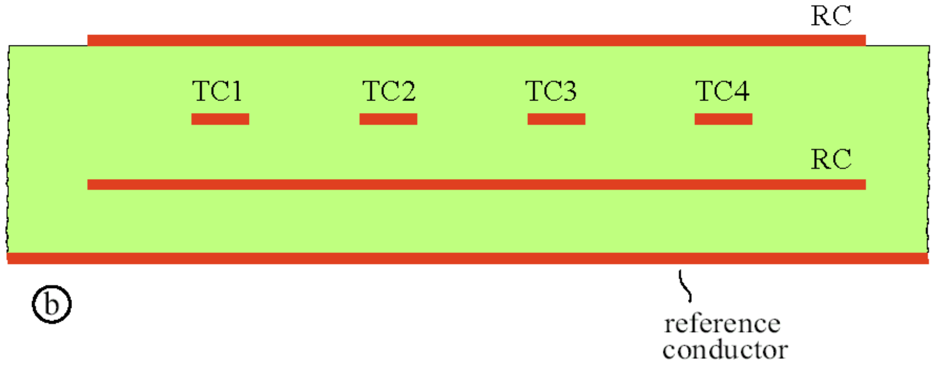
◆ Operates as intended only if the requirement for the second type is met and a variation of the ZXtalk method is used to reduce internal crosstalk.

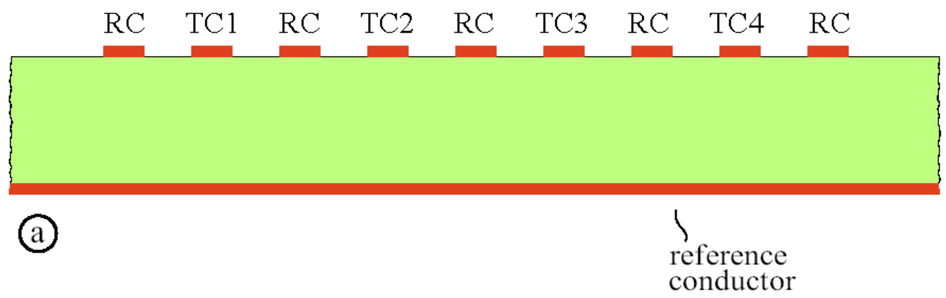
◆ External crosstalk and internal crosstalk can be effectively reduced.

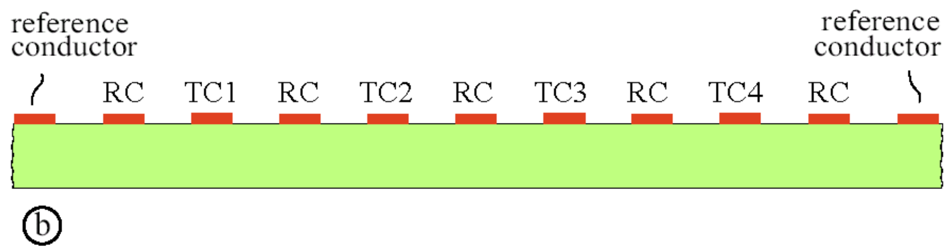
◆ Compatible with the CCMC architecture.

❒ Example of 4 interconnection-ground structures compatible with floating (i.e. type 2 or type 3) termination circuits:









❒ An interconnection comprising n TCs and a CC or RC may be modeled as a $(n + 2)$-conductor MTL, this MTL using natural voltages referenced to ground and natural currents as variables.

❒ For such a model, we use, at a given abscissa z along the interconnection:

- for any integer α such that $1 \leq \alpha \leq n$, the natural current i_α ;
- the current flowing in the RC, denoted by i_{n+1} ;
- for any integer α such that $1 \leq \alpha \leq n$, the voltage between the TC number α and the GC, denoted by $v_{G\,\alpha}$;
- the voltage between the RC and the GC, denoted by $v_{G\,n+1}$.

❒ Later, we will also need:

- the common-mode current $i_{CM} = i_1 + \ldots + i_{n+1}$;
- for any integer α such that $1 \leq \alpha \leq n$, the voltage between the TC number α and the RC, denoted by $v_{R\,\alpha}$ and given by $v_{R\,\alpha} = v_{G\,\alpha} - v_{G\,n+1}$.

❒ For the (n + 2)-conductor MTL model, the telegrapher's equations are:

$$\begin{cases} \dfrac{d\,\mathbf{v}_G}{dz} = -\mathbf{Z}'_G\,\mathbf{i}_G \\ \dfrac{d\,\mathbf{i}_G}{dz} = -\,\mathbf{Y}'_G\,\mathbf{v}_G \end{cases} \tag{106}$$

where

$\mathbf{v}_G$ is the column vector of the natural voltages referenced to ground, the entries of which are $v_{G\,1},\ldots,\,v_{G\,n+1}$;

$\mathbf{i}_G$ is the column vector of the natural currents, the entries of which are $i_{G\,1},\ldots,$ $i_{G\,n+1}$;

$\mathbf{Z}'_G$ is the p.u.l. impedance matrix with respect to ground;

$\mathbf{Y}'_G$ is the p.u.l. admittance matrix with respect to ground.

❒ $\mathbf{Z}'_G$ and $\mathbf{Y}'_G$ are symmetric matrices of size (n + 1) × (n + 1).

❒ The interconnection can also be described by an equivalent set of equations [86]:

$$\begin{cases} \dfrac{d\,\mathbf{v}_R}{dz} = -\mathbf{Z}'_R\,\mathbf{i}_R + i_{MC}\,\mathbf{Z}'_E \\ \dfrac{d\,\mathbf{i}_R}{dz} = -\mathbf{Y}'_R\,\mathbf{v}_R - v_{Gn+1}\,\mathbf{Y}'_E \end{cases} \quad \text{and} \quad \begin{cases} \dfrac{d\,v_{Gn+1}}{dz} = {}^t\mathbf{Z}'_E\,\mathbf{i}_R - i_{MC}\,Z'_{EE} \\ \dfrac{d\,i_{MC}}{dz} = -{}^t\mathbf{Y}'_E\,\mathbf{v}_R - v_{Gn+1}\,Y'_{EE} \end{cases} \tag{107}$$

where

$\mathbf{v}_R$ is the column vector of the natural voltages referenced to the RC, the entries of which are $v_{R\,1},\ldots,\ v_{R\,n}$;
$\mathbf{i}_R$ is the column vector of the natural currents $i_1,\ldots,\ i_n$;
$\mathbf{Z}'_R$ is the p.u.l. impedance matrix with respect to the RC, of size $n \times n$;
$\mathbf{Y}'_R$ is the p.u.l. admittance matrix with respect to the RC, of size $n \times n$;
$\mathbf{Z}'_E$ is the p.u.l. transfer impedance vector, of size $n \times 1$;
$\mathbf{Y}'_E$ is the p.u.l. transfer admittance vector, of size $n \times 1$;
Z'_{EE} is the p.u.l. external circuit impedance; and
Y'_{EE} is the p.u.l. external circuit admittance.

❒ If the RC behaves as a good electromagnetic screen, norms of $\mathbf{Z}'_E$ and $\mathbf{Y}'_E$ are small, so that we may use the following approximation

$$\begin{cases} \dfrac{d\,\mathbf{v}_R}{dz} \approx -\mathbf{Z}'_R \mathbf{i}_R \\ \dfrac{d\,\mathbf{i}_R}{dz} \approx -\mathbf{Y}'_R\,\mathbf{v}_R \end{cases} \tag{108}$$

and

$$\begin{cases} \dfrac{d\,v_{Gn+1}}{dz} \approx -i_{MC}\,Z'_{EE} \\ \dfrac{d\,i_{MC}}{dz} \approx -v_{Gn+1}\,Y'_{EE} \end{cases} \tag{109}$$

❒ By (108), the propagation of signals in the interconnection may be modeled as a propagation in a $(n+1)$-conductor MTL.

❒ The *ZXnoise method* is the combination of a floating termination circuit (type 2 or type 3 termination circuit) and an appropriate interconnection-ground structure [54] [69] [70] [85].

❒ The interconnection is proportioned such that all conductors other than the n TCs and the RC may be neglected when one models propagation in the interconnection, at the design stage.

❒ More precisely, the design stage involves the assumption that the interconnection may be modeled with a sufficient accuracy as a $(n + 1)$-conductor MTL.

❒ This assumption leads to a floating termination circuit.

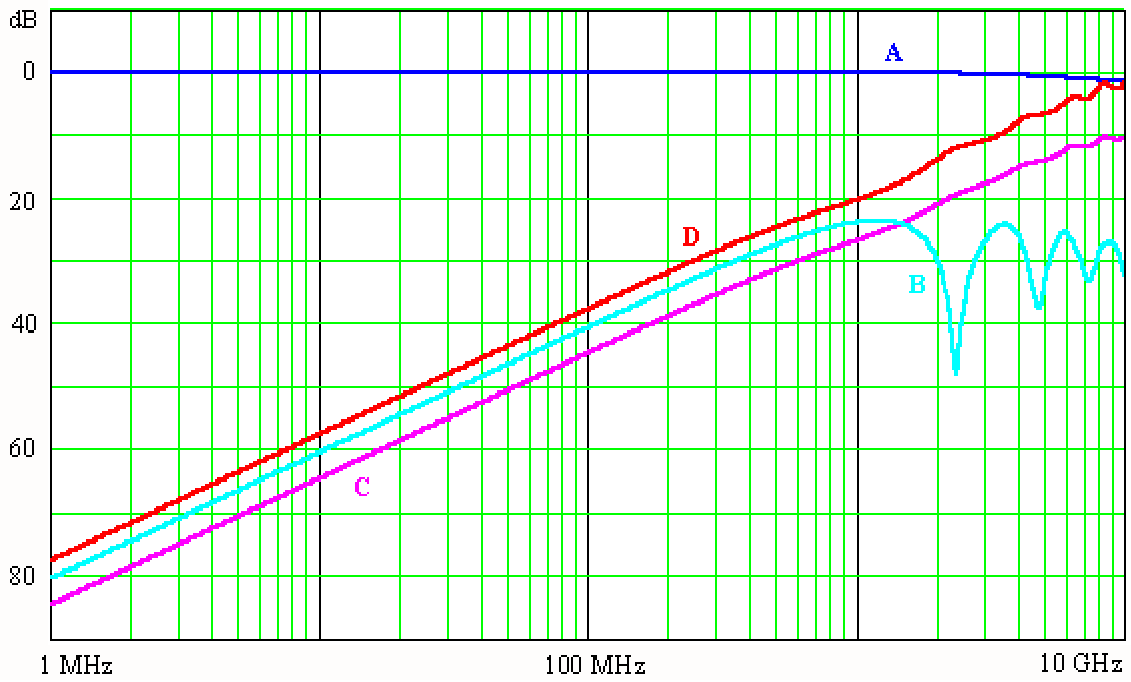
❒ A more accurate analysis of course requires a $(n + 2)$-conductor MTL model.

❒ If a type 3 termination circuit is used, the designer combines the ZXnoise and ZXtalk methods.

❒ Example 1: a compact 4-channel 0.03-m long PDL using single-ended line drivers, a RC grounded at the near end (VDCC) and type 2 termination circuits.

The TCs are very close to each other, so that internal crosstalk is relatively high. However, echo is low and a good rejection of external crosstalk is obtained.

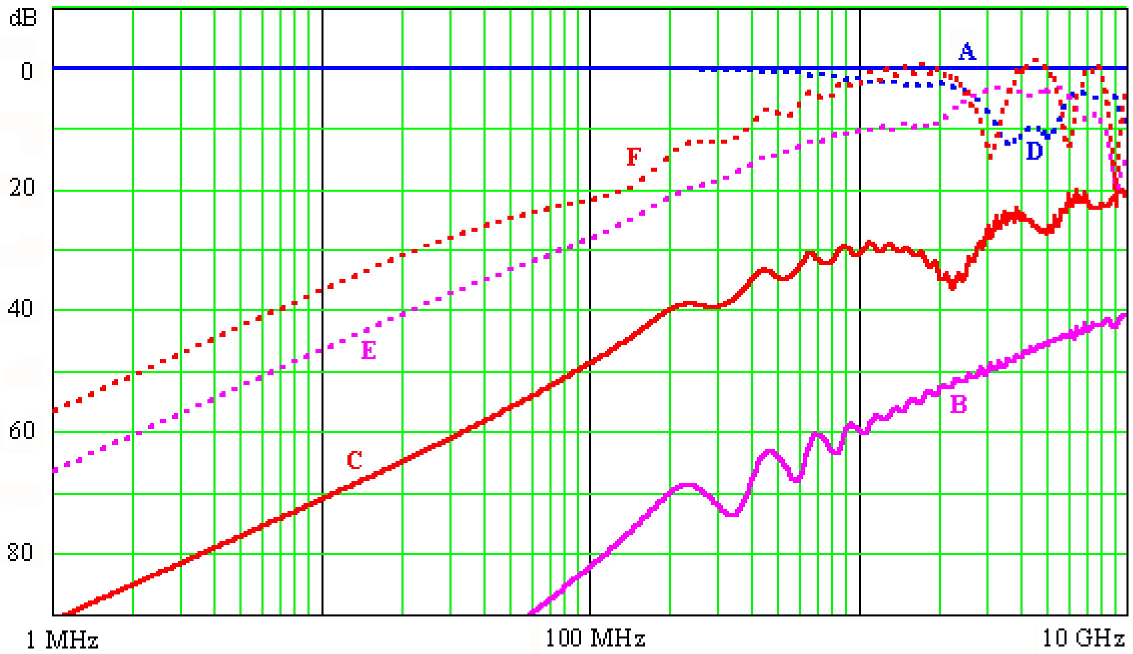
Attenuation of transmitted signal when TC1 is excited: curve A. NEXT loss on TC2 when TC1 is excited: curve B. FEXT loss on TC2 when TC1 is excited: curve C. Far-end external crosstalk loss on TC1 for ground bounce in the TX circuit: curve D. **Note that the far-end external crosstalk loss would be 0 dB for a single-ended link !**



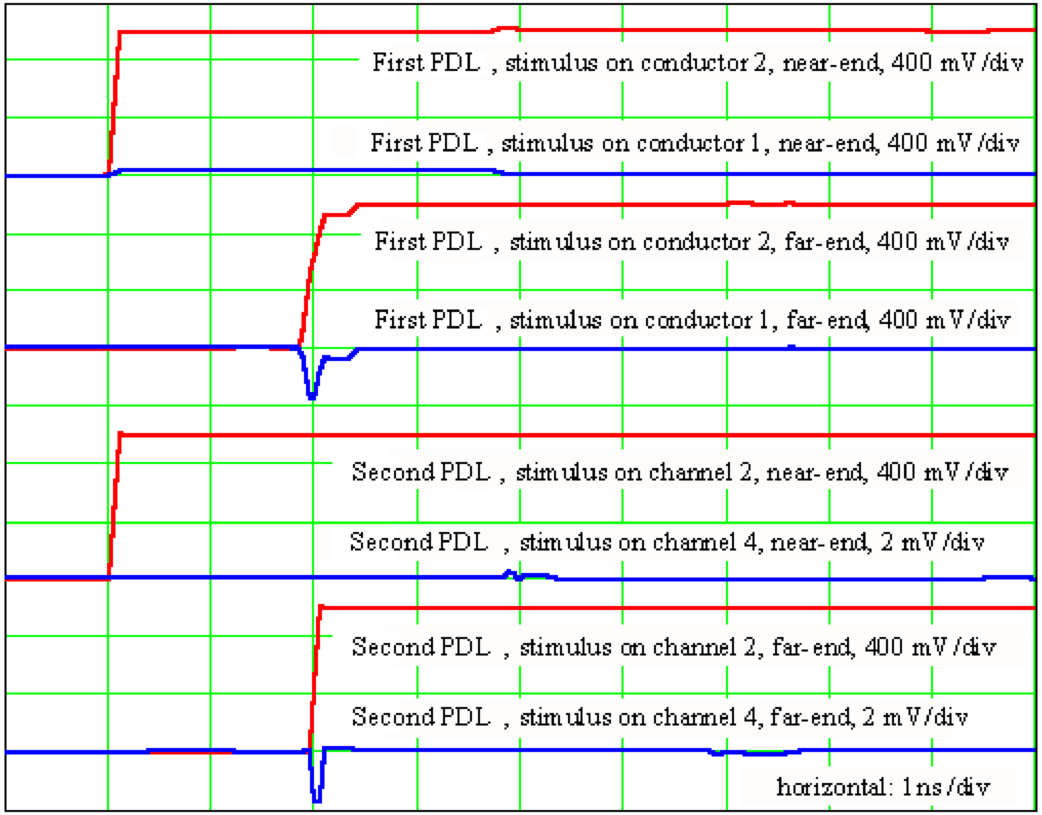
❒ Example 2: Two compact 4-channel 0.3-m long PDLs using the same interconnection-ground structure, single-ended line drivers and VDCC.

The First PDL uses a type 2 termination circuit (ZXnoise method). The Second PLD is a ZXnoise + ZXtalk design (using a type 3 termination circuit).

When a type 3 termination circuit is used, in channel 2: attenuation of transmitted signal (curve A), lowest FEXT loss when channel 3 or 4 are excited (curve B), attenuation of ground bounce (curve C). When a type 2 termination circuit is used, in TC2: attenuation of transmitted signal (curve D), lowest FEXT loss (curve E), attenuation of ground bounce (curve F).



Time domain voltages in the PDL, measured on conductor 2 in the case of a type 2 termination circuit (First PDL) or on channel 2 in the case of a type 3 termination circuit (Second PDL), the stimulus being a 1V step having a 100 ps rise time. In the first PDL, conductor 1 is the one that produces the highest peak crosstalk voltage on conductor 2. In the second PDL, channel 4 is the one that produces the highest peak crosstalk voltage on channel 2.



❐ From the presentation of pseudo-differential signaling, we see that:

◆ pseudo-differential signaling effectively reduces external crosstalk using $m + 1$ conductors instead of $2m$ conductors for m differential links;

◆ the ZXnoise method effectively reduces external crosstalk and echo;

◆ the ZXnoise + ZXtalk combination also effectively reduces internal crosstalk.

❐ There are 12 pseudo-differential transmission schemes corresponding to the compatible combinations of an architecture and a type of termination circuit.

Termination circuit	Architecture of the PDL			
	VDCC (unidirectional)	SW circuit (bidirectional)	Unidirectional CCMC	Bidirectional CCMC
Type 0	Prior Art	New		
Type 1	Prior Art	New		
Type 2 (ZXnoise)	New	New	New	New
Type 3 (ZXnoise)	New	New	New	New

❒ Several authors have introduced other multichannel transmission schemes, in which one or more of the output signals of the RX circuit are mainly determined by the voltages between two TCs, sometimes with the addition of a code maintaining a constant current [29] [50] [75] [76] [79] [81].

❒ Even though some such transmission schemes are sometimes referred to as pseudo-differential, they have not been considered in this presentation.

❒ Suggested additional reading on pseudo-differential signaling [22] [27] [28] [30] [36] [45] [47] [51] [52] [53] [61] [64] [65] [66] [67] [68] [71] [72] [73] [77] [78] [80] [82] [90] [92].

Appendix:
Inventions of Excem quoted in this tutorial

❒ Cross-reference:

Internal No.	Reference	Internal No.	Reference	Internal No.	Reference	Internal No.	Reference
P26	[31]	P36	[52]	P41	[66]	P46	[78]
P27	[32]	P37	[53]	P42	[67]	P47	[84]
P28	[33]	P38	[54]	P43	[69]	P48	[85]
P30	[46]	P39	[64]	P44	[70]	P49	[90]
P35	[51]	P40	[65]	P45	[77]		

❒ Inventions on the ZXtalk method (essential inventions in red):

ZXtalk method	Link	
	Not pseudo-differential	Pseudo-differential (ZXnoise + ZXtalk)
General	P26, P27, P47, P39, P40, P42	P39, P40, P43, P48, P42, P45, P46
Special	P28, P47 P30, P36, P39, P40, P41, P42	P39, P40, P44, P48 P30, P36, P41, P42, P45, P46

❒ Inventions on pseudo-differential signaling (essential inventions in red):

Termination circuit	Architecture of the PDL			
	VDCC (unidirectional)	SW circuit (bidirectional)	Unidirectional CCMC	Bidirectional CCMC
Type 0	Prior Art P39, P46, P49	P37 P39, P46, P49		
Type 1	Prior Art P39, P46, P49	P37 P39, P46, P49		
Type 2 (ZXnoise)	P35, P38, P48 P39, P41, P46, P49	P35, P37, P38, P48 P39, P41, P46, P49	P35, P36, P38, P48 P39, P41, P42, P46, P49	
Type 3 (ZXnoise+ZXtalk)	P39, P40, P43, P44, P48 P30, P41, P46, P49	P39, P40, P43, P44, P45 P48, P30, P41, P46, P49	P36, P39, P40, P42, P43, P44, P48 P30, P41, P46, P49	

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Annexes

Annex A: Computation of the eigenvectors and the characteristic impedance matrix

Authors: Frédéric Broydé and Evelyne Clavelier.
Prepared with Mathcad 2000 professional (Mathcad is a registered trademark of its owner).

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File: Worksheet A of Tutorial v3a.mcd

1) DEFINITION OF THE MULTICONDUCTOR TRANSMISSION LINE PARAMETERS

$\mathrm{ORIGIN} := 1 \qquad n := 8$

$$L0 := \begin{pmatrix} 531.38820 & 302.97719 & 196.55629 & 135.03626 & 96.48416 & 71.25132 & 54.28431 & 42.80081 \\ 302.97719 & 516.34491 & 294.94966 & 191.73007 & 132.03762 & 94.67304 & 70.33634 & 54.28431 \\ 196.55629 & 294.94966 & 512.06563 & 292.43477 & 190.27838 & 131.34951 & 94.67304 & 71.25132 \\ 135.03626 & 191.73007 & 292.43477 & 510.69139 & 291.81073 & 190.27838 & 132.03762 & 96.48416 \\ 96.48416 & 132.03762 & 190.27838 & 291.81073 & 510.69139 & 292.43477 & 191.73007 & 135.03626 \\ 71.25132 & 94.67304 & 131.34951 & 190.27838 & 292.43477 & 512.06563 & 294.94966 & 196.55629 \\ 54.28431 & 70.33634 & 94.67304 & 132.03762 & 191.73007 & 294.94966 & 516.34491 & 302.97719 \\ 42.80081 & 54.28431 & 71.25132 & 96.48416 & 135.03626 & 196.55629 & 302.97719 & 531.38820 \end{pmatrix} \cdot 10^{-9}$$

H/m

$\mathrm{norme}\left(L0 - L0^T\right) = 0$

$$C := \begin{pmatrix} 51.180621 & -23.614594 & -2.461115 & -0.774112 & -0.348395 & -0.203589 & -0.144816 & -0.167753 \\ -23.614594 & 63.054502 & -22.474417 & -2.105959 & -0.615859 & -0.258450 & -0.146408 & -0.144816 \\ -2.461115 & -22.474417 & 63.174880 & -22.436402 & -2.089464 & -0.607986 & -0.258450 & -0.203589 \\ -0.774112 & -2.105959 & -22.436402 & 63.186687 & -22.432098 & -2.089464 & -0.615859 & -0.348395 \\ -0.348395 & -0.615859 & -2.089464 & -22.432098 & 63.186687 & -22.436402 & -2.105959 & -0.774112 \\ -0.203589 & -0.258450 & -0.607986 & -2.089464 & -22.436402 & 63.174880 & -22.474417 & -2.461115 \\ -0.144816 & -0.146408 & -0.258450 & -0.615859 & -2.105959 & -22.474417 & 63.054502 & -23.614594 \\ -0.167753 & -0.144816 & -0.203589 & -0.348395 & -0.774112 & -2.461115 & -23.614594 & 51.180621 \end{pmatrix} \cdot 10^{-12}$$

F/m

$\mathrm{norme}\left(C - C^T\right) = 0$

2) DETERMINATION OF ASSOCIATED EIGENVECTORS

$i := 1..n \qquad j := 1..n$

$D1 := \mathrm{eigenvals}\left(10^{18}\cdot C\cdot L0\right) \qquad D2 := \mathrm{eigenvals}\left(10^{18}\cdot L0\cdot C\right)$

vectors of the eigenvalues

$$D1 = \begin{pmatrix} 25.35799 \\ 20.63546 \\ 18.54192 \\ 17.51615 \\ 16.12124 \\ 16.52110 \\ 16.91569 \\ 16.26602 \end{pmatrix} \qquad D2 = \begin{pmatrix} 25.35799 \\ 20.63546 \\ 18.54192 \\ 17.51615 \\ 16.12124 \\ 16.91569 \\ 16.52110 \\ 16.26602 \end{pmatrix}$$

BEWARE ! the order of the eigenvalues is NOT identical in D1 and D2, we choose the order of D1

Propagation velocity of the eigenmodes

$$ci_i := \frac{1}{\sqrt{10^{-18}\cdot D1_i}}$$

$$ci = \begin{pmatrix} 1.986\times 10^{8} \\ 2.201\times 10^{8} \\ 2.322\times 10^{8} \\ 2.389\times 10^{8} \\ 2.491\times 10^{8} \\ 2.460\times 10^{8} \\ 2.431\times 10^{8} \\ 2.479\times 10^{8} \end{pmatrix}$$ m/s

$Munsurci_{i,j} := 0 \qquad Munsurci_{i,i} := \frac{1}{ci_i}$

Definition of the change of variables for the currents, i.e. transition matrix from modal currents to natural currents (matrix T)

$$T_{i,j} := \left(\mathrm{eigenvec}\left(10^{18}\cdot C\cdot L0, D1_j\right)\right)_i$$

$$T = \begin{pmatrix} 0.41534 & -0.54029 & -0.50948 & -0.44297 & 0.10052 & 0.28765 & -0.36808 & -0.19704 \\ 0.32623 & -0.37448 & -0.18359 & 0.09610 & -0.27671 & -0.48644 & 0.35047 & 0.45949 \\ 0.32954 & -0.24578 & 0.17435 & 0.47288 & 0.41565 & 0.10000 & 0.34421 & -0.46202 \\ 0.33537 & -0.08629 & 0.41992 & 0.26632 & -0.49047 & 0.41308 & -0.35102 & 0.19128 \\ 0.33537 & 0.08629 & 0.41992 & -0.26632 & 0.49047 & -0.41308 & -0.35102 & 0.19128 \\ 0.32954 & 0.24578 & 0.17435 & -0.47288 & -0.41565 & -0.10000 & 0.34421 & -0.46202 \\ 0.32623 & 0.37448 & -0.18359 & -0.09610 & 0.27671 & 0.48644 & 0.35047 & 0.45949 \\ 0.41534 & 0.54029 & -0.50948 & 0.44297 & -0.10052 & -0.28765 & -0.36808 & -0.19704 \end{pmatrix}$$

Definition of the change of variables for the voltages, i.e. transition matrix from modal voltages to natural voltages (matrix S)

$ck := 10^{-10}$ arbitrary constant

$S := ck \cdot C^{-1} \cdot T$

$$S = \begin{pmatrix} 2.054301 & -1.895924 & -1.155304 & -0.702251 & 0.083579 & 0.290674 & -0.444683 & -0.174771 \\ 2.274451 & -1.724031 & -0.462953 & 0.247219 & -0.275695 & -0.609739 & 0.559119 & 0.498559 \\ 2.414446 & -1.187700 & 0.576552 & 0.976135 & 0.405178 & 0.114345 & 0.532082 & -0.483689 \\ 2.484387 & -0.422463 & 1.314582 & 0.543835 & -0.480948 & 0.500468 & -0.515995 & 0.206748 \\ 2.484387 & 0.422463 & 1.314582 & -0.543835 & 0.480948 & -0.500468 & -0.515995 & 0.206748 \\ 2.414446 & 1.187700 & 0.576552 & -0.976135 & -0.405178 & -0.114345 & 0.532082 & -0.483689 \\ 2.274451 & 1.724031 & -0.462953 & -0.247219 & 0.275695 & 0.609739 & 0.559119 & 0.498559 \\ 2.054301 & 1.895924 & -1.155304 & 0.702251 & -0.083579 & -0.290674 & -0.444683 & -0.174771 \end{pmatrix}$$

3) CHARACTERISTIC IMPEDANCE MATRIX

modal characteristic impedances
(for the associated eigenvectors)

$$zC_i := \frac{1}{ck \cdot ci_i}$$

$$zC = \begin{pmatrix} 50.357 \\ 45.426 \\ 43.060 \\ 41.852 \\ 40.151 \\ 40.646 \\ 41.129 \\ 40.331 \end{pmatrix} \quad \Omega$$

modal characteristic impedance matrix

$Zmc := \mathrm{diag}(zC)$

characteristic impedance matrix

$Zc := S \cdot Zmc \cdot T^{-1}$

$$Zc = \begin{pmatrix} 116.390 & 61.099 & 37.310 & 24.238 & 16.431 & 11.564 & 8.446 & 6.438 \\ 61.099 & 113.169 & 59.453 & 36.347 & 23.653 & 16.084 & 11.390 & 8.446 \\ 37.310 & 59.453 & 112.326 & 58.969 & 36.072 & 23.525 & 16.084 & 11.564 \\ 24.238 & 36.347 & 58.969 & 112.066 & 58.852 & 36.072 & 23.653 & 16.431 \\ 16.431 & 23.653 & 36.072 & 58.852 & 112.066 & 58.969 & 36.347 & 24.238 \\ 11.564 & 16.084 & 23.525 & 36.072 & 58.969 & 112.326 & 59.453 & 37.310 \\ 8.446 & 11.390 & 16.084 & 23.653 & 36.347 & 59.453 & 113.169 & 61.099 \\ 6.438 & 8.446 & 11.564 & 16.431 & 24.238 & 37.310 & 61.099 & 116.390 \end{pmatrix} \quad \Omega$$

control of the equations for computing the characteristic impedance matrix:

$$S \cdot Munsurci^{-1} \cdot S^{-1} \cdot L0 = \begin{pmatrix} 116.390 & 61.099 & 37.310 & 24.238 & 16.431 & 11.564 & 8.446 & 6.438 \\ 61.099 & 113.169 & 59.453 & 36.347 & 23.653 & 16.084 & 11.390 & 8.446 \\ 37.310 & 59.453 & 112.326 & 58.969 & 36.072 & 23.525 & 16.084 & 11.564 \\ 24.238 & 36.347 & 58.969 & 112.066 & 58.852 & 36.072 & 23.653 & 16.431 \\ 16.431 & 23.653 & 36.072 & 58.852 & 112.066 & 58.969 & 36.347 & 24.238 \\ 11.564 & 16.084 & 23.525 & 36.072 & 58.969 & 112.326 & 59.453 & 37.310 \\ 8.446 & 11.390 & 16.084 & 23.653 & 36.347 & 59.453 & 113.169 & 61.099 \\ 6.438 & 8.446 & 11.564 & 16.431 & 24.238 & 37.310 & 61.099 & 116.390 \end{pmatrix}$$

$$S \cdot Munsurci \cdot S^{-1} \cdot C^{-1} = \begin{pmatrix} 116.390 & 61.099 & 37.310 & 24.238 & 16.431 & 11.564 & 8.446 & 6.438 \\ 61.099 & 113.169 & 59.453 & 36.347 & 23.653 & 16.084 & 11.390 & 8.446 \\ 37.310 & 59.453 & 112.326 & 58.969 & 36.072 & 23.525 & 16.084 & 11.564 \\ 24.238 & 36.347 & 58.969 & 112.066 & 58.852 & 36.072 & 23.653 & 16.431 \\ 16.431 & 23.653 & 36.072 & 58.852 & 112.066 & 58.969 & 36.347 & 24.238 \\ 11.564 & 16.084 & 23.525 & 36.072 & 58.969 & 112.326 & 59.453 & 37.310 \\ 8.446 & 11.390 & 16.084 & 23.653 & 36.347 & 59.453 & 113.169 & 61.099 \\ 6.438 & 8.446 & 11.564 & 16.431 & 24.238 & 37.310 & 61.099 & 116.390 \end{pmatrix}$$

$$C^{-1} \cdot T \cdot Munsurci \cdot T^{-1} = \begin{pmatrix} 116.390 & 61.099 & 37.310 & 24.238 & 16.431 & 11.564 & 8.446 & 6.438 \\ 61.099 & 113.169 & 59.453 & 36.347 & 23.653 & 16.084 & 11.390 & 8.446 \\ 37.310 & 59.453 & 112.326 & 58.969 & 36.072 & 23.525 & 16.084 & 11.564 \\ 24.238 & 36.347 & 58.969 & 112.066 & 58.852 & 36.072 & 23.653 & 16.431 \\ 16.431 & 23.653 & 36.072 & 58.852 & 112.066 & 58.969 & 36.347 & 24.238 \\ 11.564 & 16.084 & 23.525 & 36.072 & 58.969 & 112.326 & 59.453 & 37.310 \\ 8.446 & 11.390 & 16.084 & 23.653 & 36.347 & 59.453 & 113.169 & 61.099 \\ 6.438 & 8.446 & 11.564 & 16.431 & 24.238 & 37.310 & 61.099 & 116.390 \end{pmatrix}$$

$$L0 \cdot T \cdot Munsurci^{-1} \cdot T^{-1} = \begin{pmatrix} 116.390 & 61.099 & 37.310 & 24.238 & 16.431 & 11.564 & 8.446 & 6.438 \\ 61.099 & 113.169 & 59.453 & 36.347 & 23.653 & 16.084 & 11.390 & 8.446 \\ 37.310 & 59.453 & 112.326 & 58.969 & 36.072 & 23.525 & 16.084 & 11.564 \\ 24.238 & 36.347 & 58.969 & 112.066 & 58.852 & 36.072 & 23.653 & 16.431 \\ 16.431 & 23.653 & 36.072 & 58.852 & 112.066 & 58.969 & 36.347 & 24.238 \\ 11.564 & 16.084 & 23.525 & 36.072 & 58.969 & 112.326 & 59.453 & 37.310 \\ 8.446 & 11.390 & 16.084 & 23.653 & 36.347 & 59.453 & 113.169 & 61.099 \\ 6.438 & 8.446 & 11.564 & 16.431 & 24.238 & 37.310 & 61.099 & 116.390 \end{pmatrix}$$

4) MATCHED LOADS

$$Zc^{-1} = \begin{pmatrix} 0.012 & -6.073\times10^{-3} & -6.123\times10^{-4} & -2.154\times10^{-4} & -1.085\times10^{-4} & -6.856\times10^{-5} & -5.111\times10^{-5} & -6.325\times10^{-5} \\ -6.073\times10^{-3} & 0.015 & -5.767\times10^{-3} & -5.064\times10^{-4} & -1.629\times10^{-4} & -7.643\times10^{-5} & -4.724\times10^{-5} & -5.111\times10^{-5} \\ -6.123\times10^{-4} & -5.767\times10^{-3} & 0.015 & -5.756\times10^{-3} & -5.011\times10^{-4} & -1.602\times10^{-4} & -7.643\times10^{-5} & -6.856\times10^{-5} \\ -2.154\times10^{-4} & -5.064\times10^{-4} & -5.756\times10^{-3} & 0.015 & -5.754\times10^{-3} & -5.011\times10^{-4} & -1.629\times10^{-4} & -1.085\times10^{-4} \\ -1.085\times10^{-4} & -1.629\times10^{-4} & -5.011\times10^{-4} & -5.754\times10^{-3} & 0.015 & -5.756\times10^{-3} & -5.064\times10^{-4} & -2.154\times10^{-4} \\ -6.856\times10^{-5} & -7.643\times10^{-5} & -1.602\times10^{-4} & -5.011\times10^{-4} & -5.756\times10^{-3} & 0.015 & -5.767\times10^{-3} & -6.123\times10^{-4} \\ -5.111\times10^{-5} & -4.724\times10^{-5} & -7.643\times10^{-5} & -1.629\times10^{-4} & -5.064\times10^{-4} & -5.767\times10^{-3} & 0.015 & -6.073\times10^{-3} \\ -6.325\times10^{-5} & -5.111\times10^{-5} & -6.856\times10^{-5} & -1.085\times10^{-4} & -2.154\times10^{-4} & -6.123\times10^{-4} & -6.073\times10^{-3} & 0.012 \end{pmatrix} \quad S$$

$$R_{i,j} := \frac{-1}{\left(Zc^{-1}\right)_{i,j}} \qquad R_{i,i} := \frac{1}{\sum_{j=1}^{n} \left(Zc^{-1}\right)_{i,j}}$$

Values of the resistances of a network of resistors having an impedance matrix equal to Zc, presented as a matrix (the diagonal entries are the values of the grounded resistors).

$$R = \begin{pmatrix} 205.831 & 164.674 & 1.633\times 10^3 & 4.643\times 10^3 & 9.215\times 10^3 & 1.459\times 10^4 & 1.956\times 10^4 & 1.581\times 10^4 \\ 164.674 & 373.546 & 173.403 & 1.975\times 10^3 & 6.140\times 10^3 & 1.308\times 10^4 & 2.117\times 10^4 & 1.956\times 10^4 \\ 1.633\times 10^3 & 173.403 & 407.954 & 173.740 & 1.996\times 10^3 & 6.243\times 10^3 & 1.308\times 10^4 & 1.459\times 10^4 \\ 4.643\times 10^3 & 1.975\times 10^3 & 173.740 & 418.071 & 173.784 & 1.996\times 10^3 & 6.140\times 10^3 & 9.215\times 10^3 \\ 9.215\times 10^3 & 6.140\times 10^3 & 1.996\times 10^3 & 173.784 & 418.071 & 173.740 & 1.975\times 10^3 & 4.643\times 10^3 \\ 1.459\times 10^4 & 1.308\times 10^4 & 6.243\times 10^3 & 1.996\times 10^3 & 173.740 & 407.954 & 173.403 & 1.633\times 10^3 \\ 1.956\times 10^4 & 2.117\times 10^4 & 1.308\times 10^4 & 6.140\times 10^3 & 1.975\times 10^3 & 173.403 & 373.546 & 164.674 \\ 1.581\times 10^4 & 1.956\times 10^4 & 1.459\times 10^4 & 9.215\times 10^3 & 4.643\times 10^3 & 1.633\times 10^3 & 164.674 & 205.831 \end{pmatrix} \Omega$$

5) ORTHOGONALITY OF ASSOCIATED EIGENVECTORS

$$S\cdot S^T = \begin{pmatrix} 9.962 & 7.766 & 5.775 & 4.303 & 3.254 & 2.508 & 1.977 & 1.604 \\ 7.766 & 9.430 & 7.388 & 5.547 & 4.166 & 3.173 & 2.467 & 1.977 \\ 5.775 & 7.388 & 9.220 & 7.277 & 5.487 & 4.138 & 3.173 & 2.508 \\ 4.303 & 5.547 & 7.277 & 9.165 & 7.253 & 5.487 & 4.166 & 3.254 \\ 3.254 & 4.166 & 5.487 & 7.253 & 9.165 & 7.277 & 5.547 & 4.303 \\ 2.508 & 3.173 & 4.138 & 5.487 & 7.277 & 9.220 & 7.388 & 5.775 \\ 1.977 & 2.467 & 3.173 & 4.166 & 5.547 & 7.388 & 9.430 & 7.766 \\ 1.604 & 1.977 & 2.508 & 3.254 & 4.303 & 5.775 & 7.766 & 9.962 \end{pmatrix}$$

$$T\cdot T^T = \begin{pmatrix} 1.1874 & 0.0015 & 0.0063 & 0.0150 & 0.0187 & 0.0185 & 0.0175 & 0.0254 \\ 0.0015 & 0.9368 & -0.0423 & -0.0101 & 0.0045 & 0.0100 & 0.0114 & 0.0175 \\ 0.0063 & -0.0423 & 0.9377 & -0.0409 & -0.0101 & 0.0041 & 0.0100 & 0.0185 \\ 0.0150 & -0.0101 & -0.0409 & 0.9382 & -0.0410 & -0.0101 & 0.0045 & 0.0187 \\ 0.0187 & 0.0045 & -0.0101 & -0.0410 & 0.9382 & -0.0409 & -0.0101 & 0.0150 \\ 0.0185 & 0.0100 & 0.0041 & -0.0101 & -0.0409 & 0.9377 & -0.0423 & 0.0063 \\ 0.0175 & 0.0114 & 0.0100 & 0.0045 & -0.0101 & -0.0423 & 0.9368 & 0.0015 \\ 0.0254 & 0.0175 & 0.0185 & 0.0187 & 0.0150 & 0.0063 & 0.0015 & 1.1874 \end{pmatrix}$$

Thus, the column-vectors of S (eigen-voltages) are not orthogonal, and the column-vectors of T (eigen-currents) are not orthogonal.

6) MODAL INDUCTANCE MATRIX AND MODAL CAPACITANCE MATRIX, FOR ASSOCIATED EIGENVECTORS

$$Lm := S^{-1}\cdot L0\cdot T \qquad\qquad Cm := T^{-1}\cdot C\cdot S$$

$$10^{9}Lm = \begin{pmatrix} 253.580 & 4.144\times10^{-14} & -1.425\times10^{-14} & -1.868\times10^{-14} & 1.865\times10^{-15} & -9.632\times10^{-15} & 2.242\times10^{-14} & 2.630\times10^{-15} \\ 7.461\times10^{-14} & 206.355 & -2.740\times10^{-15} & -1.724\times10^{-14} & 6.420\times10^{-15} & 1.527\times10^{-14} & 1.767\times10^{-15} & 3.221\times10^{-15} \\ -5.558\times10^{-14} & 2.882\times10^{-15} & 185.419 & -2.416\times10^{-14} & -5.492\times10^{-15} & -4.199\times10^{-15} & 7.819\times10^{-15} & -1.985\times10^{-14} \\ -9.002\times10^{-14} & 7.063\times10^{-15} & -3.810\times10^{-15} & 175.161 & 2.113\times10^{-14} & 5.256\times10^{-15} & 1.709\times10^{-15} & 4.110\times10^{-15} \\ 2.012\times10^{-14} & -5.549\times10^{-15} & 1.588\times10^{-14} & 1.786\times10^{-14} & 161.212 & 3.096\times10^{-14} & 2.227\times10^{-14} & 4.156\times10^{-15} \\ 5.023\times10^{-14} & -7.112\times10^{-15} & -7.566\times10^{-15} & 1.637\times10^{-15} & 9.008\times10^{-15} & 165.211 & 3.187\times10^{-14} & 1.583\times10^{-14} \\ 6.744\times10^{-14} & 1.087\times10^{-14} & 2.054\times10^{-14} & -1.762\times10^{-14} & -1.087\times10^{-15} & 1.020\times10^{-14} & 169.157 & 1.754\times10^{-14} \\ -5.079\times10^{-14} & 3.738\times10^{-14} & -2.568\times10^{-14} & -1.411\times10^{-14} & 1.052\times10^{-14} & 2.797\times10^{-14} & 3.299\times10^{-14} & 162.660 \end{pmatrix}$$

nH/m

$$10^{12}\cdot Cm = \begin{pmatrix} 100 & -2.031\times10^{-14} & 1.002\times10^{-14} & 2.112\times10^{-15} & -3.549\times10^{-15} & 8.423\times10^{-15} & 1.208\times10^{-15} & 1.04\times10^{-14} \\ -2.667\times10^{-14} & 100 & 1.805\times10^{-15} & 9.49\times10^{-16} & -4.554\times10^{-16} & 3.399\times10^{-15} & -4.195\times10^{-15} & 1.54\times10^{-16} \\ 3.556\times10^{-15} & -1.184\times10^{-14} & 100 & 5.186\times10^{-15} & 3.411\times10^{-15} & 7.134\times10^{-15} & -5.636\times10^{-15} & 1.153\times10^{-14} \\ 3.775\times10^{-14} & 1.247\times10^{-14} & 1.079\times10^{-14} & 100 & -4.045\times10^{-15} & -2.608\times10^{-15} & 9.185\times10^{-15} & -9.26\times10^{-15} \\ -2.154\times10^{-14} & -2.146\times10^{-15} & -3.452\times10^{-15} & 1.085\times10^{-16} & 100 & -7.198\times10^{-15} & 3.846\times10^{-15} & -1.016\times10^{-14} \\ -1.251\times10^{-14} & 4.651\times10^{-15} & 4.396\times10^{-15} & -1.749\times10^{-14} & -2.807\times10^{-15} & 100 & -5.786\times10^{-15} & -1.322\times10^{-14} \\ -2.574\times10^{-14} & -1.388\times10^{-14} & -1.531\times10^{-14} & 1.494\times10^{-14} & 2.89\times10^{-15} & -4.734\times10^{-15} & 100 & 5.916\times10^{-15} \\ 1.822\times10^{-14} & 2.758\times10^{-15} & 2.206\times10^{-14} & -1.423\times10^{-14} & -1.159\times10^{-14} & -1.328\times10^{-14} & -9.23\times10^{-16} & 100 \end{pmatrix}$$

pF/m

As expected, Lm and Cm are diagonal matrices, within the accuracy of our computation. Here Cm is

$$10^{12}ck\cdot identity(n) = \begin{pmatrix} 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix}$$

pF/m

7) MODAL CHARACTERISTIC IMPEDANCE MATRIX, FOR BIORTHONORMAL EIGENVECTORS

$$Zmc := \mathrm{diag}(ci)\cdot T^{T}\cdot L0\cdot T$$

$$Zmc = \begin{pmatrix}
324.707 & 1.600\times 10^{-14} & 2.132\times 10^{-14} & 4.545\times 10^{-16} & -1.662\times 10^{-14} & -1.006\times 10^{-16} & 4.386\times 10^{-14} & 4.533\times 10^{-15} \\
2.935\times 10^{-14} & 181.553 & -1.962\times 10^{-14} & -4.343\times 10^{-14} & 1.242\times 10^{-14} & 3.046\times 10^{-14} & 5.426\times 10^{-15} & -1.509\times 10^{-15} \\
1.950\times 10^{-14} & -2.335\times 10^{-14} & 114.207 & -3.102\times 10^{-14} & -8.240\times 10^{-18} & -2.320\times 10^{-15} & 1.004\times 10^{-14} & -4.305\times 10^{-14} \\
-2.021\times 10^{-14} & -4.805\times 10^{-14} & -2.759\times 10^{-14} & 78.788 & 1.016\times 10^{-14} & 2.057\times 10^{-15} & 3.634\times 10^{-16} & 5.609\times 10^{-15} \\
-2.125\times 10^{-14} & 1.213\times 10^{-14} & 1.395\times 10^{-15} & 9.188\times 10^{-15} & 39.267 & 2.394\times 10^{-14} & 5.023\times 10^{-15} & -2.359\times 10^{-14} \\
-1.211\times 10^{-15} & 2.854\times 10^{-14} & -2.963\times 10^{-15} & 1.894\times 10^{-15} & 2.408\times 10^{-14} & 48.644 & 5.093\times 10^{-15} & 2.283\times 10^{-14} \\
5.125\times 10^{-14} & 9.272\times 10^{-16} & 8.773\times 10^{-15} & 2.068\times 10^{-15} & 2.918\times 10^{-15} & 3.946\times 10^{-15} & 59.546 & 1.262\times 10^{-14} \\
3.545\times 10^{-15} & -1.064\times 10^{-15} & -4.801\times 10^{-14} & 2.408\times 10^{-15} & -2.332\times 10^{-14} & 1.946\times 10^{-14} & 1.275\times 10^{-14} & 42.472
\end{pmatrix} \quad \Omega$$

In this example, Zmc is a diagonal matrix, within the accuracy of our computation.
We note that Zmc is different in section 3 and in section 7.

8) MODAL INDUCTANCE MATRIX AND MODAL CAPACITANCE MATRIX, FOR BIORTHONORMAL EIGENVECTORS

$$Lm := T^{T}\cdot L0\cdot T \qquad\qquad Cm := T^{-1}\cdot C\cdot\left(T^{-1}\right)^{T}$$

$$10^{9}Lm = \begin{pmatrix}
1.635\times 10^{3} & 1.381\times 10^{-13} & 1.073\times 10^{-13} & -3.588\times 10^{-14} & -1.217\times 10^{-13} & -4.291\times 10^{-15} & 1.756\times 10^{-13} & 1.886\times 10^{-14} \\
7.491\times 10^{-14} & 824.728 & -9.175\times 10^{-14} & -2.120\times 10^{-13} & 5.101\times 10^{-14} & 1.589\times 10^{-13} & -1.839\times 10^{-14} & 2.617\times 10^{-15} \\
1.044\times 10^{-13} & -7.941\times 10^{-14} & 491.780 & -1.048\times 10^{-13} & -1.650\times 10^{-14} & -7.290\times 10^{-15} & 5.081\times 10^{-14} & -2.023\times 10^{-13} \\
-3.553\times 10^{-14} & -1.998\times 10^{-13} & -1.172\times 10^{-13} & 329.746 & 4.044\times 10^{-14} & 5.477\times 10^{-15} & 1.965\times 10^{-15} & 1.640\times 10^{-14} \\
-9.223\times 10^{-14} & 5.066\times 10^{-14} & 7.593\times 10^{-17} & 3.546\times 10^{-14} & 157.663 & 9.741\times 10^{-14} & 1.459\times 10^{-14} & -8.741\times 10^{-14} \\
-8.327\times 10^{-15} & 1.405\times 10^{-13} & -1.665\times 10^{-14} & 1.018\times 10^{-14} & 9.787\times 10^{-14} & 197.718 & 2.192\times 10^{-14} & 8.884\times 10^{-14} \\
2.212\times 10^{-13} & 2.003\times 10^{-16} & 3.467\times 10^{-14} & 3.315\times 10^{-15} & 2.408\times 10^{-14} & 1.398\times 10^{-14} & 244.906 & 5.111\times 10^{-14} \\
1.510\times 10^{-14} & -6.921\times 10^{-15} & -1.934\times 10^{-13} & 1.474\times 10^{-14} & -9.384\times 10^{-14} & 8.847\times 10^{-14} & 4.211\times 10^{-14} & 171.293
\end{pmatrix} \quad \text{nH/m}$$

$$10^{12}\cdot Cm = \begin{pmatrix} 15.508 & -3.295\times10^{-15} & -4.422\times10^{-16} & -3.410\times10^{-15} & 3.831\times10^{-17} & 7.541\times10^{-15} & -2.624\times10^{-15} & 3.093\times10^{-15} \\ -3.701\times10^{-15} & 25.021 & 2.773\times10^{-15} & 1.177\times10^{-14} & -8.526\times10^{-15} & -9.900\times10^{-15} & -4.224\times10^{-16} & -1.647\times10^{-16} \\ -6.409\times10^{-16} & 1.892\times10^{-15} & 37.704 & 1.144\times10^{-14} & 1.904\times10^{-15} & 6.926\times10^{-15} & -8.489\times10^{-15} & 4.172\times10^{-14} \\ -2.699\times10^{-15} & 1.063\times10^{-14} & 1.169\times10^{-14} & 53.120 & -4.637\times10^{-15} & -3.911\times10^{-15} & 6.848\times10^{-15} & -1.407\times10^{-14} \\ -1.347\times10^{-15} & -9.571\times10^{-15} & 1.539\times10^{-15} & -5.944\times10^{-15} & 102.251 & -5.048\times10^{-14} & -3.976\times10^{-15} & 4.703\times10^{-14} \\ 8.307\times10^{-15} & -9.552\times10^{-15} & 8.964\times10^{-15} & -4.823\times10^{-15} & -4.906\times10^{-14} & 83.559 & -5.401\times10^{-15} & -4.648\times10^{-14} \\ -2.580\times10^{-15} & -1.466\times10^{-15} & -8.872\times10^{-15} & 7.046\times10^{-15} & -4.360\times10^{-16} & -4.062\times10^{-15} & 69.070 & -1.454\times10^{-14} \\ 8.539\times10^{-16} & 3.708\times10^{-16} & 4.264\times10^{-14} & -1.471\times10^{-14} & 4.269\times10^{-14} & -4.578\times10^{-14} & -1.673\times10^{-14} & 94.960 \end{pmatrix}$$

pF/m

In this example, Lm and Cm are diagonal matrices, within the accuracy of our computation.
We note that Lm and Cm are different in section 6 and in section 8.

9) GENERALIZED ASSOCIATED EIGENVECTORS GIVEN BY EQUATION (50)

$zk := 50$ Ω

new matrix S

$$S := \frac{1}{zk}\cdot Zc\cdot T$$

$$S = \begin{pmatrix} 2.069 & -1.722 & -0.995 & -0.588 & 0.067 & 0.236 & -0.366 & -0.141 \\ 2.291 & -1.566 & -0.399 & 0.207 & -0.221 & -0.496 & 0.46 & 0.402 \\ 2.432 & -1.079 & 0.497 & 0.817 & 0.325 & 0.093 & 0.438 & -0.39 \\ 2.502 & -0.384 & 1.132 & 0.455 & -0.386 & 0.407 & -0.424 & 0.167 \\ 2.502 & 0.384 & 1.132 & -0.455 & 0.386 & -0.407 & -0.424 & 0.167 \\ 2.432 & 1.079 & 0.497 & -0.817 & -0.325 & -0.093 & 0.438 & -0.39 \\ 2.291 & 1.566 & -0.399 & -0.207 & 0.221 & 0.496 & 0.46 & 0.402 \\ 2.069 & 1.722 & -0.995 & 0.588 & -0.067 & -0.236 & -0.366 & -0.141 \end{pmatrix}$$

Back to the link "Annex A" of § 7

Annex B: Computation of the eigenvectors and the characteristic impedance matrix

Authors: Frédéric Broydé and Evelyne Clavelier.
Prepared with Mathcad 2000 professional (Mathcad is a registered trademark of its owner).

date: 6 Dec. 2014

File: Worksheet B of Tutorial v3a.mcd

1) DEFINITION OF THE MULTICONDUCTOR TRANSMISSION LINE PARAMETERS

$ORIGIN := 1 \qquad n := 8$

$\varepsilon 0 := 8.854188 \cdot 10^{-12}$ F/m $\qquad \mu 0 := 4 \cdot \pi \cdot 10^{-7}$ H/m

$$L0 := \begin{pmatrix} 446.404 & 220.552 & 120.338 & 67.269 & 37.881 & 21.415 & 12.19 & 7.099 \\ 220.552 & 432.448 & 213.654 & 116.608 & 65.225 & 36.801 & 20.936 & 12.19 \\ 120.338 & 213.654 & 429.05 & 211.844 & 115.666 & 64.819 & 36.801 & 21.415 \\ 67.269 & 116.608 & 211.844 & 428.134 & 211.454 & 115.666 & 65.225 & 37.881 \\ 37.881 & 65.225 & 115.666 & 211.454 & 428.134 & 211.844 & 116.608 & 67.269 \\ 21.415 & 36.801 & 64.819 & 115.666 & 211.844 & 429.05 & 213.654 & 120.338 \\ 12.19 & 20.936 & 36.801 & 65.225 & 116.608 & 213.654 & 432.448 & 220.552 \\ 7.099 & 12.19 & 21.415 & 37.881 & 67.269 & 120.338 & 220.552 & 446.404 \end{pmatrix} \cdot 10^{-9} \quad \text{H/m}$$

$$\mathrm{norme}\left(L0 - L0^{T}\right) = 0$$

$\varepsilon r := 2.8 \qquad C := \varepsilon r \cdot \varepsilon 0 \cdot \mu 0 \cdot L0^{-1}$

$$10^{12} C = \begin{pmatrix} 93.415 & -45.995 & -2.952 & -0.61 & -0.141 & -0.033 & -7.907\times 10^{-3} & -2.178\times 10^{-3} \\ -45.995 & 118.313 & -44.538 & -2.653 & -0.541 & -0.124 & -0.03 & -7.907\times 10^{-3} \\ -2.952 & -44.538 & 118.408 & -44.519 & -2.649 & -0.54 & -0.124 & -0.033 \\ -0.61 & -2.653 & -44.519 & 118.412 & -44.517 & -2.649 & -0.541 & -0.141 \\ -0.141 & -0.541 & -2.649 & -44.517 & 118.412 & -44.519 & -2.653 & -0.61 \\ -0.033 & -0.124 & -0.54 & -2.649 & -44.519 & 118.408 & -44.538 & -2.952 \\ -7.907\times 10^{-3} & -0.03 & -0.124 & -0.541 & -2.653 & -44.538 & 118.313 & -45.995 \\ -2.178\times 10^{-3} & -7.907\times 10^{-3} & -0.033 & -0.141 & -0.61 & -2.952 & -45.995 & 93.415 \end{pmatrix} \quad \text{pF/m}$$

2) DETERMINATION OF ASSOCIATED EIGENVECTORS

$i := 1..n \qquad j := 1..n$

In this worksheet, we use the *a priori* knowledge concerning completely degenerate interconnections.

$D1_i := \varepsilon r \cdot \varepsilon 0 \cdot \mu 0$

Propagation velocity of the eigenmodes

$$ci_i := \frac{1}{\sqrt{D1_i}}$$

$$ci = \begin{pmatrix} 1.792\times 10^8 \\ 1.792\times 10^8 \\ 1.792\times 10^8 \\ 1.792\times 10^8 \\ 1.792\times 10^8 \\ 1.792\times 10^8 \\ 1.792\times 10^8 \\ 1.792\times 10^8 \end{pmatrix}$$ m/s

$Munsurci_{i,j} := 0 \qquad Munsurci_{i,i} := \frac{1}{ci_i}$

Definition of the change of variables for the currents, i.e. transition matrix from modal currents to natural currents (matrix T)

Here, we can use $T := identity(n)$

$$T = \begin{pmatrix} 1.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 1.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 1.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 \end{pmatrix}$$

Definition of the change of variables for the voltages, i.e. transition matrix from modal voltages to natural voltages (matrix S)

$ck := 10^{-10}$ arbitrary constant

$S := ck \cdot C^{-1} \cdot T$

$$S = \begin{pmatrix} 1.432885 & 0.707937 & 0.386266 & 0.215923 & 0.121592 & 0.068739 & 0.039128 & 0.022787 \\ 0.707937 & 1.388089 & 0.685795 & 0.374293 & 0.209362 & 0.118125 & 0.067201 & 0.039128 \\ 0.386266 & 0.685795 & 1.377182 & 0.679985 & 0.371269 & 0.208059 & 0.118125 & 0.068739 \\ 0.215923 & 0.374293 & 0.679985 & 1.374242 & 0.678733 & 0.371269 & 0.209362 & 0.121592 \\ 0.121592 & 0.209362 & 0.371269 & 0.678733 & 1.374242 & 0.679985 & 0.374293 & 0.215923 \\ 0.068739 & 0.118125 & 0.208059 & 0.371269 & 0.679985 & 1.377182 & 0.685795 & 0.386266 \\ 0.039128 & 0.067201 & 0.118125 & 0.209362 & 0.374293 & 0.685795 & 1.388089 & 0.707937 \\ 0.022787 & 0.039128 & 0.068739 & 0.121592 & 0.215923 & 0.386266 & 0.707937 & 1.432885 \end{pmatrix}$$

3) CHARACTERISTIC IMPEDANCE MATRIX

modal characteristic impedances
(for the associated eigenvectors)

$zC_i := \dfrac{1}{ck \cdot ci_i}$

$$zC = \begin{pmatrix} 55.816 \\ 55.816 \\ 55.816 \\ 55.816 \\ 55.816 \\ 55.816 \\ 55.816 \\ 55.816 \end{pmatrix} \quad \Omega$$

modal characteristic impedance matrix $Zmc := \mathrm{diag}(zC)$

characteristic impedance matrix $Zc := S \cdot Zmc \cdot T^{-1}$

$$Zc = \begin{pmatrix} 79.978 & 39.514 & 21.560 & 12.052 & 6.787 & 3.837 & 2.184 & 1.272 \\ 39.514 & 77.477 & 38.278 & 20.892 & 11.686 & 6.593 & 3.751 & 2.184 \\ 21.560 & 38.278 & 76.869 & 37.954 & 20.723 & 11.613 & 6.593 & 3.837 \\ 12.052 & 20.892 & 37.954 & 76.705 & 37.884 & 20.723 & 11.686 & 6.787 \\ 6.787 & 11.686 & 20.723 & 37.884 & 76.705 & 37.954 & 20.892 & 12.052 \\ 3.837 & 6.593 & 11.613 & 20.723 & 37.954 & 76.869 & 38.278 & 21.560 \\ 2.184 & 3.751 & 6.593 & 11.686 & 20.892 & 38.278 & 77.477 & 39.514 \\ 1.272 & 2.184 & 3.837 & 6.787 & 12.052 & 21.560 & 39.514 & 79.978 \end{pmatrix} \quad \Omega$$

control of the equations for computing the characteristic impedance matrix:

$$S\cdot Munsurci^{-1}\cdot S^{-1}\cdot L0 = \begin{pmatrix} 79.978 & 39.514 & 21.560 & 12.052 & 6.787 & 3.837 & 2.184 & 1.272 \\ 39.514 & 77.477 & 38.278 & 20.892 & 11.686 & 6.593 & 3.751 & 2.184 \\ 21.560 & 38.278 & 76.869 & 37.954 & 20.723 & 11.613 & 6.593 & 3.837 \\ 12.052 & 20.892 & 37.954 & 76.705 & 37.884 & 20.723 & 11.686 & 6.787 \\ 6.787 & 11.686 & 20.723 & 37.884 & 76.705 & 37.954 & 20.892 & 12.052 \\ 3.837 & 6.593 & 11.613 & 20.723 & 37.954 & 76.869 & 38.278 & 21.560 \\ 2.184 & 3.751 & 6.593 & 11.686 & 20.892 & 38.278 & 77.477 & 39.514 \\ 1.272 & 2.184 & 3.837 & 6.787 & 12.052 & 21.560 & 39.514 & 79.978 \end{pmatrix}$$

$$S\cdot Munsurci\cdot S^{-1}\cdot C^{-1} = \begin{pmatrix} 79.978 & 39.514 & 21.560 & 12.052 & 6.787 & 3.837 & 2.184 & 1.272 \\ 39.514 & 77.477 & 38.278 & 20.892 & 11.686 & 6.593 & 3.751 & 2.184 \\ 21.560 & 38.278 & 76.869 & 37.954 & 20.723 & 11.613 & 6.593 & 3.837 \\ 12.052 & 20.892 & 37.954 & 76.705 & 37.884 & 20.723 & 11.686 & 6.787 \\ 6.787 & 11.686 & 20.723 & 37.884 & 76.705 & 37.954 & 20.892 & 12.052 \\ 3.837 & 6.593 & 11.613 & 20.723 & 37.954 & 76.869 & 38.278 & 21.560 \\ 2.184 & 3.751 & 6.593 & 11.686 & 20.892 & 38.278 & 77.477 & 39.514 \\ 1.272 & 2.184 & 3.837 & 6.787 & 12.052 & 21.560 & 39.514 & 79.978 \end{pmatrix}$$

$$C^{-1}\cdot T\cdot Munsurci\cdot T^{-1} = \begin{pmatrix} 79.978 & 39.514 & 21.560 & 12.052 & 6.787 & 3.837 & 2.184 & 1.272 \\ 39.514 & 77.477 & 38.278 & 20.892 & 11.686 & 6.593 & 3.751 & 2.184 \\ 21.560 & 38.278 & 76.869 & 37.954 & 20.723 & 11.613 & 6.593 & 3.837 \\ 12.052 & 20.892 & 37.954 & 76.705 & 37.884 & 20.723 & 11.686 & 6.787 \\ 6.787 & 11.686 & 20.723 & 37.884 & 76.705 & 37.954 & 20.892 & 12.052 \\ 3.837 & 6.593 & 11.613 & 20.723 & 37.954 & 76.869 & 38.278 & 21.560 \\ 2.184 & 3.751 & 6.593 & 11.686 & 20.892 & 38.278 & 77.477 & 39.514 \\ 1.272 & 2.184 & 3.837 & 6.787 & 12.052 & 21.560 & 39.514 & 79.978 \end{pmatrix}$$

$$L0\cdot T\cdot Munsurci^{-1}\cdot T^{-1} = \begin{pmatrix} 79.978 & 39.514 & 21.560 & 12.052 & 6.787 & 3.837 & 2.184 & 1.272 \\ 39.514 & 77.477 & 38.278 & 20.892 & 11.686 & 6.593 & 3.751 & 2.184 \\ 21.560 & 38.278 & 76.869 & 37.954 & 20.723 & 11.613 & 6.593 & 3.837 \\ 12.052 & 20.892 & 37.954 & 76.705 & 37.884 & 20.723 & 11.686 & 6.787 \\ 6.787 & 11.686 & 20.723 & 37.884 & 76.705 & 37.954 & 20.892 & 12.052 \\ 3.837 & 6.593 & 11.613 & 20.723 & 37.954 & 76.869 & 38.278 & 21.560 \\ 2.184 & 3.751 & 6.593 & 11.686 & 20.892 & 38.278 & 77.477 & 39.514 \\ 1.272 & 2.184 & 3.837 & 6.787 & 12.052 & 21.560 & 39.514 & 79.978 \end{pmatrix}$$

4) MATCHED LOADS

$$Zc^{-1} = \begin{pmatrix} 0.017 & -8.241\times 10^{-3} & -5.288\times 10^{-4} & -1.093\times 10^{-4} & -2.518\times 10^{-5} & -5.943\times 10^{-6} & -1.417\times 10^{-6} & -3.902\times 10^{-7} \\ -8.241\times 10^{-3} & 0.021 & -7.979\times 10^{-3} & -4.754\times 10^{-4} & -9.696\times 10^{-5} & -2.225\times 10^{-5} & -5.309\times 10^{-6} & -1.417\times 10^{-6} \\ -5.288\times 10^{-4} & -7.979\times 10^{-3} & 0.021 & -7.976\times 10^{-3} & -4.746\times 10^{-4} & -9.680\times 10^{-5} & -2.225\times 10^{-5} & -5.943\times 10^{-6} \\ -1.093\times 10^{-4} & -4.754\times 10^{-4} & -7.976\times 10^{-3} & 0.021 & -7.976\times 10^{-3} & -4.746\times 10^{-4} & -9.696\times 10^{-5} & -2.518\times 10^{-5} \\ -2.518\times 10^{-5} & -9.696\times 10^{-5} & -4.746\times 10^{-4} & -7.976\times 10^{-3} & 0.021 & -7.976\times 10^{-3} & -4.754\times 10^{-4} & -1.093\times 10^{-4} \\ -5.943\times 10^{-6} & -2.225\times 10^{-5} & -9.680\times 10^{-5} & -4.746\times 10^{-4} & -7.976\times 10^{-3} & 0.021 & -7.979\times 10^{-3} & -5.288\times 10^{-4} \\ -1.417\times 10^{-6} & -5.309\times 10^{-6} & -2.225\times 10^{-5} & -9.696\times 10^{-5} & -4.754\times 10^{-4} & -7.979\times 10^{-3} & 0.021 & -8.241\times 10^{-3} \\ -3.902\times 10^{-7} & -1.417\times 10^{-6} & -5.943\times 10^{-6} & -2.518\times 10^{-5} & -1.093\times 10^{-4} & -5.288\times 10^{-4} & -8.241\times 10^{-3} & 0.017 \end{pmatrix}$$

S

$$R_{i,j} := \frac{-1}{\left(Zc^{-1}\right)_{i,j}} \qquad R_{i,i} := \frac{1}{\sum_{j=1}^{n}\left(Zc^{-1}\right)_{i,j}}$$

Values of the resistances of a network of resistors having an impedance matrix equal to Zc, presented as a matrix (the diagonal entries are the values of the grounded resistors).

$$R = \begin{pmatrix} 127.799 & 121.351 & 1.891\times 10^{3} & 9.152\times 10^{3} & 3.972\times 10^{4} & 1.683\times 10^{5} & 7.059\times 10^{5} & 2.563\times 10^{6} \\ 121.351 & 228.535 & 125.322 & 2.104\times 10^{3} & 1.031\times 10^{4} & 4.493\times 10^{4} & 1.884\times 10^{5} & 7.059\times 10^{5} \\ 1.891\times 10^{3} & 125.322 & 242.123 & 125.377 & 2.107\times 10^{3} & 1.033\times 10^{4} & 4.493\times 10^{4} & 1.683\times 10^{5} \\ 9.152\times 10^{3} & 2.104\times 10^{3} & 125.377 & 245.005 & 125.380 & 2.107\times 10^{3} & 1.031\times 10^{4} & 3.972\times 10^{4} \\ 3.972\times 10^{4} & 1.031\times 10^{4} & 2.107\times 10^{3} & 125.380 & 245.005 & 125.377 & 2.104\times 10^{3} & 9.152\times 10^{3} \\ 1.683\times 10^{5} & 4.493\times 10^{4} & 1.033\times 10^{4} & 2.107\times 10^{3} & 125.377 & 242.123 & 125.322 & 1.891\times 10^{3} \\ 7.059\times 10^{5} & 1.884\times 10^{5} & 4.493\times 10^{4} & 1.031\times 10^{4} & 2.104\times 10^{3} & 125.322 & 228.535 & 121.351 \\ 2.563\times 10^{6} & 7.059\times 10^{5} & 1.683\times 10^{5} & 3.972\times 10^{4} & 9.152\times 10^{3} & 1.891\times 10^{3} & 121.351 & 127.799 \end{pmatrix} \Omega$$

5) ORTHOGONALITY OF ASSOCIATED EIGENVECTORS

$$S \cdot S^{T} = \begin{pmatrix} 2.772 & 2.380 & 1.783 & 1.253 & 0.846 & 0.556 & 0.358 & 0.226 \\ 2.380 & 3.102 & 2.537 & 1.858 & 1.287 & 0.861 & 0.561 & 0.358 \\ 1.783 & 2.537 & 3.178 & 2.573 & 1.874 & 1.293 & 0.861 & 0.556 \\ 1.253 & 1.858 & 2.573 & 3.195 & 2.580 & 1.874 & 1.287 & 0.846 \\ 0.846 & 1.287 & 1.874 & 2.580 & 3.195 & 2.573 & 1.858 & 1.253 \\ 0.556 & 0.861 & 1.293 & 1.874 & 2.573 & 3.178 & 2.537 & 1.783 \\ 0.358 & 0.561 & 0.861 & 1.287 & 1.858 & 2.537 & 3.102 & 2.380 \\ 0.226 & 0.358 & 0.556 & 0.846 & 1.253 & 1.783 & 2.380 & 2.772 \end{pmatrix}$$

$$T \cdot T^{T} = \begin{pmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Thus, the column-vectors of S (eigen-voltages) are not orthogonal, but the column-vectors of T (eigen-currents) are orthogonal.

6) MODAL INDUCTANCE MATRIX AND MODAL CAPACITANCE MATRIX, FOR ASSOCIATED EIGENVECTORS

$$Lm := S^{-1} \cdot L0 \cdot T \qquad\qquad Cm := T^{-1} \cdot C \cdot S$$

$$10^{9} Lm = \begin{pmatrix} 311.542 & -3.139\times 10^{-14} & -1.777\times 10^{-14} & -1.614\times 10^{-14} & -2.104\times 10^{-15} & -7.114\times 10^{-15} & -2.852\times 10^{-15} & -2.331\times 10^{-15} \\ -1.478\times 10^{-14} & 311.542 & -4.832\times 10^{-14} & -3.299\times 10^{-15} & -7.094\times 10^{-15} & -6.145\times 10^{-15} & -1.917\times 10^{-15} & 1.262\times 10^{-15} \\ -2.636\times 10^{-14} & -3.212\times 10^{-14} & 311.542 & -6.829\times 10^{-15} & -1.030\times 10^{-14} & 9.968\times 10^{-17} & 1.243\times 10^{-15} & -1.027\times 10^{-15} \\ -1.031\times 10^{-14} & -7.195\times 10^{-15} & -2.901\times 10^{-14} & 311.542 & -2.227\times 10^{-14} & -1.029\times 10^{-14} & -8.843\times 10^{-15} & -1.596\times 10^{-14} \\ 5.078\times 10^{-15} & -1.065\times 10^{-14} & 7.759\times 10^{-15} & 1.607\times 10^{-14} & 311.542 & -6.588\times 10^{-15} & 3.888\times 10^{-15} & 3.115\times 10^{-14} \\ 9.722\times 10^{-15} & 1.581\times 10^{-14} & 3.657\times 10^{-14} & 4.203\times 10^{-14} & 1.199\times 10^{-14} & 311.542 & 6.764\times 10^{-14} & 1.356\times 10^{-14} \\ -8.609\times 10^{-15} & -1.310\times 10^{-14} & -2.748\times 10^{-14} & -3.632\times 10^{-14} & -6.019\times 10^{-14} & -1.816\times 10^{-13} & 311.542 & -1.231\times 10^{-13} \\ 4.160\times 10^{-17} & 1.414\times 10^{-16} & 2.730\times 10^{-15} & -1.049\times 10^{-14} & 5.616\times 10^{-15} & -5.125\times 10^{-15} & 1.092\times 10^{-14} & 311.542 \end{pmatrix}$$

nH/m

$$10^{12} \cdot Cm = \begin{pmatrix} 100 & -2.265\times 10^{-15} & -2.577\times 10^{-15} & 1.473\times 10^{-15} & -8.685\times 10^{-16} & 9.493\times 10^{-16} & 1.246\times 10^{-16} & 2.548\times 10^{-16} \\ 2.435\times 10^{-15} & 100 & 2.201\times 10^{-15} & -9.185\times 10^{-16} & 2.908\times 10^{-15} & 5.915\times 10^{-16} & 9.604\times 10^{-17} & -5.367\times 10^{-16} \\ 5.506\times 10^{-15} & 1.519\times 10^{-14} & 100 & 1.53\times 10^{-14} & -1.117\times 10^{-15} & 1.549\times 10^{-15} & 7.077\times 10^{-16} & 1.087\times 10^{-15} \\ 8.469\times 10^{-16} & 7.196\times 10^{-16} & 6.63\times 10^{-15} & 100 & 1.493\times 10^{-14} & 3.615\times 10^{-16} & 1.232\times 10^{-15} & 1.5\times 10^{-15} \\ 1.685\times 10^{-15} & 2.124\times 10^{-15} & -4.243\times 10^{-15} & 1.797\times 10^{-14} & 100 & 1.54\times 10^{-15} & -1.909\times 10^{-16} & -1.45\times 10^{-15} \\ -1.256\times 10^{-15} & -6.702\times 10^{-16} & -3.756\times 10^{-15} & -6.098\times 10^{-15} & -2.805\times 10^{-15} & 100 & -4.289\times 10^{-15} & -1.52\times 10^{-15} \\ 1.465\times 10^{-15} & 2.315\times 10^{-15} & 4.614\times 10^{-15} & 7.937\times 10^{-15} & 1.305\times 10^{-14} & 3.112\times 10^{-14} & 100 & 1.799\times 10^{-14} \\ -1.061\times 10^{-16} & -9.506\times 10^{-17} & 6.035\times 10^{-17} & 7.534\times 10^{-16} & 1.251\times 10^{-15} & -3.253\times 10^{-15} & -4.146\times 10^{-15} & 100 \end{pmatrix}$$

pF/m

As expected, Lm and Cm are diagonal matrices, within the accuracy of our computation. Here Cm is

$$10^{12} ck \cdot \mathrm{identity}(n) = \begin{pmatrix} 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix}$$

pF/m

7) MODAL CHARACTERISTIC IMPEDANCE MATRIX, FOR BIORTHONORMAL EIGENVECTORS

$$Zmc := \mathrm{diag}(ci)\cdot T^{T}\cdot L0\cdot T$$

$$Zmc = \begin{pmatrix} 79.978 & 39.514 & 21.560 & 12.052 & 6.787 & 3.837 & 2.184 & 1.272 \\ 39.514 & 77.477 & 38.278 & 20.892 & 11.686 & 6.593 & 3.751 & 2.184 \\ 21.560 & 38.278 & 76.869 & 37.954 & 20.723 & 11.613 & 6.593 & 3.837 \\ 12.052 & 20.892 & 37.954 & 76.705 & 37.884 & 20.723 & 11.686 & 6.787 \\ 6.787 & 11.686 & 20.723 & 37.884 & 76.705 & 37.954 & 20.892 & 12.052 \\ 3.837 & 6.593 & 11.613 & 20.723 & 37.954 & 76.869 & 38.278 & 21.560 \\ 2.184 & 3.751 & 6.593 & 11.686 & 20.892 & 38.278 & 77.477 & 39.514 \\ 1.272 & 2.184 & 3.837 & 6.787 & 12.052 & 21.560 & 39.514 & 79.978 \end{pmatrix} \qquad \Omega$$

In this example, Zmc is not a diagonal matrix.
We note that Zmc is different in section 3 and in section 7.

8) MODAL INDUCTANCE MATRIX AND MODAL CAPACITANCE MATRIX, FOR BIORTHONORMAL EIGENVECTORS

$$Lm := T^{T}\cdot L0\cdot T \qquad Cm := T^{-1}\cdot C\cdot\left(T^{-1}\right)^{T}$$

$$10^{9}Lm = \begin{pmatrix} 446.404 & 220.552 & 120.338 & 67.269 & 37.881 & 21.415 & 12.190 & 7.099 \\ 220.552 & 432.448 & 213.654 & 116.608 & 65.225 & 36.801 & 20.936 & 12.190 \\ 120.338 & 213.654 & 429.050 & 211.844 & 115.666 & 64.819 & 36.801 & 21.415 \\ 67.269 & 116.608 & 211.844 & 428.134 & 211.454 & 115.666 & 65.225 & 37.881 \\ 37.881 & 65.225 & 115.666 & 211.454 & 428.134 & 211.844 & 116.608 & 67.269 \\ 21.415 & 36.801 & 64.819 & 115.666 & 211.844 & 429.050 & 213.654 & 120.338 \\ 12.190 & 20.936 & 36.801 & 65.225 & 116.608 & 213.654 & 432.448 & 220.552 \\ 7.099 & 12.190 & 21.415 & 37.881 & 67.269 & 120.338 & 220.552 & 446.404 \end{pmatrix} \qquad \text{nH/m}$$

$$10^{12}\cdot Cm = \begin{pmatrix} 93.415 & -45.995 & -2.952 & -0.610 & -0.141 & -0.033 & -7.907\times 10^{-3} & -2.178\times 10^{-3} \\ -45.995 & 118.313 & -44.538 & -2.653 & -0.541 & -0.124 & -0.030 & -7.907\times 10^{-3} \\ -2.952 & -44.538 & 118.408 & -44.519 & -2.649 & -0.540 & -0.124 & -0.033 \\ -0.610 & -2.653 & -44.519 & 118.412 & -44.517 & -2.649 & -0.541 & -0.141 \\ -0.141 & -0.541 & -2.649 & -44.517 & 118.412 & -44.519 & -2.653 & -0.610 \\ -0.033 & -0.124 & -0.540 & -2.649 & -44.519 & 118.408 & -44.538 & -2.952 \\ -7.907\times 10^{-3} & -0.030 & -0.124 & -0.541 & -2.653 & -44.538 & 118.313 & -45.995 \\ -2.178\times 10^{-3} & -7.907\times 10^{-3} & -0.033 & -0.141 & -0.610 & -2.952 & -45.995 & 93.415 \end{pmatrix}$$

pF/m

In this example, Lm and Cm are not diagonal matrices.
We note that Lm and Cm are different in section 6 and in section 8.

9) GENERALIZED ASSOCIATED EIGENVECTORS GIVEN BY EQUATION (50)

$zk := 50$ Ω

new matrix S

$$S := \frac{1}{zk}\cdot Zc\cdot T$$

$$S = \begin{pmatrix} 1.6 & 0.79 & 0.431 & 0.241 & 0.136 & 0.077 & 0.044 & 0.025 \\ 0.79 & 1.55 & 0.766 & 0.418 & 0.234 & 0.132 & 0.075 & 0.044 \\ 0.431 & 0.766 & 1.537 & 0.759 & 0.414 & 0.232 & 0.132 & 0.077 \\ 0.241 & 0.418 & 0.759 & 1.534 & 0.758 & 0.414 & 0.234 & 0.136 \\ 0.136 & 0.234 & 0.414 & 0.758 & 1.534 & 0.759 & 0.418 & 0.241 \\ 0.077 & 0.132 & 0.232 & 0.414 & 0.759 & 1.537 & 0.766 & 0.431 \\ 0.044 & 0.075 & 0.132 & 0.234 & 0.418 & 0.766 & 1.55 & 0.79 \\ 0.025 & 0.044 & 0.077 & 0.136 & 0.241 & 0.431 & 0.79 & 1.6 \end{pmatrix}$$

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Seminar 32

Tutorial on Echo and Crosstalk in Printed Circuit Boards and Multi-Chip Modules — Lecture Slides

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